MANUAL

EVALUATION OF AQUIFER PARAMETERS

GOVERNMENT OF INDIA

CENTRAL GROUND WATER BOARD

MINISTRY OF IRRIGATION

1992

Table of uW(u)

u	uW (u)	u	uW (u)
8	3·014(—4)	8(—6)	8·928(—5)
6	2·161(—3)	6(—6)	6·870(—5)
4	1·512(—2)	4(—6)	4·740(—5)
2	9·780(—1)	2(—6)	2·510(—5)
1	2·194(—1)	1(—6)	1·324(—5)
8(—1)	2·485(—1)	8(—7)	1·077(—5)
6(—1)	2·726(—1)	6(—7)	8·250(—6)
4(—1)	2·810(—1)	4(—7)	5·660(—6)
2(—1)	2·446(—1)	2(—7)	2·970(—6)
1(—1)	1·823(—1)	1(—7)	1·554(—6)
8(—2)	1·622(—1)	8(—8)	1·261(—6)
6(—2)	1·377(—1)	6(—8)	9·630(—7)
4(—2)	1·072(—1)	4(—8)	6·584(—7)
2(—2)	6·710(—2)	2(—8)	3·430(—7)
1(—2)	4·038(—2)	1(—8)	1·784(—7)
8(—3)	3·407(—2)	8(—9)	1·446(—7)
6(—3)	2·727(—2)	6(—9)	1·101(—7)
4(—3)	1·979(—2)	4(—9)	7·504(—8)
2(—3)	1·128(—2)	2(—9)	3·890(—8)
1(—3)	6·332(—3)	1(—9)	2·015(—8)
8(—4)	5·244(—3)	8(—10)	1·630(—8)
6(—4)	4·105(—3)	6(—10)	1·240(—8)
4(—4)	2·899(—3)	4(—10)	8·424(—9)
2(—4)	1·588(—3)	2(—10)	4·352(—9)
1(—4)	8·633(—4)	1(—10)	2·245(—9)
8(—5)	7·085(—4)	8(—11)	1·824(—9)
6(—5)	5·486(—4)	6(—11)	1·378(—9)
4(—5)	3·820(—4)	4(—11)	9·344(—10)
2(—5)	2·048(—4)	2(—11)	4·812(—10)
1(—5)	1·094(—4)	1(—11)	2·475(—10)

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SOME FREQUENTLY USED SYMBOLS AND THEIR DIMENSIONS

Symbol	Definition	Dimensions
b	Thickness of aquifer	L
b', b"	Thickness of confining beds	L
B	Drainage Factor of an unconfined aquifer with delayed yield.	L
C	Hydraulic Resistance of a confining bed.	T
l/C	Leakance of a confining bed	T^{-1}
D	Hydraulic Diffusivity of aquifer	L^2T^{-1}
K	Hydraulic Conductivity	LT^{-1}
K', K"	Vertical Hydraulic Conductivity of confining beds	LT^{-1}
K_x, K_y, K_z	Hydraulic Conductivity in the three mutually perpendicular coordinate directions.	LT^{-1}
L	Leakage Factor of a semi-confined aquifer.	L
N	A factor in unconfined aquifer with delayed yield, being equal to $1+\dfrac{S_Y}{S_A}$	Dimensionless
Q	Discharge rate	L^3T^{-1}
r_i, r_p	Radial distances of an observation well from an image well and pumped well respectively	L
s	Drawdown	L
s'	Residual drawdown	L
S	Storativity	Dimensionless
S', S"	Storativities of confining beds	Dimensionless
Ss, Ss', Ss"	Specific Storages of aquifer and confining beds	Dimensionless
S_A	Early time Storativity of an unconfined aquifer with delayed yield.	Dimensionless
S_Y	Specific Yield	Dimensionless
S_r	Specific Retention	Dimensionless
T	Transmissivity	L^2T^{-1}
t	Time since pumping started	T
t'	Time since pumping stopped	T
twt	Time after delayed yield ceases to affect the drawdown.	T
u	Variable of integration $=r^2S/4Tt$	Dimensionless
Vx, Vy, Vz	Velocities in the three mutually perpendicular co-ordinate directions.	LT^{-1}
W (u)	Well function of u.	Dimensionless
$\frac{1}{\alpha}$	Delay index of Boulton	T
θ	Porosity	Dimensionless

1. INTRODUCTION

1.1 In pursuance of the decision of the internal meeting of the Central Ground Water Board, Government of India constituted a Committee vide its Order No. 3-23/78-MI(A) dated the 26th May, 1978 of the following members, to review the present technology for determination of aquifer constants with a view to suggesting improvements.

1. Dr. B. D. Pathak, Director, Central Ground Water Board, Northern Region, Lucknow. — Leader
2. Shri B. P. C. Sinha, Director, Central Ground Water Board, Western Region, Jaipur. — Member
3. Shri R. K. Prasad, Senior Hydrologist, Central Ground Water Board, North Western Region, Chandigarh. — Member
4. Shri R. S. Saxena, Superintending Engineer (MI), Ministry of Irrigation, Krishi Bhavan, New Delhi. — Member
5. Dr. A. S. Chawla, Professor, Water Resources Development Training Centre, Roorkee University, Roorkee. — Member
6. Dr. B. B. S. Singhal, Professor, Department of Geology and Geophysics, Roorkee University, Roorkee. — Member

Dr. B. D. Pathak, Leader of the Committee, convened and presided over the first meeting which was held on 6th/7th July, 1978 in the office of the Director, Central Ground Water Board, Northern Region, Lucknow and was attended by Dr. B. B. S. Singhal and Dr. A. S. Chawla of Roorkee University, Roorkee.

The objectives of the meeting were to discuss the problems met with in determination of aquifer constants, to review the present technology for determination of aquifer constants, research needed to improve the present technology and collection and compilation of the already available data on aquifer parameters.

An open discussion with the hydrogeologists of Central Ground Water Board, Northern Region, Lucknow helped in defining the general problems in determination of aquifer constants. Various methods for determination of the aquifer parameters were reviewed and problem was assessed. It was felt by the Committee that the job entrusted has much wide range and needs active participation of a few more members. On the recommendations of the Committee, Government of India vide its Order No. 3-23/78-MI(A) dated 3-10-78 coopted the following persons as member of the Committee.

1. Dr. A. Achuta Rao, Director, Central Ground Water Board, Vedavati River Basin Project, Bangalore.
2. Shri K. R. Karanth, Director, Central Ground Water Board, Western Region, Jaipur.
3. Shri A. R. Bakshi, Junior Hydrogeologist, Central Ground Water Board, Northern Region, Lucknow.

The second meeting of the Committee was held on 22nd and 23rd August, 1978 in the Guest House of Roorkee University, Roorkee. After reviewing the methods of pumping test data analysis, the Committee was of the view that:

(i) The pre-test trend of water level should be taken at least for two days in all the observation wells, pumping well and other wells within a radius of one kilometre.

(ii) The duration of pumping for the test purpose should be at least 24 hours in confined, 48 hours in semi-confined and 72 hours in the unconfined aquifers.

(iii) In order to obtain reliable and representative values of aquifer and confining bed parameters by time draw-down and distance-draw-down relationship and so also to detect anisotropy and possible inter-connection of aquifers, there should be desirably five observation wells in different directions and distances at each site. The observation wells should be designed by putting three of them in the aquifer being tested and one each in the underlying and overlying aquifers. The distance of the observation wells may vary from 10 to 100 metres and in some cases upto 250 metres or more depending upon the thickness of the aquifers and local conditions.

(iv) Although a large number of tubewells are being constructed by State and Private agencies, there is no arrangement for measurements of the piezometric levels of the aquifer tapped. Therefore, it is recommended that while constructing the tubewells there should be adequate provision for measuring the water levels of the tapped aquifer.

It was also decided in the second meeting of the Committee that a working note on the following be prepared for the consideration of the Committee;

(i) Short cut methods for determination of aquifer constants.

(ii) Inverse method for determination of aquifer parameters by using water level data.

(iii) Tracer technique for determination of aquifer parameters.

The Committee was further of the opinion that for the research needed to improve the present technology the following is to be done ;

(i) Gaps in knowledge are to be defined.

(ii) To evolve suitable method for determination of aquifer parameters by testing dug wells, in the unconsolidated and semi-consolidated rocks with primary porosity and in consolidated rocks with secondary porosity.

(iii) Hydraulics of flow in cavity wells, dug-cum-bored wells and dug wells be discussed.

(iv) Determination of aquifer constants from water level data.

(v) Grain size parameters and its relationship with aquifer parameters.

(vi) Borehole geophysical method for determination of aquifer parameters.

A format for the final report of the Committee was also finalised.

The progress of the work of the Committee was reviewed by Shri J. K. Jain, Chairman, Central Ground Water Board who in a letter to Dr. B. D. Pathak, Leader of the Committee, commended as follows:

"From the proceedings of the Committee to review the present technology for determination of aquifer constants with a view to suggesting improvements, I find that keen interest is being taken on the subject by all the members. I convey my thanks and gratitude for the same. As you would be aware, a large number of pumping tests are being performed annually by Central Ground Water Board and the State Ground Water Organisations. It is also true that at present the methodology employed is not always rigorously valid for the actual situation encountered.

There is, thus, no doubt that a publication in the form of a technical manual which would give the essentials of the pump tests technology and also mention the assumptions/limitation involved in each procedure together with an indication of the areas where further research and development is called for, would go a long way in bringing an all round improvement in the technology of water evaluation. In the above context, I was wondering if the members of the present Committee, after in dividually putting in so much of labour would jointly undertake to produce such a publication. I shall be grateful if the suggestion is considered in the next meeting of the Committee and a favourable decision taken".

As decided in the second meeting, the third meeting of the Committee was held on 26th October, 1978 in the Committee Hall of Central Ground Water Board, Jamnagar House, Mansingh Road, New Delhi. Shri J. K. Jain, Chairman, Shri B. K. Baweja, Chief Hydrogeologist and Shri Ajit Singh, Chief Engineer, Central Ground Water Board were the special invitees. While addressing the members of the Committee, they have their considered views on the aims and objectives of the Committee and utility of a technical manual on evaluation of aquifer parameters. As the consensus was in favour of preparation of the manual, the Committee was entrusted with the job of preparation of the draft manual too.

Shri J. K. Jain, Chairman, Central Ground Water Board proposed the name of Dr. H. S. Chauhan, Professor and Head of Agriculture Engineering, Gobind Ballabh Pant University, Pant Nagar, Nainital for the consideration of the Committee to include him as a member of the Committee. The Committee decided to co-opt him as a member.

The outline of the technical manual on evaluation of aquifer parameters was discussed and finalised. The Chapters were distributed amongst the members of the Committee.

The fourth meeting of the Committee was held in the Chamber of Chairman, Central Ground Water Board, New Delhi on 20th-21st April, 1979. The Committee reviewed the progress made in the preparation of the manual. In conclusion, it recommended that testing of shallow aquifers by constructing shallow tubewells may be taken up by Central Ground Water Board, in its normal exploration and testing programmes.

The fifth and sixth meeting of the Committee were held in the office of Central Ground Water Board, Lucknow on 15th-16th June, 1979 and 13th to 15th September, 1979 respectively in which the draft chapters of the manual submitted by the members were reviewed.

Seventh and last meeting of the Committee was held in the office of the Central Ground Water Board New Delhi on 5th August, 1980, in which the various draft chapters of the manual were carefully scrutinised. The comments of Shri V. M. Sikka, Junior Hydrogeologist, Central Ground Water Board, North Western Region, Chandigarh were accepted after the minor modificatons. Shri B. P. C. Sinha, Director (HQ) Central Ground Water Board proposed the name of Shri V. M. Sikka to be co-opted as member of the Committee, which was accepted. It was felt that the manual needs careful editing and revision of some of the chapters. As a consequence, it constituted an editorial board comprising the following members to bring the manual in a suitable form for final submission to the Government for approval, publication and issuance:

1. Shri V. M. Sikka,
   Junior Hydrogeologist,
   Central Ground Water Board,
   North-Western Region,
   Chandigarh.

2. Shri A. R. Bakshi,
   Junior Hydrogeologist,
   Central Ground Water Board,
   Northern Region,
   Lucknow.

3. Shri R. K. Prasad,
   Senior Hydrologist,
   Central Ground Water Board,
   North-Western Region,
   Chandigarh,

4. Shri B. P. C. Sinha,
   Director (HQ),
   Central Ground Water Board,
   Faridabad.
   (Haryana).

In the finalisation of this Manual Shri V. M. Sikka put in special efforts not only in recasting Chapter 5 by selecting and re-interpreting pumping test data in the light of the recommendations of the Committee, but also prepared final plates, drawings and type curves which accompany this Manual.

The final report of the Committee was intended to serve as a- compendium of Committee's view and recommendation on various aspects of the over all problem referred to it by the Government and was submitted to the Government separately.

The manual is designed for the professionals engaged in the exploration and assessment of ground water resources for which determination of the aquifer parameters is a basic prerequisite.

The manual has been organised on the following lines. In the preparation of this text, back ground knowledge on the fundamentals of geology and hydrology on the part of the reader has been presumed.

Chapter 1 deals with the evaluation of the manual and its historical background.

Chapter 2 defines the various aquifer types and their hydrogeologic and geohydrologic characteristics.

Chapter 3 deals with the hydraulics of ground water flow in general and radial flow towards wells in particular. Starting from the basic equations of Darcy and continuity, it develops the general equation of ground water flow from which the rardial flow towards wells in different types of aquifers are derived. Solution of governing differential equations in each case is presented in detail.

Chapter 4 of the manual provides detailed guidelines for proper design and lay out of the aquifer tests. It provides adequate guidelines for planning and construction of the observation and test wells and procedure and frequency of observations for proper evaluation of aquifer parameters.

Analysis of pumping test data in different formations are presented in Chapter 5. Typical Indian examples with step by step procedure to analyse aquifer test data for sedimentary formations with primary porosity and hard rock formations with dominantly secondary porosity to determine aquifer parameters.

The frequently used approximate methods for determination of aquifer parameters are described in Chapter 6.

Chapter 7 deals with less conventional methods like laboratory tests and tracer techniques for determination of aquifer parameters. It also introduces the essential features of inverse technique for parameter identification.

A comparative study of the various existing methods for determination of aquifer parameters is presented in Chapter 8. It also brings out the limitations of each technique and provides adequate guide lines on suitable method for various field conditions.

In Chapter 9, guidelines for preparation of test reports are discussed.

The recommended symbols, units, other useful tables and annexures are also given separately.

In addition to relevant references at the end of each Chapter, the manual provides an exhaustive bibliography on the subject for detailed reading and reference.

2. AQUIFER TYPES AND THEIR HYDRAULIC PROPERTIES

2.1 Introduction

Water which occurs below the ground surface is known as sub-surface water. Sub-surface water occurs under two broad hydraulic situations; saturated and unsaturated. The word groundwater is used to denote all interstitial water below the water table which is the upper surface of the completely saturated ground. Above the water table lies the zone of unsaturated soil or the zone of aeration. It extends right upto the ground surface. At some places, there might be localised zone of saturation within zone of aeration which forms perched water.

2.2 Hydrological Classification of Geological Formations

The occurrence and movement of groundwater depends on the geohydrological characteristics of the sub-surface formations. These natural deposits vary greatly in their lithology, texture and structure which in turn influence their hydrological characteristics. The geological formations are classified into the following four types depending upon their hydrological properties:

(i) Aquifer: An aquifer is a formation or a geological structure which has good permeability to supply sufficient quantity of water to a well or spring. Unconsolidated sedimentary formations like gravel and sand form excellent aquifers. Fractured igneous and metamorphic rocks and carbonate rocks with solution cavities also form good aquifers.

(ii) Aquitard: It is a formation which has low to medium permeability which is not sufficient to be a source of water supply but allows certain quantity of water to flow on a regional scale from one aquifer to the other due to leakage. Formations having predominance of silt and clay alongwith *kankar* form aquitards. Aquitards behave as semi-confining layers.

(iii) Aquiclude: It is a formation which may have high porosity but the permeability is very low, e.g. clay and shale.

(iv) Aquifuge: It is a formation which is neither porous nor permeable e.g. massive igneous and metamorphic rocks.

The classifications of a formation as aquifer or aquitard also depends upon the availability of water in an area. A rock type in an area may be regarded as an aquitard but in another region it may be regarded as an aquifer on account of the non-availability of more permeable formations.

The aquifer serves both as a conduit for the movement of water from the zone of recharge to the place of discharge and as a storage reservoir.

Table 2·1 gives a classification of rocks based on their hydrological proprties.

TABLE : 2·1

Hydrological Classification of Rocks

Rock types	Porosity: Primary (grain) %	Porosity: Secondary (fracture)[1]	Permeability range (cm/sec): 10^2 10^0 10^{-2} 10^{-4} 10^{-5} 10^{-8}	Well yields: High Meduim Low	Type of water-bearing unit
Sediments unconsolidated					
Gravel	30-40				Aquifer
Coarse sand	30-40				Aquifer
Medium to fine sand	30-35				Aquifer
Silt	40-50	Occasional			Aquiclude
Clay, till	45-55	Rare (mud cracks)			Aquiclude
Sediments consolidated					
Limestone dolomite	1-50	Solution joints, planes			Aquifer or aquifuge
Coarse meduim sandstone	<20	Joints and fractures			Aquifer or aquiclude
Fine sandstone argillite	<10	Joints and fractures			Aquifer or aquifuge
Shale, siltstone	—	Joints and fractures			Aquifuge or aquifer
Volcanic rocks					
Basalt	—	Joints, fractures			Aqifer or aquifuge
Acid volcanic rocks	—				Aquifuge or aquifer
Crystalline rocks					
Plutonic and metamorphic		Weathering and fractures decreasing as depth increases			Aquifuge or aquifer

1. Rarely exceeds 10 per cent.

TYPES OF AQUIFERS

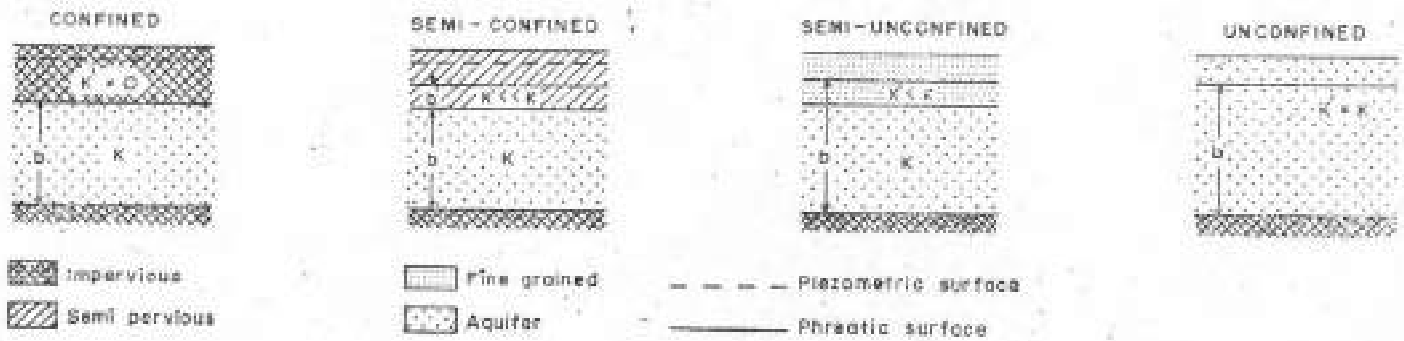

FIG. 2.1

FROM - Kruseman & DeRidder (1970)

2.3 Types of Aquifers

Aquifers are huge ground water storage reservoirs. The lateral continuity and vertical boundaries are often not well defined. The aquifers may be either of localised nature or may extend over distances of several hundred kilometres. Aquifers in Ganga basin, India; Great Australian Artesian Basin and in Sahara (Nubian sandstones) have been traced over distances of several hundred kilometres. Based on the hydraulic characteristics, the aquifers can be classified into following four types (Fig. 2·1).

(i) Confined aquifers;

(ii) Semi-confined or leaky confined aquifers;

(iii) Unconfined aquifers; and

(iv) Semi-unconfined aquifers.

2.3.1 Confined Aquifers

These are also termed as artesian aquifers. A confined aquifer is overlain and underlain by a confining layer (aquiclude or aquifuge). water in the confined aquifers occurs under pressure which is more than the atmospheric pressure. The piezometric surface which is an imaginary surface to which the water will rise in wells penetrating the confined aquifer, should lie above the top of the aquifer, i.e. above the base of the overlying confining layer. A particular aquifer at one place may be a confined aquifer while at another place it may behave as an unconfined aquifer where the water level falls below the base of the overlying confining layer. Similarly, at one particular place an aquifer may change from confined to unconfined character with time.

Following three types of artesian aquifer systems have been identified by Tolman (1937).

(i) Stratiform aquifers: These occur as either gently dipping beds, monoclines or synclines. They usually form high pressure systems e.g. Dakota sandstones of U.S.A., Cuddalore sandstones of Tamilnadu, Upper Cretaceous sandstones of Kutch.

(ii) Fractures and joints: In igneous and metamorphic rocks, ground water may occur in confined conditions in joints and fractures. In volcanic rocks, e.g. Deccan traps, interflow spaces and vesicular horizons may form confined aquifers.

(iii) Solution Cavities: Ground water in soluble rocks, at times, also occurs under confined conditions, e.g. Lias Limestones of Londer Basin, Permian limestone in the Roswell Basin of New Mexico and Vindhyan limestones of Barunda in Rajasthan.

Wells which tap confined aquifers may be either of flowing or of non-flowing conditions. In India, the well known examples of flowing wells are in Tarai belt at the foot of Himalaya in Nainital district, Cuddalore sandstones of Tamilnadu, alluvial terraces of river Yamuna, and Rajamundry Sandstones, Andhra Pradesh.

2.3.2 Semi-Confined Aquifers

In nature, truly confined aquifers are rare because the confining layers are not exactly impervious. In semi-confined or leaky confined aquifers, the aquifer is overlain or underlain by an aquitard or semi-pervious layer through which vertical leakage takes place due to head difference. The permeability of the semi-confining layer is small so that, any horizontal component of flow in it can be neglected.

2.3.3 Unconfined Aquifers

An un-confined aquifer is not overlain by any confining layer but it has a confining layer at its bottom. It is normally partly saturated with water and the upper surface of saturation is termed as water table which is under atmospheric pressure. Water in an unconfined aquifer is called unconfined or phreatic water.

In unconfined aquifer the gravity drainage is often not instantaneous and therefore there is some time lag in the lowering of water table and the drainage of the aquifer. The delay effect is more in fine grained aquifers as compared to coarse grained aquifers.

2.3.4 Semi-Unconfined Aquifers

These aquifers exhibit characters in between semi-confined and unconfined aquifers as the permeability of the fine grained overlying layers is more than in a semi-confined aquifer and the horizontal flow component in it cannot be neglected.

Based on the permeability of the covering layer Kruseman and De Ridder (1970) have given the following distinguishing features of the different type of aquifers.

TABLE 2.2

Classification of aquifers based on the Permeability of the Covering Layer

Covering Layer	Aquifer type
1. Impervious	Confined
2. Semi-pervious, so that the horizontal flow can be neglected	Semi-confined
3. Less pervious than the main part of the aquifer, but the horizontal flow is not negligible	Semi-unconfined
4. Same as the main part of the aquifer	Unconfined

The distinction between different types of aquifers is, at times, difficult. The subsurface lithology, water levels and other hydrological parameters of both the aquifers and confining layers should be studied carefully in order to ascertain the nature of the aquifers. The distinction between different types of aquifers is important because their capacity to release water from storage differs. This is also of relevance from the point of view of ground water balance and management studies.

2.4 Geological Controls on Formation of Aquifers

2.4.1 Sedimentary Formations

The unconsolidated alluvial deposits form potential aquifers. The Indo-Gangetic alluvium of India, Pakistan and Bangladesh represents a vast reservoir of groundwater. Limited extent of alluvium in intermontane valleys, e.g. Doon Valley, Kashmir Valley, also form potential aquifers. Alluvial fans at the foot of the Himalaya formed of clastic material especially in the upper and middle reaches of the cones should form productive horizons.

Semi-consolidated sandstones, e.g. Dakota sandstones of USA, Cuddalore sandstones of South India, Lathi sandstones of Rajasthan, upper Bhuj Sandstone of Kutch (Gujarat) and other similar formations are also potential source of ground water. However, with greater cementation, permeability decreases, and therefore sandstons of Vindhyan age are usually poor aquifers. The sandstones of Gondwana age which vary in grain size and degree of cementation and compaction, form poor to medium aquifers.

The carbonate rocks have generally low values of primary porosity and permeability but due to fracturing and subsequent dissolution, they may develop high permeabilities as indicated in Table (2.1.) Limestones and dolomites on account of extensive dissolution give rise to typical geomorphological feature which are characterised as Karst.

2.4.2 Hard Rocks

In crystalline (igneous and metamorphic) rocks, the development of porosity and permeability is due to fracturing and weathering. Weathering and fracturing of such rocks will increase overall permeabilities by two to four orders of magnitude. Weathering and fracturing is more near the surface and therefore, the permeability of hard rocks generally decreases with depth. In tropical countries, weathering may extend to about 100 m depth but in temperate regions, the effect of weathering is prominent to about 50 m.

The volcanic rocks, such as basalts, show great variation in permeabilities. Basalts of Columbia-Snake River area of USA and of Hawaii islands form good aquifers. Walton and Stewart (1961) have given values of transmissivity to be as high as $2 \cdot 0 \times 10^3$ to $2 \cdot 25 \times 10^5$ m^2/day for the basalts in Snake river area. The permeabilities of surface basalts of Hawaii area are so high, that in spite of high rainfall, perennial surface streams are rare.

Basalts of Deccan Trap have usually medium to low permeabilities depending on the presence of primary and secondary fratures. Pumping tests have given transmissivity values varying from 15m^2/day to 150 m^2/day.

2.5 Hydraulic Properties

The important hydraulic properties of aquifers and confining layers are porosity, hydraulic conductivity, transmissivity, storage co-efficient, hydraulic diffusivity, leakage co-efficient, leakage factor, drainage factor, specific capacity and specific capacity index.

2.5.1 Porosity (θ)

It is an important hydrological characteristic of a formation. The porosity is the measure of the interstices present in a formation. It is defined as the ratio of the volume of voids to its total volume and can be expressed either as a percentage or as decimal fraction.

Porosity is usually of two types; primary porosity and secondary porosity. Primary porosity is the inherent character of a rock which is developed during the formation of the rock. In sedimentary rocks and unconsolidated formations, porosity is of primary nature and is due to the intergranular spaces. In volcanic rocks, e.g. basalts, the primary porosity is due to the presence of gas cavities (vesicles) and also lava tubes and lava tunnels. Secondary porosity is developed due to subsequent processes such as fracturing and jointing in hard rocks (Igneous and metamorphic rocks) and dissolution in carbonate rocks (limestone). In shales also, at places secondary porosity due to fracturing is developed.

The porosity of natural materials may range from almost zero in hard massive rocks to as much as 50 percent in clays. Typical values of porosity for different aquifer materials as given by McWhorter and Sunada (1977) are presented in Table (2.3.)

TABLE 2·3

Porosity of Aquifer Materials

Aquifer material	No. of Analysis	Range	Arithmatic Mean
Igneous Rocks			
Weathered granite	8	0·34—0·57	0·45
Weathered gabbro	4	0·42—0·45	0·43
Basalt	94	0·03—0·35	0·17
Sedimentary Materials			
Sandstone	65	0·14—0·49	0·34
Siltstone	7	0·21—0·41	0·35
Sand (fine)	243	0·[illegible]—0·53	0·43
Sand (coarse)	26	0·31—0·46	0·39
gravel (fine)	38	0·25—0·38	0·34
gravel (coarse)	15	0·24—0·36	0·28
Silt	281	0·34—0·61	0·46
Clay	74	0·34—0·57	0·42
Limestone	74	0·07—0·56	0·30
Metamorphic Rocks			
Schists	18	0·04—0·49	0·38

Porosity in rocks is mainly controlled by the following factors:

1. Shape and arrangement of constituent grains
2. Degree of sorting
3. Cementation and compaction
4. Fracturing
5. Dissolution

The influence of arrangement of grains (packing) on porosity has been studied by Graton and Fraser (1935). They have considered different geometry of spherical aggregates. It has been found that the cubic packing shows maximum porosity (0.476) while the rhombohedral packing is closest to natural packing and has a porosity of 0·26. Angularity of the particles permits bridging with a resulting increase of porosity.

Sorting has a very important influence on porosity. Well sorted clastic material has higher porosity irrespective of grain size. In poorly sorted material, the porosity is less as the small size grains occupy the pore-spaces between bigger grains.

Cementation and compaction reduces porosity. In unconsolidated alluvial formations, the porosity at deeper levels is less due to greater compaction. In volcanic rocks porosity decreases with the deposition of secondary minerals in vesicles in the form of amygdales.

Fracturing and jointing results in higher values of secondary porosity and therefore is of greater importance in hard rocks. Some shales and clays may also acquire high secondary porosity and permeability due to joints and fractures, e.g. shales of Pennsylvanian age in the Central States of USA.

Table (2.3) indicates that limestones show a wide variation in porosity which is due to development of solution cavities to varying extent.

The porosity of an aquifer is the sum of specific retention (Sr) and specific yield (S_Y). Specific retention is a measure of the volume of water which is retained by the aquifer material against gravity on account of cohesive and inter-granular forces. Specific yield is the water yielding capacity and is also termed as effective porosity.

Specific yield is expressed quantitatively as the percentage of the total volume of rock occupied by the water which can be drained out by gravity.

Specific yield increases with increase in grain size and sorting while specific retention increases with decrease in grain size and assortment.

2.5.2 Hydraulic Conductivity (K)

The hydraulic conductivity also known as permeability is a measure of the ease with which a fluid moves through a formation and is defined as the amount of flow per unit cross sectional area under the influence of a unit gradient. It has the dimensions of Velocity (LT^{-1}) and is usually expressed in m/day.

The hydraulic conductivity depends both upon the properties of the fluid as well as on the properties of the aquifer. The specific permeability, k which is the permeability of the porous medium depends only on the property of the medium and is independent of the fluid properties. Specific permeability is related with hydraulic conductivity by the relation $K = k\frac{\gamma}{\mu}$

where μ is the co-efficient of viscosity and γ is the specific weight of the fluid. Specific permeability has the dimensions of L^2 and is usually expressed in darcy units. The Darcy is defined as follows. A porous medium is said to have a permeability of one darcy if a single phase fluid of one centipose viscosity that completely fills the porespace of the medium will flow through it at a rate of 1 cm^3/sec per cm^2 of cross-sectional area under a pressure gradient of 1 atm. per cm. Substituting appropriate unit in the above definition, it can be determined that

$$1 \text{ darcy} = 0{\cdot}987 \times 10^{-8} \text{ cm}^2$$

For water at 20°C, medium of permeability of 1 darcy would have a hydraulic conductivity (K) of $9{\cdot}613 \times 10^{-4}$ cm/sec.

The range of hydraulic conductivity values for various types of geological formations is given in table (2.4.)

TABLE 2·4

Some Typical Values of Hydraulic Conductivity

Soil Type	Hydraulic Conductivity, (K) in cm/sec.
Clean gravel	1—10^2
Clean coarse sand	1·0—0·01
Sand mixture, clayey Sand	0·01—0·005
Fine Sand	0·05—0·001
Sandy Loam	0·005—0·003
Silty Sand	0·002—0·0001
Peat, little decomposed	0·0006—0·002
Peat, moderately decomposed	0·0008—0·0002
Silt	0·0005—0·00001
Clay	<0·00001

2.5.3 Transmissivity (T)

Transmissivity or coefficient of transmissibility, is a hydraulic characteristic of the aquifer which was first introduced in ground water literature by C.V. Theis in 1935. It is defined as the rate of flow of water at the prevailing field temperature under a unit hydraulic gradient through a vertical strip of aquifer of unit width and extending through the entire saturated thickness of the aquifer. It is, therefore, a product of the average permeability and the saturated thickness of the aquifer i.e. T = Kb where b is the thickness of the aquifer. Transmissivity has the dimensions of $L^2 T^{-1}$ and is usually expressed in m^2/day.

The concept of transmissivity holds good in confined aquifer but in unconfined aquifer, as the saturated thickness of the aquifer changes with time, the T will also change accordingly.

2.5.4 Co-efficient of Storage or Storativity (S)

The Storage Coefficient of an aquifer is defined as the volume of water that a vertical column of the aquifer of unit cros-sectional area releases from storage or takes into storage as the average head within this column declines or rises a unit distance. It is dimensionless. In artesian aquifers where water released from or taken into storage is entirely due to compressibility of aquifer and of water, the storage co-efficient S is given by $S = bSs$, where b is the thickness of the aquifer an Ss (specific storage) which has the dimensions of L^{-1} and is defined as the volume of water which a unit volume of the aquifer releases from storage because of expansion of water and compression of the aquifer under a unit decline in the average head within the unit volume of the aquifer. The storage coefficient in confined aquifers has the order of magnitude of 10^{-3} to 10^{-6}.

The Storage Coefficient Sw for a water table aquifer is given by $Sw = Sy + bSs$ where b is the height of water table above the base of the free aquifer, and Sy is the specific yield of the aquifer. Usually $Sy \geqslant bSs$, thus Sw for all practical purposes be regarded as the Specific Yield. The Specific Yield is defined as the ratio of the volume of water that a rock or soil will yield by gravity to its own volume. In other words, it represents very closely the effective porosity. The Storage Coefficient in unconfined aquifers, Sw, ranges from 0·05 to 0·30.

In confined aquifer the storage coefficient depends upon the compressibility of the aquifer and the expansion of water. Jacob (1940) has given the following equation to relate S with the aquifer compressibility and fluid properties.

$$S = \theta\gamma b \left(\frac{1}{Ew} + \frac{C}{\theta Es}\right)$$

θ = porosity (dimensionless)

γ = Specific weight of water ($ML^{-2}T^{-2}$)

b = Thickness of saturated medium (L)

Ew = bulk modulus of elasticity of water ($ML^{-1}T^{-2}$)

Es = bulk modulus of elasticity of the solid skeleton of the aquifer $ML^{-1}T^{-2}$)

C = a dimensionless ratio, which may be considered unity in uncemented granular material.

Since the unconfined aquifer is not bounded by confining layers the Specific Yield or storage coefficient does not depend upon the compressibility of either the aquifer or the fluid. The Specific Yield for all practical purposes is same as effective porosity or drainable porosity.

Both S and Sy are important hydrological properties and their accurate determination is important for ground water balance studies.

2.5.5 Hydraulic Diffusivity (D)

Hydraulic diffusivity is defined as the ratio of transmissivity and storativity and is given as,

$$D = \frac{T}{S} = \frac{K}{Ss}$$

$$= \frac{K}{\theta\gamma\left(\frac{1}{Ew} + \frac{C}{\theta Es}\right)}$$

Diffusivity has the dimensions of L^2T^{-1} and is generally expressed in m²/day.

For unconfined conditions, the hydraulic diffusivity term is directly proportional to the transmissivity of the aquifer, obtained as the product of the hydraulic conductivity of the water bearing material and the average saturated thickness b_{av} of the aquifer and is inversely proportional to the storage coefficient.

In unconfined aquifer, transmissivity can be expressed as $T = Kb_{av}$, where b_{av} is the average saturated thickness of the unconfined aquifer. The diffusivity in unconfined aquifer will, therefore, be

$$D = \frac{T}{Sy} = \frac{Kb}{Sy}$$

Where Sy is the specific yield of unconfined aquifer and $b_{av} = b - \frac{s_o}{2}$

Where b is the saturated thickness of the aquifer under static condition and s_o is the drawdown in aquifer just outside the pumping well, i.e. at the outer face of the well.

2.5.6 Leakage Coefficient or Leakance $\left(\frac{1}{C}\right)$

It is the property of semi-confining layer. It is the ratio of the vertical permeability of semi-confining layer to its thickness, i.e. K'/b'. It has the dimensions of T^{-1}.

2.5.7 Hydraulic Resistance (C)

It is also called reciprocal leakage coefficient or resistance against vertical flow and is a property of confining layers of leaky aquifers. It is equal to b'/K'. It characterises the resistance of the semi-pervious layer to upward or downward leakage. It has the dimensions of time. If hydraulic resistance $C = \infty$, the aquifer is confined.

2.5.8 Leakage Factor (L)

The leakage factor $L = \sqrt{T.C.} = \sqrt{T/K'/b'}$. It determines the distribution of the leakage into the leaky (semi-confined) aquifer. High value of L indicates a great resistance of the semi-pervious strata to flow. The factor L has the dimensions of length and is usually expressed in metres.

2.5.9 Delay Index $\left(\frac{1}{\alpha}\right)$

It is a measure of the delayed drainage of an unconfined aquifer and has the dimension of time. (T)

2.5.10 Drainage Factor (B)

The drainage factor $B = \sqrt{Kb/\alpha S_Y}$. It is a property of unconfined aquifer. Large values of B indicate a fast drainage. The drainage factor has the dimensions of length (L) and is expressed in metres.

2.5.11 Specific Capacity

It is a measure of both the effectiveness of a well and also of the aquifer characteristics (T and S). It is defined as the ratio of the pumping rate and the drawdown and is usually expressed in litres per minute per metre of drawdown for a specific period of pumping.

2.5. 12 Specific Capacity Index

It is a measure of the formation characteristics. It is obtained by dividing the specific capacity by the saturated thickness of the aquifer. The specific capacity index values are of use in determining the relative productivity of different units in a multi-unit aquifer and also in predicting well yield from a given thickness of aquifer. Unit area specific capacity values are obtained by dividing the specific capacity by the cross sectional area of the well. The specific capacity can also be divided by $2\pi r_w b$ (where r_w is the radius of the well) to account for variation in the well radii and depth.

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3. GROUND WATER FLOW AND WELL HYDRAULICS

3.1 Introduction

A judicious exploration of ground water resources is not possible without the basic knowledge of ground water hydraulics. Therefore, a number of fundamental laws governing the flow of ground water through permeable formations and practical formulae for flow through aquifers are discussed in this Chapter.

3.2 Darcy's Law

Darcy's law states that the filtration velocity of seeping fluid depends linearly on the loss of head over the given path and can be expressed by the following relation.

$$V = -K\frac{dh}{ds} \qquad (3 \cdot 1)$$

Where,

V = Filtration velocity or specific discharge, the discharge per unit cross-sectional area of the soil through which the water is flowing.

K = Co-efficient of permeability of the soil, also termed the hydraulic conductivity or the transmission constant.

h = Head of the groundwater at the point in consideration, measured with respect to a given reference level.

$\frac{dh}{ds}$ = Hydraulic gradient at that point, i.e. the loss of head dh divided by the distance ds along the direction of flow.

The piezometric head of the ground water or ground water potential, can be expressed as

$$h = \frac{P}{\rho g} + Z + \frac{V^2}{2g} \qquad (3.2)$$

Where,

$\frac{P}{\rho g}$ = The pressure head of the groundwater at the point in consideration.

P = The water pressure at that point.

ρ = The density of the water.

g = The acceleration of gravity.

Z = The elevation of the point with respect to the given reference level. It is also termed the elevation head or geodetic head.

V = Velocity of flow at the point under consideration.

The piezometric head can therefore be defined as the sum of pressure head, elevation head and velocity head. In case of groundwater, velocity of flow is small and hence velocity head is comparatively much smaller than other heads and hence is neglected.

A porous medium may have permeability which vary with flow direction. Such a condition is termed anisotropy and velocity components in a rectangular coordinate system are given by

$$V_x = -K_x\frac{\partial h}{\partial x} \qquad (3 \cdot 3)$$

$$V_y = -K_y\frac{\partial h}{\partial y} \qquad (3 \cdot 4)$$

$$V_z = -K_z\frac{\partial z}{\partial z} \qquad (3 \cdot 5)$$

Where K_x, K_y and K_z are coefficients of permeability in the x, y and z directions respectively. The velocity at any point in an aquifer may be taken as the vector sum of the component velocities. To simplify mathematical treatment, aquifers will be assumed to be homogeneous and isotropic. Hence,

$$V_x = -K\frac{\partial h}{\partial x} \qquad (3 \cdot 6)$$

$$V_y = -K\frac{\partial h}{\partial y} \qquad (3 \cdot 7)$$

$$V_z = -K\frac{\partial h}{\partial z} \qquad (3 \cdot 8)$$

In hydrodynamics, a velocity potential $\varnothing$ is defined as a scalar function of space and time such that its derivative with respect to any direction is the fluid velocity in that direction. If, for the present purpose $\varnothing = -Kh$, then it follows from above equation.

$$V_x = \frac{\partial \varnothing}{\partial x} \qquad (3 \cdot 9)$$

$$V_y = \frac{\partial \varnothing}{\partial y} \qquad (3 \cdot 10)$$

$$V_z = \frac{\partial \varnothing}{\partial z} \qquad (3 \cdot 11)$$

Existence of velocity potential implies irrotational flow.

The groundwater head h has the dimension of length, (L). The specific discharge (or the filtration velocity) V has the dimension of a velocity, (L/T).

There exists a great variety of units for expression K. It is recommended that m/day be used as unit. This will lead to a convenient order of magnitude of the values of both K and the other formation constants.

Darcy's law is valid for laminar flows. For Reynold's number (N_R) upto 1, the flow is fully laminar. The flow is fully turbulent at Reynold's number greater than 1000. For $1<N<1000$, the flow is in a transition zone. Reynold's number N_R is defined as

$$N_R = \frac{Vd}{u}$$

Where V is the filtration velocity defined by Darcy's Law, d is a characteristic grain diameter of the soil and u is the kinematic viscosity of the water.

Muskat (1937) and others obtained good comparative results for sand if the effective grain diameter is substituted in the above equation for ground water fl ow. The resultant N_R was less than 1.

The question of the existence of a lower limit to validity of Darcy's Law is still subject of discussion. A lower limit being of much less practical importance than the upper limit, the problem can be left on one side.

3.3 Differential Equation Governing Groundwater Flow

Ground water satisfies the equation of continuity. It expresses the principle of conservation of matter, i.e. the net inward flux through an elemental volume of an aquifer in the flow field must equal the rate at which matter is accumulating within the element (DeWiest, 1965). The Continuity equation, in its general form may be expressed as

$$\left[\frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} + \frac{\partial(\rho V_z)}{\partial z}\right] = \rho Ss \frac{\partial h}{\partial t} \quad (3.12)$$

Where Ss is the specific storage of the aquifer and is defined as the volume of the water which a unit volume of the aquifer releases from storage because of expansion of water and compression of aquifer under a unit decline in head. ρ is the density of the fluid.

Expressing the velocity components in terms of hydraulic gradients from Darcy's Law and simplifying equation 3.12, yield,

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S_s}{K} \frac{\partial h}{\partial t} \quad (3.13)$$

In the special case of a confined aquifer of thickness b, the storage coefficient $S = Ssb$ and the transmission T of the aquifer, $T = Kb$ may be introduced. Equation (3.13) then becomes,

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S}{T} \frac{\partial h}{\partial t} \quad (3.14)$$

Under steady-state flow conditions, the velocity and pressure distribution do not change with time and equation 3.14 reduces to :

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad (3.15)$$

This is known as Laplace equation which governs the steady-state flow of groundwater in a homogeneous and isotropic aquifer. The differential equation governing the unsteady state radial flow in nonleaky confined aquifer in polar co-ordinates is given by :

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{S}{T} \frac{\partial h}{\partial t} \quad (3.16)$$

where r is the distance from the well to the point of observation.

3.4 Steady-state Flow

3.4.1 Confined Aquifer :

Steady radial flow to a well through the aquifer at a distance r from the central axis can be expressed as

$$Q = AV = -2\pi r Kb \frac{dh}{dr} \quad (3.17)$$

where Q is the rate of flow.

Re arranging and integrating for the boundary conditions at the well, $h = h_w$ at $r = r_w$ and at the edge of equi-potential boundary, $h = h_o$ at $r = r_o$, yields

$$h_o - h_w = \frac{Q}{2\pi Kb} \ln \frac{r_o}{r_w} \quad (3.18)$$

or

$$Q = \frac{2\pi Kb(h_o - h_w)}{\ln\left(\frac{r_o}{r_w}\right)} \quad (3.19)$$

This is known as the equilibrium or Thiem's equation. The drawdown s, at a radial distance r from the well is given as

$$s = h_o - h = \frac{Q}{2\pi Kb} \ln \frac{r_o}{r} \quad (3.20)$$

3.4.2 Unconfined Aquifer

Consider the case of pumped well located in an unconfined aquifer and discharging steady discharge Q. The flow at radial distance r from the well is given

NONLEAKY ARTESIAN AQUIFER WITH FULLY PENETRATING WELLS

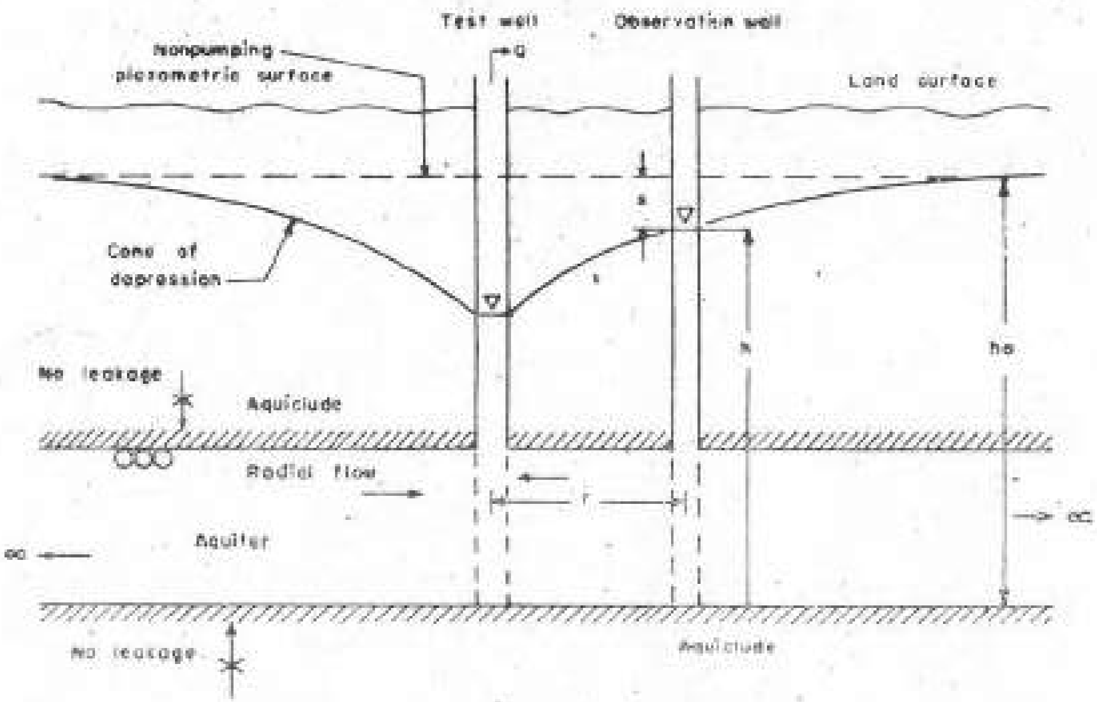


FIG. 3-1

FROM- Walton

by the following equation under the simplifying assumptions made by Dupuit.

$$Q = AV = 2\pi r K \ h \frac{dh}{dr} \qquad (3\cdot21)$$

where h denotes the height of water table above the lower impermeable boundary at any distance r, from the well. Rearranging and integrating for the boundary conditions at the well, $h = h_w$ for $r = r_w$ and h = head at a distance r from the well.

$$h^2 - h^2_w = \frac{Q}{\pi K} \, l_n \frac{r}{r_w} \qquad (3\cdot22)$$

This equation can be used to determine the distribution of head radially from the well. The maximum value of h is given by the initial head existing before pumping and is usually denoted by ho or H. The piezometric head or water table elevation at any distance r can also be expressed in terms of ho

$$h^2_o - h^2 = \frac{Q}{\pi K} \, l_n \frac{r_o}{r} \qquad (3\cdot23)$$

where h_o denotes the head at radial distance r_o

Discharge to a well penetrating an unconfined aquifer is given by

$$\frac{\pi K(h^2_o - h^2_w)}{\ln \frac{r_o}{r_w}} \qquad (3\cdot24)$$

As stated earlier the derivation of equation of flow through unconfined aquifer is based on Dupuit's theory. In the actual case of a pumped well, the free surface will be higher than the computed free surface as the Dupuit's assumption of near horizontal stream lines is not satisfied. The maximum deviation from the computed water surface profile occurs near the well where the curvature of the stream line is most marked. The free surface, will, therefore, emerge higher than the computed water level in the well, the height of the actual free surface at the well face above the water level in the well being designated as seepage face.

Babbitt and Caldwell have suggest the following equation for computing free surface for well located in unconfined aquifer based on results of experimental study.

$$Q = \frac{\pi K h_o (h_o - h)}{C_x \ln \left(\frac{r_o}{h_o}\right)} \qquad (3\cdot25)$$

where C_x is an empirical factor closely expressed by

$$C_x = 0\cdot3 \log (r_o/r) \qquad (3\cdot26)$$

within the range of experimental data. Several other investigators have suggested approximate expressions for the free surface on the basis of model studies.

3. 5 Unsteady-State Flow

3. 5. 1 Confined Aquifers

Consider a well fully penetrating an artesian aquifer overlain and underlain by aquiclude pumped at a constant rate Q. The aquifer is assumed to be homogeneous, isotropic, infinite in areal extent, and is of the same thickness throughout. The flow is radial throughout (Fig. 3·1)

When a production well in such an aquifer is pumped, water is continuously withdrawn from storage within the aquifer as the cone of depression progresses radially farther from the well. Because of the absence of a source of recharge in the form of vertical leakage or a recharge boundary, there can be no stabilisation of water levels and the head in the aquifer will continue to decline provided the aquifer is effectively infinite in areal extent. However, the rate of decline of head continuously decreases as the cone of depression spreads. Water is released from storage by the compaction of acquifer and its associated beds and by the expansion of the water itself.

The unsteady-state radial flow in the nonleaky artesian aquifer is governed by differential equation (3. 16.)

To obtain a simple mathematical solution, the production well is replaced in the analysis by a mathematical sink of constant strength and is assumed to have an infinitesimal diameter. It is also assumed that water is released from storage instantaneously. For boundary conditions

$$h \to h_o \text{ as } r \to \infty \text{ for } t \geq O \text{ and } \lim_{r \to O} \left(r \frac{\partial h}{\partial r} \right) = \frac{Q}{2\pi T}$$

and the initial conditions $h(r, o) = h_o$ for $t \leq O$, for $t = O$, the solution to the problem given by Eq. (3·16) is as follows (Jacob, 1950):

$$s = h_o - h = \frac{Q}{4\pi T} \int_{\frac{r^2S}{4Tt}}^{\infty} \frac{e^{-u}}{u} \, d_u \qquad (3.27)$$

Theis (1936) first applied this equation to well hydraulics and the equation is known as the non-equilibrium equation. The integral is a function of the lower limit $u = \frac{r^2S}{4Tt}$ and is tabulated as the exponential integral under the symbol $-E_i(-u)$. Expanding the exponential integral in a convergent series, the drawdown, $s = h_o - h$, in an observation well at a distance r and at a time t for a constant discharge become:

$$s = \frac{Q}{4\pi T}\left(-0\cdot5772 - \ln u + u - \frac{u^2}{2\cdot2!} + \frac{u^3}{3\cdot3!} - \frac{u^4}{4\cdot4!} + \ldots\ldots\right) \qquad (3\cdot28)$$

The drawdown at the production well face is found from Eq. (3·28) for r equal to the effective radius of the well.

With the exponential integral expressed symbolically as W(u), that is,

$$W(u) = \int_u^\infty \frac{e^{-u}}{u} du$$

Equation (3.27) may be rewritten as

$$s = \frac{Q}{4\pi T} W(u) \quad (3.29)$$

where,

$$u = \frac{r^2 S}{4Tt}$$

s= drawdown, (L)

r= distance from pumped well to observation point (L)

Q= discharge (L^3/T)

t= time after pumping started, (T)

T= coefficient of transmissibility (L^2/T)

S= coefficient of storage, fraction.

W(u) is read as the "well function for nonleaky artesian aquifers fully penetrated by wells and constant discharge conditions". Values of W(u) given by Wenzel (1942), in terms of the practical range of u are presented in Annexure-II.

When u becomes small (less than, say 0.01), the sum of the terms in the series in Eq. 3.28 beyond ln u becomes insignificant. When the pumping period becomes large or when the distance r is small, values of u will be small. When $u \leq 0.01$, then (Cooper and Jacob, 1946).

$$\int_u^\infty \frac{e^{-u}}{u} du = (-0.5772 - \ln u)$$

and $$s = \frac{Q}{4\pi T}(-0.5772 - \ln u) \quad (3.30)$$

For a particular time, Eq. (3.30) is the equation of a straight line plot of s versus log r. Drawdown s_1 and s_2 on the straight line at distance r_1 and r_2 respectively as given by Eq. 3.30 are

$$s_1 = \frac{Q}{4\pi T}(-0.5772 - \ln u_1) = \frac{Q}{4\pi T}\left[\ln\left(\frac{1}{u_1}\right) - 0.5772\right] \quad (3.31)$$

$$s_2 = \frac{Q}{4\pi T}(-0.5772 - \ln u_2) = \frac{Q}{4\pi T}\left[\ln\left(\frac{1}{u_2}\right) - 0.5772\right] \quad (3.32)$$

Thus,

$$s_1 - s_2 = \frac{Q}{4\pi T} \ln \frac{4\,Tt\, r_2^2\, S}{4\,Tt\, r_1^2\, S} \quad (3.33)$$

Converting to logarithms to the base 0, Eq. (3.33) may be written as,

$$s_1 - s_2 = \frac{2.30}{2\pi T} \log\left(\frac{r_2}{r_1}\right) \quad (3.34)$$

For a log cycle, $\log \frac{r_2}{r_1} = 1$, Eq. (3.34) become

$$T = \frac{2.30\ Q}{2\pi\ \Delta s} \quad (3.35)$$

where,

Δs=drawdown difference per log cycle, (L).

For a particular distance from the pumped well, Eq. (3.30) is the equation of a straight line plot of s versus log t. Drawdown s_1 and s_2 on the straight line at time t_1 and t_2, respectively, are given in Eqs. (3.31) and (3.32.) The difference between s_2 and s_1 is

$$s_2 - s_1 = \frac{Q}{4\pi T} \ln \frac{u_2}{u_1} = \frac{Q}{4\pi T} \ln \frac{t_2}{t_1} \quad (3.36)$$

Converting to logarithms to the base 10, Eq. 3.36 may be written as

$$s_2 - s_1 = \frac{2.30Q}{4\pi T} \log \frac{t_2}{t_1} \quad (3.37)$$

For a log cycle, $\log (t_2/t_1) = 1$ and Eq. 3.37 becomes,

$$T = \frac{2.30Q}{4\pi \Delta s} \quad (3.38)$$

where

Δs=drawdown difference per log cycle, (L).

The straight line plot of distance—drawdown and time—drawdown may be extrapolated to their intersection with the zero—drawdown axis. At the zero—drawdown intercepts, s-O and from Eq. (3.30)

$$0.5772 = \ln \frac{1}{u} = \ln \frac{4T\ to}{r^2 S} \quad (3.39)$$

and

$$0.5772 = \ln \frac{4\,Tt}{r_o^2\,S} \qquad (3.40)$$

Converting the logarithms to the base 10, Eqs. (3.39) and (3.40) may be rewritten as

$$S = \frac{2.25\,T\,t_o}{r^2} \qquad (3.41)$$

and

$$S = \frac{2.25\,T\,t}{r_o^2} \qquad (3.42)$$

where t_o = intersection of time—drawdown semilog straight line with zero—drawdown axis, (T)

r_o = intersection of distance—drawdown semilog straight line with zero—drawdown axis, (L).

Other terms are as defined earlier.

3.5.2 Unconfined aquifers

The non-equilibrium equation assumes that water is released instantaneously from storage by lowering of the head. In a confined aquifer this is essentially true because the water is released by expansion of the water and compression of the aquifer. In an unconfined aquifer, however, the water comes chiefly from gravity drainage from storage whithin the void space of the aquifer. As the water drains over a finite period of time, the storage coefficient during a pumping test varies with time. It increases at a diminishing rate with time and approaches the specific yield.

Because of this difference between confined and unconfined aquifers, the non-equilibrium equation can be applied to unconfined aquifers, only if the following limitations are observed: (a) the drawdown should be small in relation to the saturated aquifer thickness, and (b) a specified minimum pumping time should have elapsed so that the drainage effect is minimized. Boulton suggested that the minimum pumping time can be determined by the equation.

$$t_{min} = \frac{5.00\,S_y h_o}{K}$$

Where t min is the time after pumping began in days, S_y is the specific yield, h_o is the saturated aquifer thickness in metres and K is the co-efficient of permeability in metres per day. It can be noted that shallow, highly permeable aquifers require a minimum period of pumping, but thick relatively tight formations require a long pumping test period in order to apply the equation.

3.5.3 Leaky Confined Aquifer

Consider a well fully penetrating an artesian aquifer overlain by an aquitard and underlain by an aquiclude. Overlying the aquitard are deposits (source bed) in which there is a water table. The aquifer is homogeneous, isotropic, infinite in areal extent, and is of the same thickness throughout. The flow in the aquifer is radial throughout. Flow in the aquifer is augmented by vertical leakage through the aquitard. The flow lines are assumed to be refracted at full right angle as they cross the aquitard-aquifer interface. The aquitard is assumed to be more or less incompressible so that water released from storages therein is negligible. The water table is not influenced appreciably by pumping (Fig. 3.2).

The discharge of water from a well in such an aquifer is supplied from storage within the aquifer, as well as from leakage through the aquitard. Because of the presence of recharge in the form of leakage water levels will stabilize when the entire discharge of the well is derived from leakage. The rate of veritical leakage is proportionate to the difference in head between the water table and the piezometric surface of the aquifer. If K′ is the coefficient of permeability of the aquitard b′ is the thickness of the aquitard and $L^2 = (T/K'b')$ where L is termed as the leakage factor, the equation :

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} - \frac{S}{L^2} = \frac{S}{T}\frac{\partial s}{\partial t} \qquad (3.43)$$

is the differential equation in polar-coordinate notation governing the unsteady-state radial flow in a leaky artesian aquifer with fully penetrating wells without water released from storage in the aquitard (Jacob, 1946).

To facilitate the solution, the production well is replaced by a mathematical sink of constant strength with the boundary conditions $s \to o$ as $r \to \infty$ for $t \geq 0$

$$\lim_{r \to 0} r\frac{\partial s}{\partial r} = -\frac{Q}{2\pi T}$$

and the initial condition $s = 0$ for $t \leq 0$ for all values of r. The solution to the problem given by Equation (3.43) is (Hantush and Jacob, 1955)

$$s = \frac{Q}{4\pi T}\int_u^\infty \frac{1}{u}\exp\left(-u - \frac{r^2}{4L^2u}\right)du \qquad (3.44)$$

With the integral in Eq. (3.44) expressed symbolically as $W\left(u, \frac{r}{L}\right)$

that is $\int^\infty \frac{1}{u}\exp\left(-u - \frac{r^2}{4L^2u}\right)du = W\left(u, \frac{r}{L}\right)$

Equation (3.44) may be rewritten in the form as

$$s = \frac{Q}{4\pi T}\,W\left(u, \frac{r}{L}\right) \qquad (3.45)$$

LEAKY ARTESIAN AQUIFER WITH FULLY PENETRATING WELLS

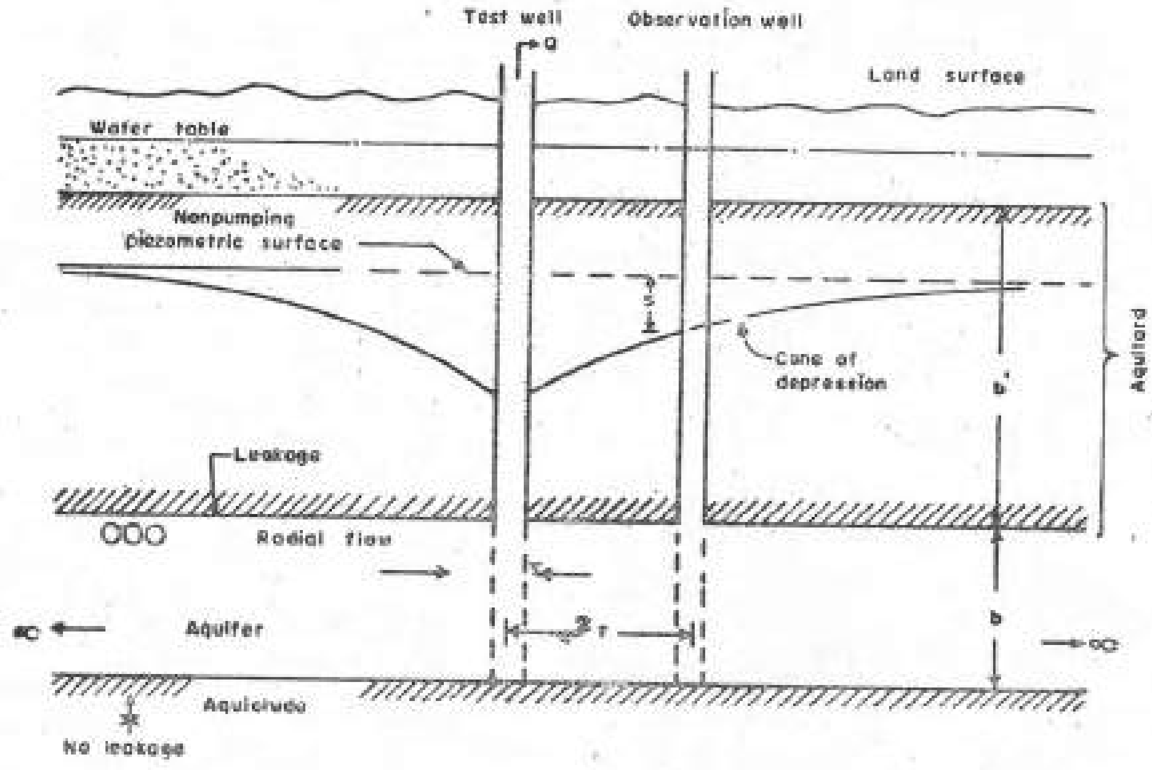


FIG. 3.2

FROM - Walton

Where $u = \dfrac{r^2 S}{4Tt}$ (3·46)

$$\frac{r}{L} = \frac{r}{\sqrt{T/(K'/b')}} \quad (3\cdot47)$$

s = drawdown (L)

r = distance from pumped well to observation point (L)

Q = discharge, (L^3/T)

t = time after pumping started (T)

T = coefficient of transmissibility (L^2/T)

S = Co-efficient of storage, fraction.

K' = co-efficient of vertical permeability of aquitard*(L/T)

b' = thickness of the aquitard, (L)

W(u, r/L) is read as the "well function for leaky artesian aquifers with fully penetrating wells without water released from storage in aquitard and constant discharge conditions". Values of W(u, r/L) in terms of the practical range of u and r/L given by Hantush (1956) are presented in Annexure-V.

When discharge is balanced by leakage and water levels stabilise at permanent stages, a steady state exists. The differential equation in polar, coordinate notation governing the steady-state radial flow in the leaky artesian aquifers is (Jacob, 1946) :

$$\frac{\partial^2 s}{dr^2} + \frac{1}{r}\frac{\partial s}{\partial r} - \frac{S}{L^2} = 0 \quad (3\cdot48)$$

For solving the above differential, the production well is replaced by a mathematical sink of constant strength. With boundary conditions $s \quad s \to o$ as $r \to \infty \quad t \geqslant 0$

$$\lim_{r \to O} r\frac{\partial s}{\partial r} = -\frac{Q}{2\pi T}$$

and the initial conditions $s = o$ for $t \leqslant o$ and for all values of r, the solution to the problem given by Equation (3·48) is (Staggewentz and Van Nos. 1939) :

$$s = \frac{Q}{2\pi T} K_o \left(\frac{r}{L}\right) \quad (3.49)$$

here $\dfrac{r}{L} = \dfrac{r}{\sqrt{(T/K'/b')}}$

s = drawdown (L)

r = distance from pumped well to observation point (L)

Q = discharge (L^3/T)

T = coefficient of transmissibility, (L^2/T)

K' = co-efficient of permeability of aquitard, (L/T)

b' = thickness of aquitard (L)

Values of Ko (r/L) in terms of the practical range of r/L are presented in Annexure-IV. The co-efficient of storage cannot be computed under steady-state conditions because, under such conditions of flow, the entire yeild of the well is derived from only leakage sources.

3·5·4 Water Table Aquifers

Consider a well fully penetrating water table aquifer underlain by an aquiclude. The aquifer is homogeneous, isotropic infinite in areal extent, and is of the same thickness throughout. Drawdown is assumed to be very small in comparison to the original saturated thickness of the aquifer (Fig. 3·3)

When a well in such an aquifer is pumped, water is continuously withdrawn from storage within the aquifer as the cone of depression progresses radially outward from the well. Because of the absence of a source of recharge, there can be no steady state flow, and the head in the aquifer will continue to decline as long as the aquifer is effectively infinite. However, the rate of decline of head continuously decreases as the cone of depression spreads. Water is released from storage by the gravity drainage of the interstices in the portion of the aquifer drained by the pumping and by the compaction of the aquifer and the expansion of the water. The gravity drainage of water through stratified sediments is not immediate; the coefficient of storage appears to vary and to increase as a diminishing rate with the time of pumping.

Un-confined stratified sediments often react to pumping for a short time after discharge starts as would an artesian aquifer. Gravity drainage of water through stratified sediments is not instantaneous but water is released instantaneously from storage by the compaction of the acquifer and by the expansion of water itself. After a short initial period, the cone of depression slows in its rate of expansion as water released from storage by gravity drainage of sediments reaches the cone. Finally, the rate of expansion of the cone of depression increases and the cone continues to expand as gravity drainage keep space with declining water levels (Walton, 1962).

If the delayed gravity drainage were absent the differential equation in polar coordinate notation governing the unsteady-state flow in the water table aquifer would be (Boulton, 1954)

WATER-TABLE AQUIFER WITH FULLY PENETRATING WELLS

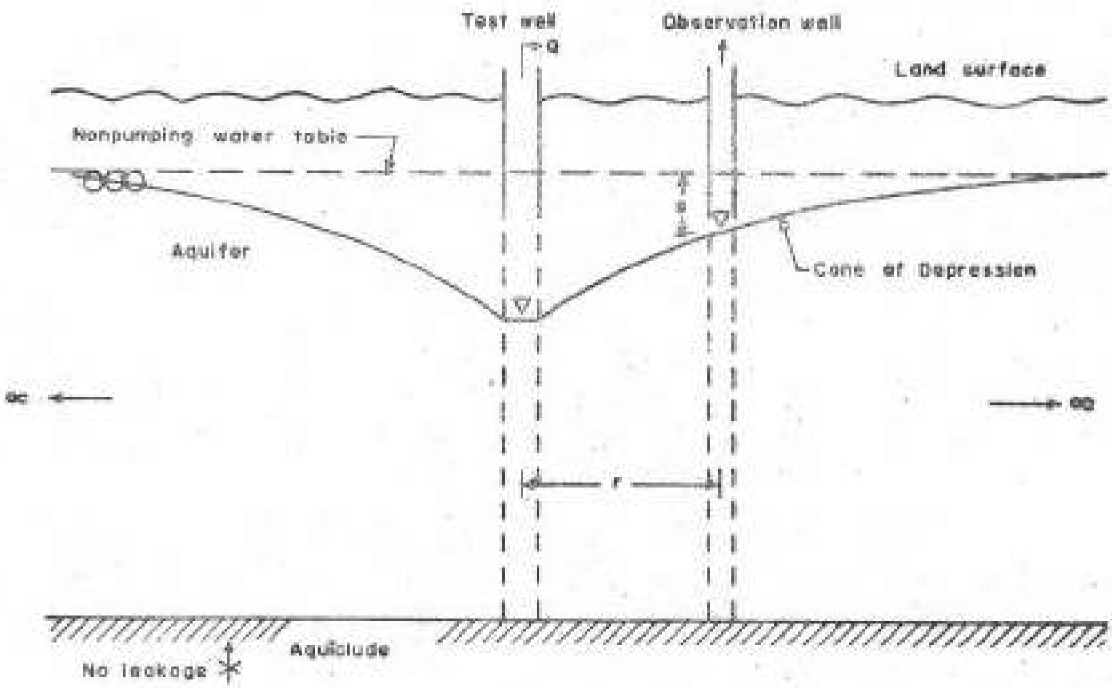


FIG. 3.3

FROM - Walton

$$T\left(\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}\right) = S\frac{\partial s}{\partial t} \qquad (3\cdot 50)$$

The amount of water derived from storage, due to an increment of drawdown Δs between times τ and $\tau + \Delta\tau$ after pumping started, consists of two components: (1) a volume of water instantaneously released from storage per unit horizontal area, Δs S, is the co-efficient of storage as defined for artesian conditions and (2) a delayed yield from storage, per unit horizontal area, at any time t ($t > \tau$) from the start of pumping $\Delta s \alpha S_Y e^{-\alpha(t-\tau)}$ (α is an empirical constant).

The total volume of delayed gravity drainage per unit area per unit drawdown is

$$\alpha S_Y \int_{\tau}^{\infty} e^{-\alpha(t-\tau)}\, dt = S_Y \qquad (3\cdot 51)$$

the effective co-efficient of storage under water table conditions is $S+S = NS$ and the time rate of delayed gravity drainage per unit area at time t from equation (3·51) is

$$\alpha S_Y \int_0^t \frac{\partial s}{\partial \tau} e^{-\alpha(t-\tau)}\, d\tau \qquad (3\cdot 52)$$

Then the differential equation governing the unsteady state flow to the fully penetrating well in the water table aquifer in polar coordinate notation, taking into account the delayed yield also, is (Boulton, 1963)

$$T\left(\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}\right) = S\frac{\partial s}{\partial t} + \alpha S_Y \int_0^t \frac{\partial s}{\partial t} e^{-\alpha(t-\tau)} d\tau(t=\tau) \qquad (3.53)$$

With the boundary conditions $s \to O$ as $r \to \infty$ for $t > O$ and the initial condition $s = O$ for $t < O$, and for all values of r, the solution of the problem given by Eq. (3·53) is (Boulton, 1963)

$$s = \frac{Q}{4\pi T}\int_0^{\infty}\frac{2}{x}\left\{1-e^{-u_1}\left(\mathrm{Cosh}\,u_2+\frac{\alpha N(1-x^2)}{2u_2}\mathrm{Sinh}\,u_2\right)\right\} J_0\left(\frac{r}{B}\right) dx. \qquad (3.54)$$

where

$$u_1 = \frac{\alpha t N(1-x^2)}{2}$$

$$u_2 = \frac{\alpha t\sqrt{N^2(1+x^2)^2-4N x^2}}{2}$$

$$x = \sqrt{\frac{N-1}{N}} = \sqrt{\frac{S_Y}{S+S_Y}}$$

$$B = \sqrt{\frac{T}{\alpha S_Y}}$$

$$N = \frac{S+S_Y}{S}$$

To cover the useful field of finite N values would require very extensive tabulation. For the case when N is large and tends to infinity, Eq. 3·54 can be reduced to (Boulton, 1963)

$$s = \frac{Q}{4\pi T}\int_0^{\infty} 2J_0\left(\frac{r}{B}\right)\left[1-\left(\frac{1}{x^2+1}\right)\exp\left(-\frac{\alpha t x^2}{x^2+1}\right)-F\right]\frac{dx}{x} \qquad (3.55)$$

where $F = \frac{x^2}{x^2+1}\exp[-\alpha Nt(x^2+1)]$

The function F vanishes when $t > O$, but is finite as t approaches zero and Nt approaches a finite value Thus, for small values of t, Eq. (3·55) reduces to. (Boulton, 1963)

$$s = \frac{Q}{4\pi T}\int_0^{\infty} 2J_0\left(\frac{r}{B}x\right)\left(\frac{x^2}{x^2+1}\right)\{1-\exp[-\alpha Nt(x^2+1)]\}\frac{dx}{x} \qquad (3\cdot 56)$$

With the integral in Eq. (3·56) expressed symbolically as $W(u_{ay}, r/B)$, that is

$$W\left(u_{ay},\frac{r}{B}\right) = \int_0^{\infty} 2J_0\left(\frac{r}{B}x\right)\left[1-\frac{1}{x^2+1}\exp\left(-\frac{\alpha t x^2}{x^2+1}\right)-F\right]\frac{dx}{x}$$

Eq. 3·55 may be written in the form as (Prickett, 1960).

$$s = \frac{Q}{4\pi T} W\left(u_{ay}, \frac{r}{B}\right) \qquad (3\cdot 57)$$

$$\text{where } u_a = \frac{r^2 S}{4Tt} \text{(applicable for small values of t)} \qquad (3\cdot 58)$$

$$u_Y = \frac{r^2 S_Y}{4Tt} \text{(Applicable for large values of t)} \qquad (3\cdot 59)$$

$$\frac{r}{B} = \frac{r}{\sqrt{T/\alpha S_Y}} \qquad (3\cdot 60)$$

$$\alpha = \frac{(r/B)^2\, 1/u_y}{4t} \qquad (3\cdot 61)$$

$$u_Y = \frac{u_a}{N-1} \qquad (3\cdot 62)$$

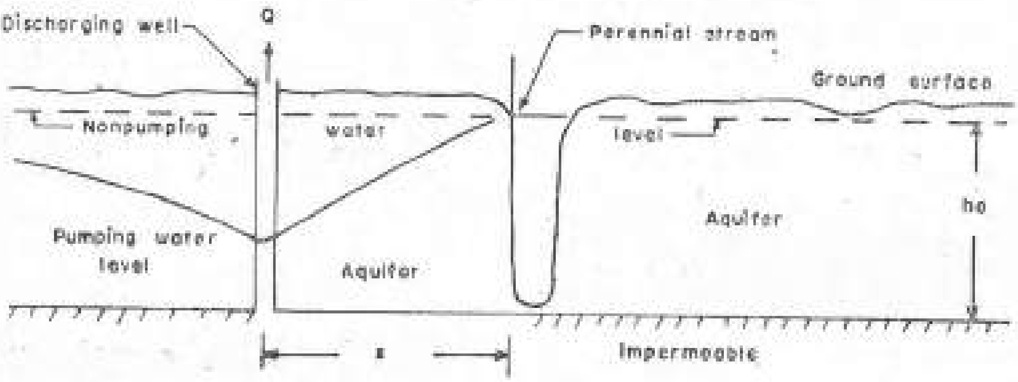
$$N = \frac{S+S_Y}{S} \qquad (3\cdot 63)$$

s = drawdown, (L)

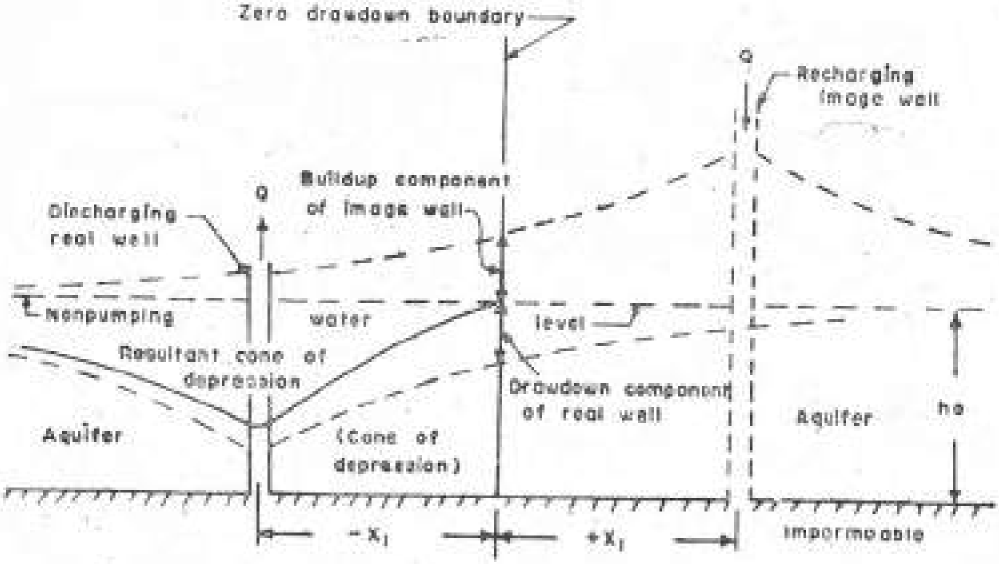
r = distance from pumped well to observation point, (L).

FIG. 3.4

SECTIONAL VIEWS OF (A) DISCHARGING WELL NEAR A PERENNIAL STREAM AND (B) THE EQUIVALENT HYDRAULIC SYSTEM AQUIFER OF INFINITE AREAL EXTENT



(A)



(B)

Aquifer thickness should be very large compared to resultant drawdown near real well.

FROM- Ferris et. al (1962)

Q = discharge, (L^3/T)

t = time after pumping started (T)

T = coefficient of transmissibility, (L^2/T)

S = Co-efficient of storage under artesian conditions (Fraction)

S_Y = Co-efficient of storage associated with gravity drainage of interstices, fraction (commonly referred to as the specific yield)

$\propto$ = reciprocal of delay index, $(T)^{-1}$

B = drainage factor (L)

$W(U_{ay}, r/B)$ is read as the "well function for water table aquifer with fully penetrating wells". Values of $W(U_{ay}, r/B)$, given by Boulton (1963), in terms of the practical range of u_a, u_y and $\frac{r}{B}$ are presented in Annexure VII.

3.6 Flow near Boundaries

In the previously described well flow equations, the aquifers were assumed to be infinite. Near an aquifer boundary the radial flow equations no longer hold. However, generally these equations are quite good except where the well is very near to the aquifer boundary.

Where an aquifer boundary is important, that is, where a well is close to the boundary, then the method of images is helpful for obtaining solutions of well flow problems. An image is an imaginary well introduced to createa hydraulic flow system which will be equivalent to the effect of a known physical boundary on the flow system. By this method aquifers of finite extent can be transformed to one's of infinite extent, so that the previous equations can be applied.

As an example of the problem of well flow near boundaries, one might consider the case of a well flowing near stream (see fig. 3·4). It is assumed that the stream is in hydraulic connection with the aquifer, then pumpage of the well will induce flow from the stream into the well. In order to solve this problem an image well is introduced on the opposite side of the stream and equal to the distance from the stream that the pumping well is located from the stream. Thus, a pumping well on one side and a recharge well on the other side of the stream provide for a line of constant elevation along the stream itself. This elevation represents the head of water in the stream. The drawdown at any point near the pumping well can be computed by determining the drawdown resulting from the pumping well and the increase in head resulting from the recharge image well on the other side of the stream. Fig. 3·4(b) illustrates how the sum of the two curves produces the desired resultant.

If a well is pumping near an impermeable boundary the hydraulic condition required is that there be no flow through this boundary. Thus a zero gradient is necessary. This zero gradient can be obtained by introducing an image pumping well on the other side of the aquifer boundary and equidistant from the boundary as the pumping well. Again, the sum of the two individual well drawdowns will give the correct drawdown at any point near the pumping well in the aquifer.

The method can be extended to other more complicated boundary problems by introducing appropriate image wells as necessary to represent any combinations or positions of permeable and impermeable aquifer boundaries.

The method of images can also be applied to locate an aquifer boundary where no information from field studies is available. This common field problem can be solved using pumping test data and applying the method of images and the non-equilibrium equation. Consider the situation of a pumping well and an observation well located near an unkown impermeable aquifer boundary. As described previously, and as shown in Fig. 3·5, an image pumping well furnishes the equivalent hydraulic flow system. It can be shown from the non-equilibrium equation that

$$\frac{r_p^2}{t_p} = \frac{r_i^2}{ti} \qquad (3\cdot64)$$

where r_p and r_i are identified in fig. 3·5. The value t_p is the time since pumping began to any selected drawdown before the boundary affects the drawdown. The time t_i is the time since pumping began to when the divergence of the drawdown curve from the curve equals the selected drawdown. The unknown distance r can be found from knowing the values of r_p, t_p and t_i. Where no information is available about a boundary, three observation wells are necessary to locate the direction and position of the boundary. The common intersection of three arcs having radii of the computed values of r_i indicate the location. The boundary would then lie along the perpendicular bisector of the line connecting the pumping well and the common intersection point.

3·7 Multiple Well Systems

Where two or more wells are located in the same aquifer and are close, the cones of depression of the individual wells may intersect. This intersection is known as interference of wells. Wells located very near each other may seriously interfere with one another in that the drawdowns are increased leading to higher pumping lifts and costs. A proper design of a well field attempts to provide the most economical pumping system, taking into account the effects of interference.

The drawdown resulting from a group of pumping wells is equal to the sum of the individual well drawdowns at a given point. Thus, for a specified location, the total drawdown is given as

$$s = s_a + s_b + s_c + \cdots\cdots + s_n \qquad (3\cdot65)$$

where s_a, s_b, s_c etc. are the drawdowns at the given location resulting from the discharge of wells, a, b, c etc. respectively. Well field drawdowns can be

WELL LOCATIONS FOR LOCATION OF AN IMPERMEABLE AQUIFER BOUNDARY

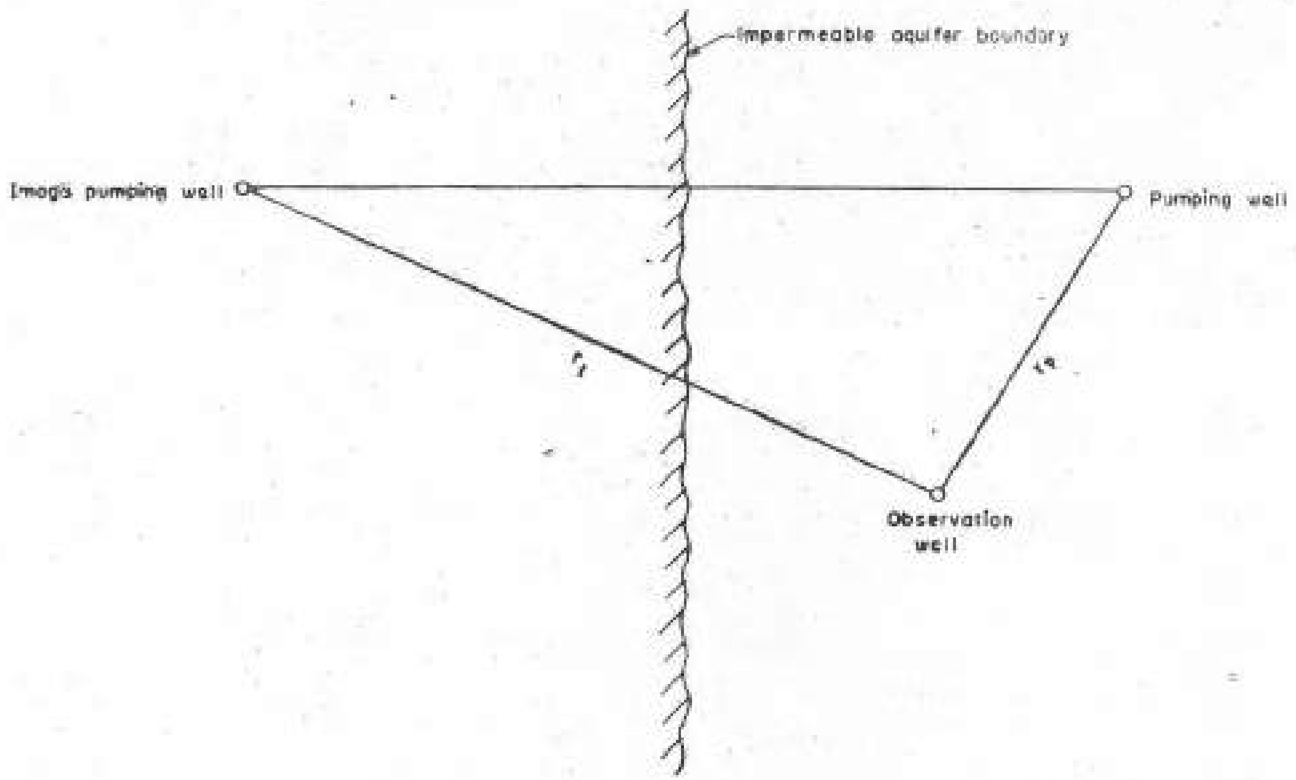


FIG. 3.5

determined by this method, using either the equilibrium or non-equilibrium equations. For an approximate steady state condition in a confined aquifer having n pumping wells, drawdown s given as

$$s = h_s - h = \sum_{i}^{n} \frac{Q^i}{2\pi Kb} l_n \frac{R_i}{r_i} \quad (3\cdot66)$$

where h_s -h_0 is the drawdown at a specified location, Ri is the distance from the $_i$th well to a point at which the drawdown becomes negligible, and r_i is the distance from the $_i$th well to the given point.

Simplified equations for drawdowns of fixed geometrical patterns of well field layouts are available in standard text books. This method presupposes that the drawdown is small in relation to the saturated thickness, if it is to be applied to an unconfined aquifer.

3.8 Partially Penetrating Wells

A partially penetrating well is one having a length of water entry which is less than the thickness of the aquifer. This is a common field occurrence because drillers when striking a satisfactory aquifer frequently make no effort to extend the well down to the bottom of the formation; in such cases the flow equation for fully penetrating wells is modified for partially penetrating wells because of the vertical convergence near these wells. As a result, the drawdowns in a partially penetrating well will be greater for the same discharge as compared to a fully penetrating well. And conversely, the discharge of a partially penetrating well will be less than that of a fully penetrating well for the same drawdowns.

The drawdown of a partially penetrating well in a confined formation can be specified by

$$h_o - h_w = \frac{Q_0}{2\pi K}\left[\frac{1}{h_s} l_n \frac{\pi h_s}{2r_w} + \frac{0\cdot10}{b} - \frac{1}{b} l_n \frac{r_o}{2b}\right] \quad (3\cdot67)$$

where h_o is the head at the radius of influence r_o, h_w is the head at the edge of the well having a radius r_w, Q_0 is the discharge of the partially penetrating well, h_s is the depth of penetration of the well, and b is the thickness of the saturated formation. If this equation is divided by the drawdown equation for a fully penetrating well, the ratio of yield for the two wells for the same drawdown can be obtained. For unconfined aquifers, the same equation can be applied by replacing b by H, the saturated thickness of the formation, and if the drawdown is small in relation to the saturated thickness.

3·9 Well losses

Well losses are losses in head which occur in the immediate vicinity of a well and are caused by flow through the well screen and by flow inside of the well to the pump intake. Because the well loss is associated with turbulent flow, it is proportional to an $_n$th power of the discharge where n>1 and probably less than 2. An exact value for n cannot be stated because of the variation among individual wells.

The total drawdown s_w at a well includes the drawdown involved in the flow through the aquifer plus the well loss itself. Thus the drawdown can be expressed by

$$sw = \frac{l_n \frac{r_o}{r_w}}{2\pi Kb} Q + CQ^n \quad (3\cdot68)$$

where C is a constant depending upon the radius, construction and condition of the well. Well losses can be minimized by building large diameter wells, by maintaining wells in good condition at all times, and by employing low pumping rates.

Specific capacity of a well is its discharge divided by its drawdown. Consideration of the last equation indicates that the specific capacity of a well is not constant as is sometimes assumed but rather decreases with increasing discharge. Writing a similar equation, it can be shown that the specific capacity also depends upon the time of pumping. A high specific capacity indicates a good yielding well located in a permeable formation. It is apparent that the higher the specific capacity the more efficient the well is in yielding water supplies from underground.

3·10 Flow to Non-Penetrating Wells (Cavity Wells)

Cavity wells are tubewells which, being without strainers, draw their supplies from one aquifer only. A cavity well does not go very deep and requires a very hard clayey stratum to form a strong and dependable roof over the cavity. The stratum immediately on top of the water bearing stratum, in which the cavity is proposed to be developed, is known as the roof of the cavity (Fig. 3·6).

The stability of the well and width of the cavity depend on the shearing strength of the clay constituting the roof of the cavity. The clay should normally be very stiff (unconfined compressive strength, qu>2 kg/cm²) and of non-disintegrating type. Even a 3m thick layer of good stiff clay will be sufficient to withstand the overlying weight without collapsing for discharges upto 0·45 m³/sec. In case of less stiff clays, 8m thick roofs have been found to be stable.

3·10·1 Steady State Flow to a Cavity Well in a Non-leaky Artesian Aquifer.

The general form of Laplace's equation in spherical coordinates (r, θ, ø) with origin at the centre of the sphere can be written as

$$\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{dh}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial h}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 h}{\partial ø^2}\right] = O \quad (3\cdot69)$$

where h is the piezometric head, and K is the hydraulic conductivity. For symetrical flow about vertical axis, Equation (3·67) reduces to

$$K\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial h}{\partial r}\right)\right] = O \quad (3\cdot70)$$

STEADY-STATE FLOW TO A CAVITY WELL IN A NONLEAKY ARTESIAN AQUIFER

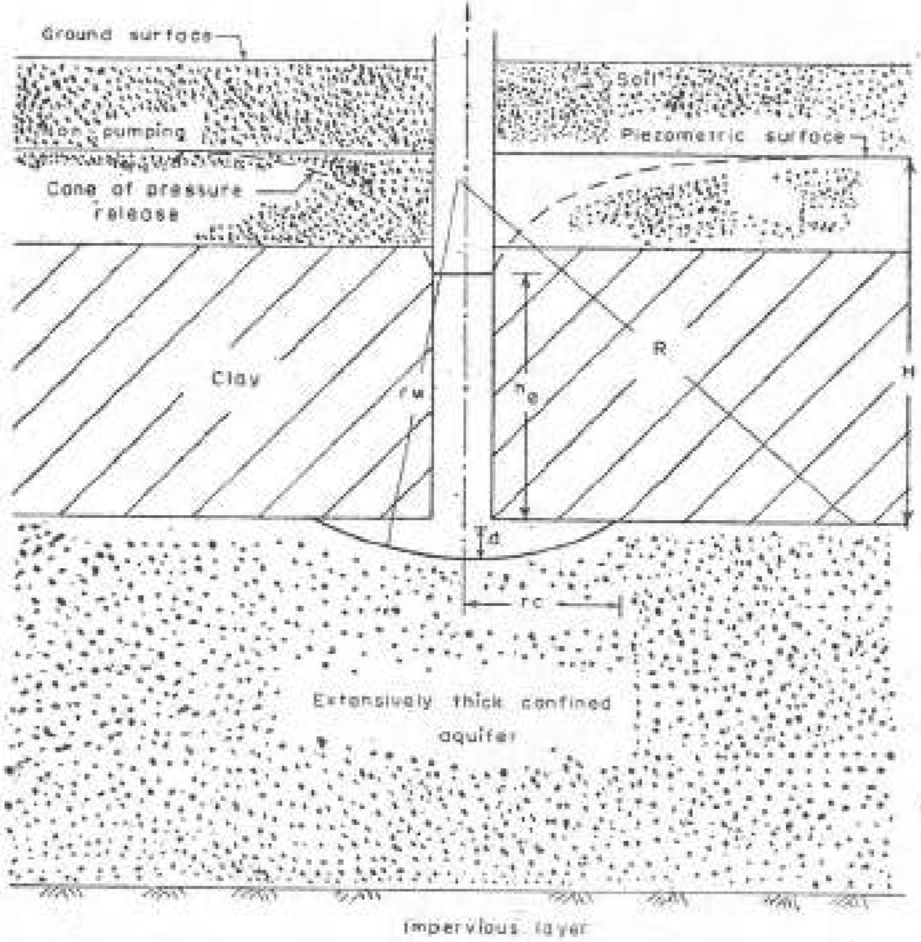


FIG. 3.6

Assuming the cavity to be segment of a sphere of radius r_w resting on the top of a confined, homogeneous and isotropic aquifer of infinite areal extent and extensive thickness, Equation (3·70) can be utilized to obtain steady state solution of a cavity well. For the boundary conditions as given below ;

$$h(r_w) = h_c \qquad (3\cdot71\ a)$$

$$h(R) = H \qquad (3\cdot71\ b)$$

$$K\frac{h}{r} = \frac{Q}{\pi d r_w} \qquad (3\cdot71\ c)$$

Solution of Equation (3·70) can be obtained as a relationship between drawdown (H-h_c) and the cavity well discharge Q

$$Q = 2\pi\, dk\,(H-h_c)/(1-\frac{r_w}{R}) \qquad (3\cdot72)$$

where, d = depth of cavity and

R = radius of influence measured from the centre of the spherical cavity.

Field study of failed cavity wells has shown that the radius of the cavity is usually of the order of 6 to 8m and depth is hardly about 30 cm ro so.

3·10·2 Unsteady State Flow to a Cavity Well

Kanwar and Khepar, Chauhan et al and Jaiswal et. al have studied the hydraulics of cavity wells for unsteady state flow conditions. Assuming a hemispherical cavity and making assumptions similar to those for unsteady state radial flow, the system can be described by the following equation.

$$\frac{\partial^2 h}{\partial r^2}+\frac{2}{r}\frac{\partial h}{\partial r} = \frac{1}{K}\frac{\partial h}{\partial t} \qquad (3\cdot73)$$

The solution of Equation (3·7) gives the following relationship for a cavity wells

$$s = H-h = \frac{Q}{2\pi Kr}\,\mathrm{erfc}\frac{r}{\sqrt[4]{K_o\ t}} \qquad (3\cdot74)$$

erfc(u) may be expended in a series and neglecting terms of higher order the Equation 3·74 reduces to,

$$s = \frac{Q\sqrt{S_s}}{4\sqrt{\pi}\,(K)^{3/2}}\ t^{-\frac{1}{2}} + \frac{Q}{2\pi Kr} \qquad (3\cdot75)$$

where, s = drawdown in the observation pipe placed at a distance r from the cavity well.

Q = constant well discharge

K = permeability of aquifer material

t = time since pumping began, and

S_s = specific storage coefficient (i.e., for unit aquife thickness)

Equation 3·75 can be used to determine the formation constants as indicated below :

If drawdown data are taken in a single observation pipe placed at a known distance r, from the cavity well, this equation takes the form of a straight line as all other terms in the equation are constant.

$$s = a\,t^{-\frac{1}{2}} + C. \qquad (3\cdot76)$$

where $a = \dfrac{Q\sqrt{S_s}}{4\sqrt{\pi}\ K^{3/2}}$

$$C = \frac{Q}{2\pi\ Kr}$$

Therefore, if a plot of observed drawdowns and the corresponding square root of time, i. e. (t) are plotted on a simple arithmatic paper, it will give a straight line and the values of constants a and c can be easily found out fromthe slope and the intercept of the straight line respectively. Once the values of the constants a and c are found, the values of K and Ss can be worked out. The values of T and S can thereafter be calculated by multiplying the values of K and S_s by the thickness of the aquifer respectively.

The mathematical model developed by Jaiswal et al 1977(a) for flow to a non-penetrating well and techniques developed may be used for determination of aquifer parameters S, T and depth of aquifer b The boundary value problem and the relatioship for flow to a non-penetrating well with hemispherical bottom of vanishing radius discharging at constant rate (Fig. 3·6) may be reproduced below, as

$$\frac{\partial^2 s}{\partial \rho^2} + \frac{2}{\rho}\frac{\partial s}{\partial \rho} = \frac{\partial s}{\partial t}$$

$$s(\rho, o) = o \quad t<0 \qquad (3\cdot77)$$

$$s(\infty, t) = o \quad t>o$$

$$\mathrm{Lim}\longrightarrow O\ (\rho\frac{2\partial s}{\partial \rho}) = -\frac{Q}{2\pi k} \quad t>o$$

where, s is drawndown at radial distance ρ and time t, l is radial distance in spherical coordinate system, $\alpha = S/T$ and Q is constant rate of flow, S is storage coefficient and T is transmissivity. The solution was obtained for finite thickness of aquifers using the method of images as,

$$s = \frac{Qb}{2\pi Tr}\ C(\sqrt{u},\ r/b) \qquad (3\cdot78)$$

where,

$$C(\sqrt{u},\ r/b) = \mathrm{erfc}\ (\sqrt{u}\,) + \sum_{n=1}^{\infty} \frac{2\ \mathrm{erfc}\ (\sqrt{1+(2nb/r)^2}\ \sqrt{u})}{\sqrt{1+(2nb/r)^2}}$$

$$u = r^2 S/4\,Tt$$

r = distance of observation well from the centre of the well, in cylindrical coordinate.

b = Thickness of the aquifer,

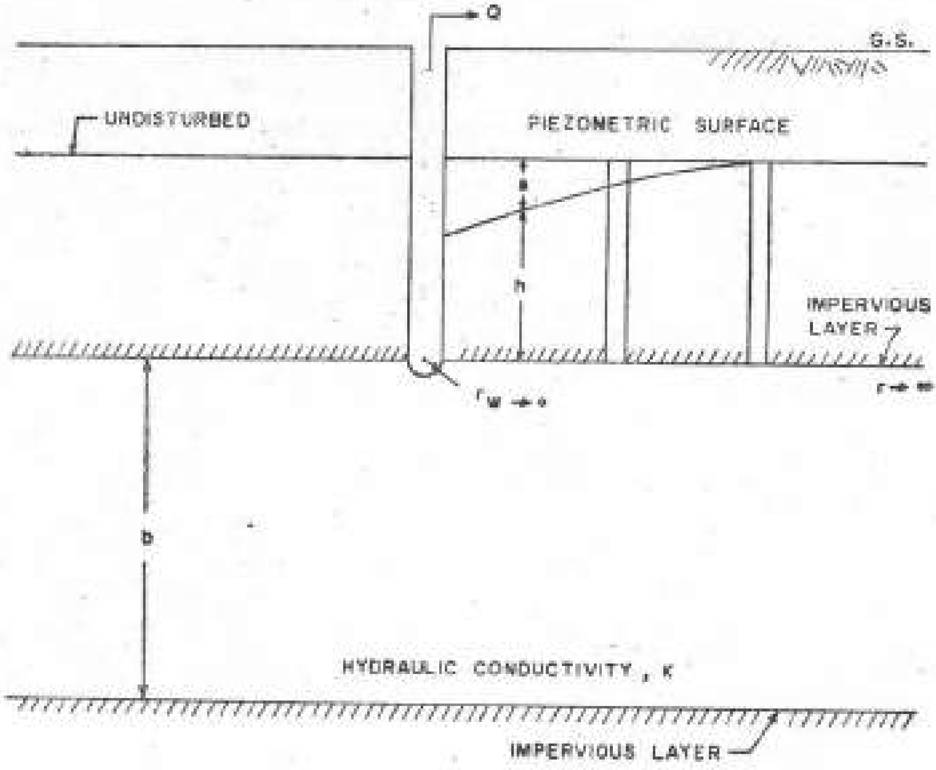
erfc = Complementary error function.

Equation (3·78) was developed for non-penetrating well and holds good for observation well at the points just below the upper confining layer.

If the pump test data, i. e. drawdown in an observation well with time with constant rate of pumping is available then aquifer parameters may be determined using equation (3·78) .

FIG. 3.7

DEFINITION SKETCH OF THE FLOW SYSTEM



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4. AQUIFER TEST-PROCEDURE FOR DESIGN AND OBSERVATION

4·1 Introduction

An aquifer test is designed to impose a hydraulic stress on the aquifer in such a way that measurements of the response to the stress will fit in a theoretical model of aquifer responses. The test setup consists of a test well in which the aquifer is subjected to stress and one or more observation wells in which the response is measured. By conducting such tests are determined :

(1) Hydraulic characters of aquifer and the confining beds

(2) The possible influence and nature of aquifer boundaries.

The most useful parameters derived by aquifer tests are the aquifer constants obtained from the analysis of observed data which make it possible to predict the maximum production of water from the aquifer and subsequent effects of such production.

Properly and correctly interpreted data of well tests are among the most useful information which can be used to plan groundwater development programmes. A lot of care is needed in following a proper procedure for the design of aquifer test especially with regard to the selection of a suitable site, construction of test and observation wells and systematic conducting of the test and recording of observations.

As a first step towards determination of aquifer constants in a selected area, full existing hydrogeological information pertaining to the aquifers and confining layers should be collected, dealing with their nature, thickness and shape, boundary condition of the flow regime and the available estimates of aquifer constants. Such data may include dip and strike of aquifers, their configuration with respect to source of recharge and discharge, slope of the water table and piezometric surface etc. From the collected data, if possible interpretation should be made as to whether the aquifers are confined, unconfined or leaky, and whether they are extensive or bounded.

4·2 Designing of Aquifer Test

Designing of an aquifer test is discussed below under the following heads:

(i) Selection of site

(ii) Construction of test well

(iii) Construction of observation well

4·2·1 Selection of Site

For determination of aquifer constants, a site should be selected which is representative of as wide an area as possible; and as far as possible it should be away from Railway lines and Roads where heavy traffic might produce fluctuations in the piezometric surface of a confined aquifer. Further, the location of site should be such that pumped water can be safely discharged to a sufficiently great distance through pipes or lined channels so that no percolation of discharged water occurs in the zone overlying the cone of depression created in the unconfined aquifers. Further more, the site should be as far away as possible from an existing pumped well, from unlined distributaries and canals and other sources of recharge or discharge. In any case, the distances to all such sources, and their operational schedules should be measured and recorded.

4·2·2 Construction of Test Wells

Wells for aquifer tests should be of as small in diameter as practicable, in order that water in storage in the well is negligible in comparison to the water produced from the aquifer during the test, and the diameter is sufficient to pump water at the desired rate.

The test well should be properly constructed in order that the stress in the aquifer immediately outside the casing is reflected in the water level changes in the well. Mud rotary wells should be fully developed. It should also be ensured that the gravel packing and screen opening should be adequate to allow desired pumping rates without much entrance and other losses.

As far as possible, testwells should be constructed separately for confined and unconfined aquifers. Alternatively, a combined well tapping both the aquifers can be constructed with a cement seal in the confining clay outside the well. As far as possible fully penetrating wells tapping the full thickness of the aquifer should be constructed. In a conbined test well, while testing, the confined aquifers are temporarily isolated for testing the unconfined aquifer. The lower part of the test well is filled by sand through a pipe upto 5—8 m depth above the top of the slotted casing installed against the Ist confined aquifer, then by 3-5 m thick layer of clay in the depth range of the confining bed. Water from the confined aquifer cannot mix with the water from the unconfined aquifer through the gravel pack owing to the presence of cement there. After testing the unconfined aquifer, the well can be cleaned of clay and sand and tested cumulatively.

After the test well is fully developed, a preliminary pumping test should be made to use the data for planning the final aquifer test. If possible, one should make reasonable assumptions regarding the relationship between hydraulic conductivity and distribution of grain size of aquifer material. Also from the lithological log, value of 'S_y' may be estimated for unconfined aquifer using grain size distribution as a basis for estimation (Johnson, 1966). For confined aquifers the range of 'S' is great, varying from 10^{-2} in the semi-confined aquifers to 10^{-5} or less in confined aquifers. In the absence of data, $S=3 \cdot 3\times10^{-6}\times b$ (Aquifer thickness in metres) would give an approximate value of 'S'. Both 'T' and 'S' estimated values should be substituted into an appropriate type curve solution in order to predict drawdown in the test well and observation wells. This will help in planning the installation of observation wells.

4. 2. 3 Construction of Observation Wells

Experience has established that a properly designed test which employs one or more observation wells has a higher chance of successfully predicting aquifer characteristics than one which relies entirely on the measurements in a pumped well even after the well losses are estimated. As far as possible, the observation well should tap the same or corresponding depth zone of the aquifer tapped in the test well. Preferably, observation wells may be screened at depth corresponding to the centre of the screened positions in the test well. Observation wells should be of small diameter in order to minimise the timelag in water level response due to storage in the well. Observation wells of 50 mm, 100 mm and 150 mm diameters are adequate for shallow, medium and deep depths. Larger diameter wells may be installed for observations by automatic recorders.

The distance of observation wells with respect to the test well should be chosen in such a way as to obtain good response curves. If the test is to provide accurate values of both 'T' and 'S', drawdown must be observed over both the steep and flat parts of the type curve. If only one observation well is to be installed, it should be located at such a distance that it provides the said desired response. However, where several observation wells are to be installed, they may be located in different directions at various distances so as to give greatest possible aerial coverage. In such a situation, the response should be plotted as t/r^2 or r^2/t for all the observation wells on one sheet of graph paper, which would provide complete definition of curve shape—the data from observation wells having small 'r' may plot in the flat, while others having greater 'r' may plot in the steep part. Further, if the pumped well is fully penetrating one, there is no restriction on distance of the observation wells; however, in case of partially penetrating test well, the nearest observation well may be placed at a distance which is at least 1·5 times the thickness of the aquifer.

In this connection, it is to be remembered that the shape of cone of depression is the guide to choose the distance of the observation wells. Lesser the permeability of the aquifer material, the smaller and steeper is the cone of depression and the higher the permeability, the larger and flatter is the cone, all other conditions being equal. On 'S' depends the speed of propagation of the cone of depression, the smaller the 'S', the faster the response at any given point. Thus, in aquifers with low 'S' values, the observation wells can be far removed from the test wells. In highly permeable water table aquifers the decline in the water levels in observation wells located at great distances from the test wells may be negligible.

In general, the number of observation wells, their depths and distances from the test well should be determined according to the geological and hydrological conditions at the site interpreted during drilling and construction of test well. With the use of several observation wells, one can analyse the data both by drawdown Vs time plot and drawdown Vs distance plot and can greatly increase the reliability of the determined aquifer parameters. When confined or semi-confined conditions are suspected, water level observation should be made both in wells tapping confined and unconfined strata. The study of drawdown in both such wells will allow interpretation of leaky effects and vertical flow so that proper formula is employed in analysis. In unconfined aquifers, regular observation wells may be installed alongwith shallow observation wells tapping near water table zone only which would provide response of water table on pumping. A "Piezometer nest" can be constructed to measure several aquifers response at a given location. If aquifer boundaries are to be located, at least 3 observation wells are required.

It is very essential to mention here that before conducting the aquifer test, all the observation wells should be tested for response time by injecting a "slug" of water, about 10 litres. The water level should return to near static condition in two to ten minutes if the well has adequate response time. This will help in knowing if the wells are porperly developed or not.

4. 3 Preparation of Site Plan

A site plan to a suitable scale should be prepared showing the location of test and observation wells, other pumping wells, and all sources of ground water recharge and discharge that are likely to effect the tests. Geological features like dykes should also be shown. The distance from the centre of the test well to the centre of the observation wells and the well directions should be recorded.

4. 4 Observations during Aquifer Test

4. 4. 1 Pre-pumping Preparation and Observations

After completing the preliminary tests, all the wells should be allowed to recoupe completely. Before attempting an aquifer test, it is very essential to understand the natural [conditions of flow system and response to the existing natural condition. In order to establish water level trend and to correct the observed drawdown due to pumping, periodic monitoring of water levels at intervals 1-2 hrs., prior to actual test need to be done for 3-5 days. A plot of regional trend, by drawing median line through the water level sinus curve should be made and the same should be extrapolated to the test period to enable corrections to be made in the test measurements. In case it is not possible to stop a neighbouring well from its pumping schedule, observations as to its effect on the test and observation wells should be recorded at specific discharge, and the same schedule should only be continued and its effect taken into account.

In confined and semi-confined aquifers, water level changes in response to fluctuations in the barometric pressure should also be recorded. Similar observation of water levels should be recorded in response to tides where the test area is close to sea shore. A relationship may be worked out between the two and, if required, corrections may be incorporated in the head observed during the test.

4.4.2 Observations during Test

These observations mainly include measurement of water levels, discharge rate and temperature of water pumped. All water level measurements should be done with reference to measuring points which should be clearly defined stable points. All water level measurements subsequently should be related to a common datum like general land surface or mean sea level.

For water level measurements, steel tapes graduated upto 1 mm interval and coated with chalk provide a reasonably accurate method of measurements. Continuous records of water level can also be obtained by using automatic water level recorders especially in wells where change in water level is small and slow. However, care is required to be taken to see that float is free of friction from the casing. Electrical sounding tapes may also be used if frequency of measurement is high. Air pressure lines are also useful in wells where heavy drawdowns are expected but the measurements are not accurate. Accuracy of field pressure gauge needs to be determined before using the same.

Since it has been suggested that observation wells should be of small diameter, so particular care is required for not using large weights on tapes which otherwise cause displacement of water causing large errors in measurements.

The method to be adopted for measurements of discharge should be decided upon and arrangements made accordingly. The usual methods are volumetric method, rectangular and 'V' notch, manometer tube and circular orifice weir and jet method—which is least accurate. The method should be appropriate for the discharge rate. The circular orifice weir method is the most common method used to measure the rate of discharge. The orifice is a perfectly round hole in the centre of a circular steel plate that is fastened to the outer end of a level discharge pipe. A manometer tube is fitted in a 1/3 or 1/4 inch hole made in the discharge pipe exactly 61 cms from the orifice plate. The water level in the manometer tube represents the pressure in the discharge pipe when water is pumped through the orifice. Standard tables (Johnson, 1966) have been published which show the flow rate for various combinations of orifice and pipe diameter (Annexure I).

Before commencement of test, all observers for the test should ensure that their watches are synchronised correct to seconds. Stop watches can be used to directly record the time since pumping started or stopped in the initial stages.

An aquifer test consists of pumping of a test well at a constant discharge and shutting off of the pump after an interval of time, and recording of water levels in the test well and observation wells, both during pumping and after stoppage of pump. Depending on the field conditions, the test operations may be varied, e.g. a pump operating continuously on a well may be stopped and restarted after an interval and the responses obtained. If there are several wells continuously operating, one may be stopped and the responses noted in others.

Immediately after starting of pump or its stoppage the water levels fall or rise very rapidly and hence the water level measurements should be very rapid during this period in order to define the drawdown or recovery curves precisely. After about 100 minutes of pumping or stoppage, 25 measurements during each log cycle of time are adequate to define the drawdown curve with sufficient accuracy.

The following time intervals of measurement are recommended (TABLE 4·1) :

TABLE 4·1

Recommended time intervals for water level measurements

Time since pumping started/stopped (in minutes)		Time interval for water level measurements (in minutes)	No. of recordings	For temperature and conductivity measurements (for risk of saline water encroachment) (in hours)
From	To			
0	10	½ or 1	10—20	
10	20	2	5	
20	60	5	8	2
60	100	10	4	
100	300	20	10	
300	1000	50	14	4
1000	3000	100	20	8
3000	Shutdown	200		12

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If the water level fluctuates erratically or if sudden changes in trend occur, the general rule of measurements recommended may be changed and frequent measurements may be taken to define the trend. An important guide to the proper measurement interval will be the plot of drawdown Vs time which should be kept correct as the test proceeds.

During first 60 minutes, the timing of measurement should be accurate to nearest second. To achieve the desired accuracy, two observers should be posted at each well, during the early part of the test. Later, one observer can do the job. Stop watches should be used during early measurements but later synchronised watches can be used.

In addition to water level measurements, temperature measurements of the discharged water should be made periodically. This will be required to affect correction to permeability value if it is to be corrected to a standard temperature. Similarly, water samples should be collected at appropriate intervals and the conductivity noted to observe changes in the quality specially if the aquifers are overlain or underlain by aquifers with different quality of ground water, or if there is lateral change in the quality. Selected samples may be subjected to complete chemical analysis.

In general, the greater the pumping rate, the more responsive is the stress generated in the aquifer. While deciding the rate of pumping, the following points should be kept in mind :

(i) Pumping rate should be large enough to produce significant drawdown in the observation wells. The shorter the duration, the larger the pumpage rate required to effect significant drawdown in the observation wells.

(ii) The maximum rate of pumping should be considerably less than the capacity of the pump.

(iii) The resulting drawdown should be within the range of pump setting.]

(iv) Entrance velocity through the well screens should not exceed 3 cm/sec.

Before deciding about the rate of pumping, trial runs with varying discharges (step drawdown test) should be conducted and resultant response in the observation wells should be recorded. This will give a fairly good idea about the rate required for conducting the test. As the stress to the aquifer has to be applied at a constant rate, the discharge in the pumped well has to be maintained constant. It is worth while to mention here the common cause of fluctuation in discharge so that continuous monitoring of pumping rate is done to avoid such changes:

(i) Pumping rate will tend to decline as pumping lift increase with increased drawdown.

(ii) Internal combustion engine will change their rate of operation in response to change in temperature.

(iii) Electric motor will change the rate of rotation in response to fluctuation in voltage.

If pumping rate changes by 10% or more during the test owing to accidental causes or negligence, the result of drawdown may contain sufficient error to vitiate the test. All changes in the disharge rate should be promptly recorded so that water level changes may be accounted for. In such cases, it may be necessary to rely on recovery data. Under conditions of erratic discharge rate, a reasonable estimate of average pumping rate should be made and only recovery data should be used for computation.

If the test well used is a large diameter well, say a dug well, the diameters at various depths in the zone of water level fluctuation should be recorded to account for contribution of water from the storage in the well.

4.5 Recording and Plotting of Data

Data collected during aquifer test should be recorded in the type of Form 4.1. Erratic water level measurements may be due to errors of measurement or fluctuations in pumping rate. These need to be corrected by immediate remeasurement.

For measurement taken in the test well, errors due to partial penetration, vertical flow and entrance losses need to be corrected. In all the wells involved in the testing, errors in water level due to regional water level trend and barometric effects should be nullified.

Plots of log t Vs s should be prepared as the test proceeds. Such plots will give an indication whether it is a case of non leaky artesian aquifer, or leaky confined aquifer. In unconfined aquifers such plots will indicate if delayed yield phenomena is occurring and how long pumping needs to be continued.

Distance drawdown plot should also be made and quantity of material dewatered in the cone of depression for 1000, 2000, 3000 and 5000 minutes of pumping should be calculated and compared with the corresponding amount of water pumped. Recovery data may also be recorded and plots of residual drawdown It Vs log t/t' should be prepared.

(FORM-4.1)

Well No. . . . .
Code No. . . . .

PUMPING TEST DATA

1. NAME OF THE PROJECT. . . . .

2. LOCATION OF SITE : District. . . . . Taluk/Tehsil. . . . . Village. . . . .
Survey No. . . . . Coordinates. . . . . Toposheet No. . . . .

3. WELL DETAILS : Pumped well. . . . . Observation wells. . . . .
Distance and bearing from pumped well to observation wells. . . . .

4. DATA OF PUMPED WELL/OBSERVATION WELL. . . . .
(a) Measuring point. . . . .(m) Height a. g. 1. . . . .(m) Altitude
(b) Method of discharge measurement. . . . .
(c) Water level measurement by tape/. . . . .

NOTE (i) Under remarks give temperature of water and indicate collection of water sample, turbidity, etc.
(ii) Recovery measurements should be continued till near pre-pumping level is reached.

Measured by	Date	Hour	Tape Reading at		Depth to water below measuring point (m)	Q (LPM)	t (min)	t' (min)	t/t'	r^2/t or t/r^2	s or s'	Remarks
			Measuring point (m)	Water level (m)								
1	2	3	4	5	6	7	8	9	10	11	12	13

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5. ANALYSIS OF PUMPING TEST DATA

5·1 Introduction

The analysis of pumping test data involves transformation of raw field data into calculated values of hydraulic properties of the tested aquifer. On completion of the pumping test, all the collected data on well discharge, drawdown recovery in the various observation wells and the pumped well with-time, pretest water level trend etc. are processed which involves following steps:

(i) *Drawdown/Recovery Corrections*

The pretest water level trend is extrapolated through the pumping/recovery period, and differences between extrapolated stages of the water levels that would have been observed if the well had not been pumped and the water levels measured during the pumping/recovery period are computed, for determination of corrected drawdown/recovery data. The effects of all extraneous factors such as changes in barometric pressure, diurnal fluctuations or other interferences should be removed from all the data by applicable correlation techniques.

(ii) *Compilation of Data in the Form of Graphs*

The corrected drawdown data for each well is plotted against the corresponding time/distance to describe the "time-drawdown" and "distance—drawdown" curves on semi-logarithmic and double logarithmic papers. The corrected recovery/residual drawdown data is plotted against corresponding $t^1/t/t^1$ values on logarithmic papers for analysis of recovery data.

(iii) *Determination of Aquifer Type*

Knowledge of subsurface lithological crossection and the form of "time-drawdown" curves, help in determination of the type of aquifer that has been pumped. Fig. (5·1) depicts typical "time-drawdown" curves for different aquifer types. The "time-drawdown" curves of shallow/deep piezotmeters installed in overlying/underlying aquifers, provide additional information on the type of aquifer. No effect of pumping on water levels in the deep piezometers is suggestive of the impervious nature of the basis confining bed. However, if the water levels in the deeper underlying aquifer dropped during the pumping, though less than the drawdown in the tested aquifer, the basis bed could be semi-previous and one may speak of a two-layered aquifer system. Further, if the basis confining layer is impervious, the drawdowns observed in the shallow piezo-meters installed in the overlying aquifer may help to determine the type of aquifer. Kruseman and De Ridder have given the following table (5·1) for the said purpose :

TABLE 5·1

Classification of Aquifers Based on Drawdown observed in the Covering Layer

Drawdown in the Covering Layer	Type of Aquifer
none	confined or semi-confined
small	semi-confined
appreciable . .	semi-unconfined
the same as in the aquifer	unconfined

(iv) *Determination of Flow Conditions at the end of Pumping*

Before using a particular method for analysis of test data, it is essential to know the flow conditions at the end of pumping. This is done with the help of "time-drawdown" curves. Steady-state is attained when the "time-drawdown" curves become straight horizontal lines and prior to that unsteady-state flow conditions exist.

Various methods are available for analysis of pumping test data of different aquifer types under different flow conditions. Before applying a particular method, the underlying assumptions vis-a-vis the natural conditions should be considered. These methods have been grouped under two categories as under:

(a) Pumping Tests Under Simple Conditions

(b) Pumping Tests Under Special Conditions

Pumping tests under simple conditions deal with aquifers which are homogeneous, isotropic, infinite in areal extent and under fully penetrating constant discharge conditions. Whereas, pumping tests performed on non-uniform aquifers of restricted extent, under partial penetration, variable discharge conditions are put under special conditions.

5·2 Pumping Tests Under Simple Conditions

Methods under this category are based on simple generalised assumptions and have been grouped and discussed separately for different aquifer types under steady and unsteady state flow conditions. Assumptions underlying all the methods in this section are :

(i) The aquifer is infinite in areal extent.

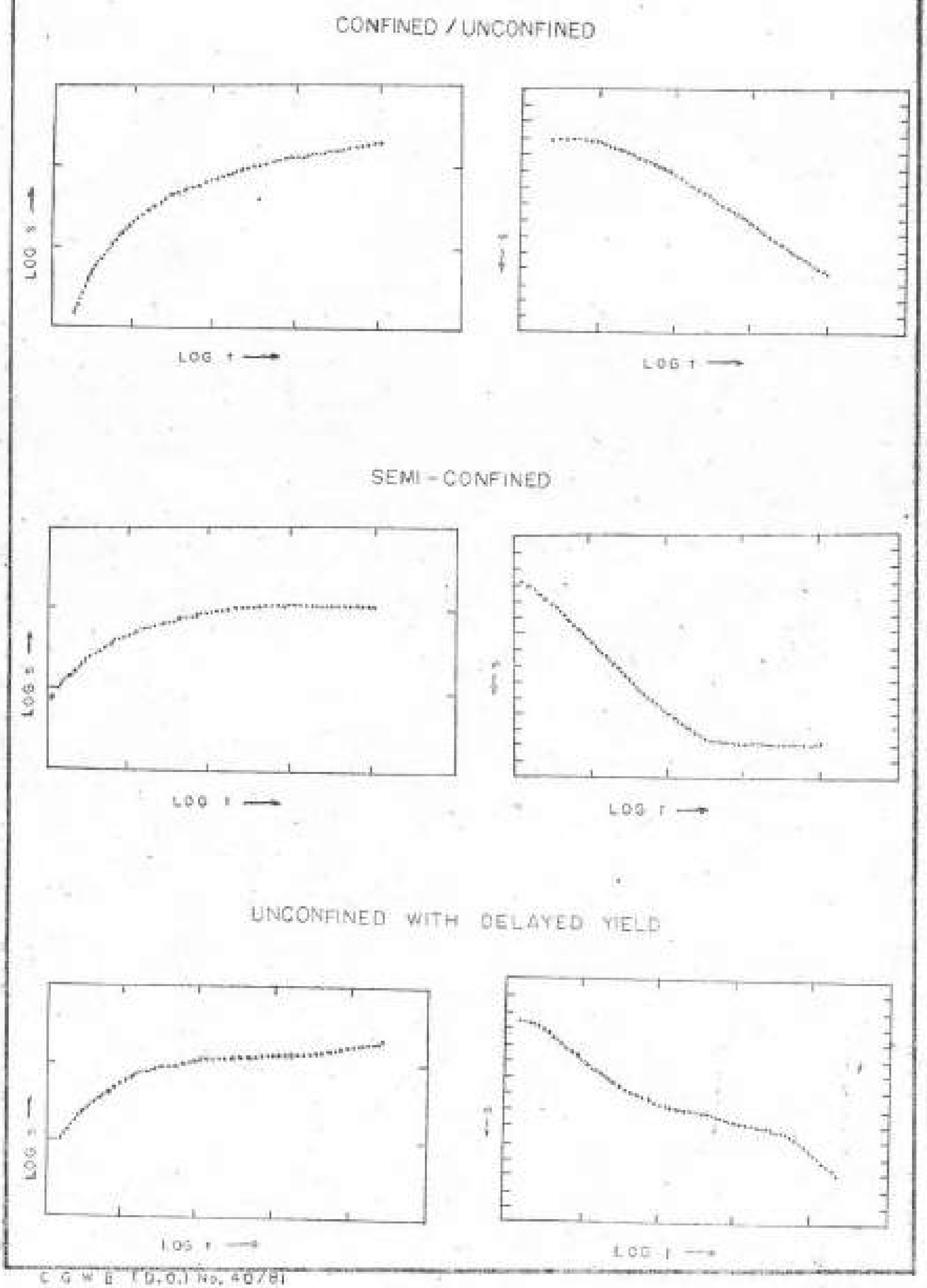
(ii) The aquifer is homogeneous, isotropic and of uniform thickness over the area influenced by the pumping test.

(iii) Prior to pumping the piezometre surface and/or phreptic surface are (nearly) horizontal over the area influenced by the pumping test.

(iv) The aquifer is pumped at a constent discharge rate.

FIG. 5.1

TIME - DRAWDOWN CURVES



(v) The pumped well penetrates the entire aquifer thickness and thus receives water from the entire thickness of the aquifer by horizontal flow.

The assumptions of homogeneity, isotropicity and infinite areal extent of the aquifer, are seldom met with, in the field and at the first glance it appears that the methods based on these assumption are of little practical value. However, for short period of pumping in uniform aquifers, these methods are being applied successfully and the results obtained have proved to be reasonably reliable for most purposes.

The methods under this category can further be divided into following types depending on the type of aquifer.

5·2·1 Confined Aquifers :

5·2·1·1 Steady-State Flow :

5·2·1·1·1 Theim's Method :

Theim's method is applicable to the drawdown data of a confined aquifer satisfying assumptions listed in section 5·2, under steady-state flow conditions. Since true steady-state conditions cannot be attained in a confined aquifer, situation where variable of the drawdown with respect to time are negligible, is taken as steady-state flow condition.

Theim (1906) showed that for an aquifer satisfying the above conditions, the well discharge can be expressed as :

$$Q = \frac{2\pi T(s_{m1} - s_{m2})}{\ln (r_2/r_1)} \qquad (5·1)$$

where,

Q is the well discharge in m^3/day (L^3T^{-1})

T is the Transmissivity in m^2/day (L^2T^{-1})

r_1 and r_2 are the respective distances of the observation wells from the pumped well in metres (L) s_{m1} and s_{m2} are the respective steady-state drawdowns in the observation wells in metres (L) Eq. (5·1) can be written as,

$$T = \frac{Q}{2\pi (s_{m1} - s_{m2})} \ln\left(\frac{r_2}{r_1}\right) \qquad (5·2)$$

Procedure—I :

Substitute the values of the steady—state drawdowns (s_m) of the two observation wells into Fig. (5·2) together with the corresponding values of r and the known Q and solve for T.

Procedure—II :

—Plot on a semi-logarithmic paper the observed steady—state drawdowns (s_m) of each observation well against the corresponding value of r(r on logarithmic scale).

—Draw the best fitting line through the plotted points to describe the "distance—drawdown" curve.

—Determine the slope of the curve, Δs_m i. e. the difference of s_m per log cycle of r, giving $r_2/r_1 = 10$ or $\log r_2/r_1 = 1$. In doing so, Eq. (5·2) reduces to,

$$T = \frac{2·30\, Q}{2\pi \Delta s_m} \qquad (5.3)$$

—Substitute the values of Q and Δs_m into Eq. (5·3) and solve for T.

Remarks ; Data of at least three observation wells are needed to get reliable results.

Example :

[After Bhatnagar, N. C. Agashe, R. M. and Sikka, V. M. (1977)].

Data of pumping test conducted on a confined aquifer at "Mathana" site, located in Upper Yamuna river basin, Haryana state, India, has been analysed by different applicable methods to illustrate a typical field case history. Here, an alluvial confined aquifer group exists in the depth range of 280 metres to 368 metres—underlain and overlain by confining clays which are 24 metres and 50 metres thick respectively Fig. (5·2). In Oct., 1976, a test of 7000 minutes pumping was conducted on a fully penetrating test-well in the said aquifer, at a constant discharge of 2725 m^3/day. The summarised water level data of all the wells and the t/t/t' Vs drawdown/recovery data collected from the testwell and two similar observation wells located at distance of 99·90 metres and 199·80 metres is given in Table (5·2).

TABLE 5·2

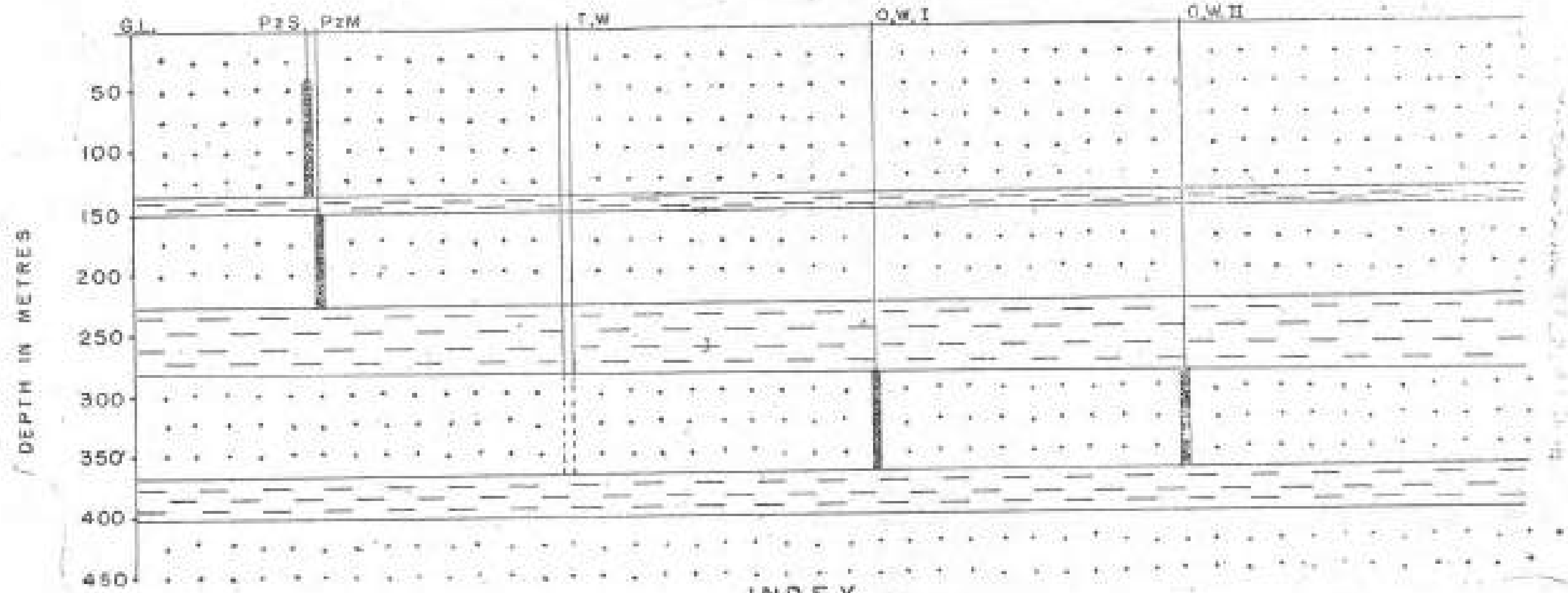
Pumping test data, "Mathana" site

(A) Water Level/Drawdown Data :

Well	Distance from the pumped well (m)	Non-pumping water level (mB. M.P.)	Maximum drawdown	Remarks
1	2	3	4	5
Pumped	—	8·738	11·515	—
OW—I	99·90	9·653	1·860	—
OW—II	199·80	9·419	1·570	—
PZ S	82·80	9·511	Nil	Taps overlying aquifer
PZ M	80·25	11·009	Nil	Do.

LITHOLOGICAL CROSS-SECTION OF THE PUMPING TEST SITE AT MATHANA

SCALE (HOR.) :- 1:2000



INDEX

Aquifer (Sand with clay lenses)..........

Aquiclude / Aquitard (Clay / Clay with silt)..........

Test Well .... T.W.

Observation Well .... O.W.

.... Pumping Screen.

.... Observation Filter.

PzS .... Shallow Piezometer.

PzM .... Medium Piezometer.

FIG. 5.2

FROM. Bhatnagar N.C., Agashe R.M., Sikka V.M. (1977)

(B) Drawdown Data :

t (min)	OW—I s (m)	OW—II s (m)
2	0 ·024	0 ·002
4	0 ·084	0 ·015
6	0 ·136	0 ·023
10	0 ·219	0 ·061
15	0 ·299	0 ·103
20	0 ·357	0 ·146
30	0 ·447	0 ·214
40	0 ·511	0 ·266
50	0 ·564	0 ·311
70	0 ·643	0 ·386
100	0 ·729	0 ·461
120	0 ·780	0 ·506
150	0 ·838	0 ·559
180	0 ·878	0 ·603
200	0 ·907	0 ·627
260	0 ·979	0 ·696
300	1 ·009	0 ·727
400	1 ·090	0 ·802
500	1 ·163	0 ·873
600	1 ·217	0 ·927
700	1 ·256	0 ·966
800	1 ·289	1 ·003
900	1 ·313	1 ·026
1000	1 ·341	1 ·053
1200	1 ·402	1 ·105
1500	1 ·450	1 ·160
1700	1 ·474	1 ·188
2000	1 ·532	1 ·239
2500	1 ·597	1 ·300
3000	1 ·635	1 ·345
3600	1 ·687	1 ·396
4000	1 ·717	1 ·424
4500	1 ·742	1 ·453
5000	1 ·782	1 ·487
5500	1 ·797	1 ·508
6000	1 ·804	1 ·514
6500	1 ·837	1 ·547
7000	1 ·860	1 ·570

(C) Recovery Data :

t/t'	Pumped Well s' (residual drawdown) (m)	OW—I s' (m)	OW—II s' (m)
1	2	3	4
7001 ·00	11 ·513	1 ·859	1 ·569
2334 ·30	3 ·539	1 ·809	1 ·564
1401 ·00	3 ·217	1 ·753	1 ·557
701 ·00	2 ·752	1 ·650	1 ·532
351 ·00	2 ·300	1 ·512	1 ·438
234 ·30	2 ·082	1 ·427	1 ·373
176 ·00	1 ·927	1 ·326	1 ·319
117 ·60	1 ·776	1 ·266	1 ·239
88 ·60	1 ·618	1 ·196	1 ·172
71 ·00	1 ·456	1 ·137	1 ·123
51 ·00	1 ·300	1 ·051	1 ·041
36 ·00	1 ·162	0 ·958	0 ·955
27 ·90	1 ·158	0 ·891	0 ·885
24 ·30	1 ·024	0 ·850	0 ·850
18 ·50	0 ·940	0 ·780	0 ·780
15 ·00	0 ·954	0 ·719	0 ·722
11 ·00	0 ·849	0 ·642	0 ·642
8 ·00	0 ·754	0 ·557	0 ·559
6 ·00	0 ·607	0 ·476	0 ·480
4 ·50	0 ·477	0 ·377	0 ·386
3 ·69	0 ·430	0 ·320	0 ·331
3 ·00	0 ·352	0 ·242	0 ·255
2 ·50	0 ·256	0 ·179	0 ·189
2 ·16	0 ·234	0 ·121	0 ·129
2 ·00	0 ·188	0 ·104	0 ·113

Procedure-I :

For analysis of drawdown data by Theim's method the maximum drawdowns observed at 7000minutes in observation wells I and II, are substituted into Eq. (5 ·2).

$$T = \frac{Q}{2\pi\ (s_{m1} - s_{m2})} \ln \frac{r_2}{r_1}$$

Where,

$Q = 2725\ m^3/day$

$r_1 = 99 \cdot 90\ m$

$r_2 = 199 \cdot 80$

$s_{m1} = 1 \cdot 860\ m$

$s_{m2} = 1 \cdot 570 m$

$$T = \frac{2725}{2x3 \cdot 14(1 \cdot 860 - 1 \cdot 570)} \ln \frac{199 \cdot 80}{99 \cdot 90}$$

$$= 1035\ m^2/day$$

Procedure-II :

Values of s_{m1} and s_{m2} are plotted against the corresponding values of r_1 and r_2 on semi-logarithmic paper Fig. (5 ·3). A straight line is fitted through these two points and its slope, $\triangle s_m$, i. e. drawndown difference over one log cycle of r is read-being 1 ·0 m. Introduction of this value and the Q value into Eq. (5 ·3) yields,

$$T = \frac{2 \cdot 30Q}{2\ \pi\ \triangle s_m}$$

$$= \frac{2 \cdot 30x2725}{2x3 \cdot 14x1 \cdot 0} = 1000\ m^2/day$$

5 ·2 ·1 ·2 Unsteady—state Flow

Theis (1935) equation of non-steady state flow in a confined aquifer is expressed as,

$$s = \frac{Q}{4\ \pi\ T} \int_{u}^{\infty} \frac{e^{-u}}{u}\ du$$

7—542CGWB/82

DISTANCE - DRAWDOWN PLOT

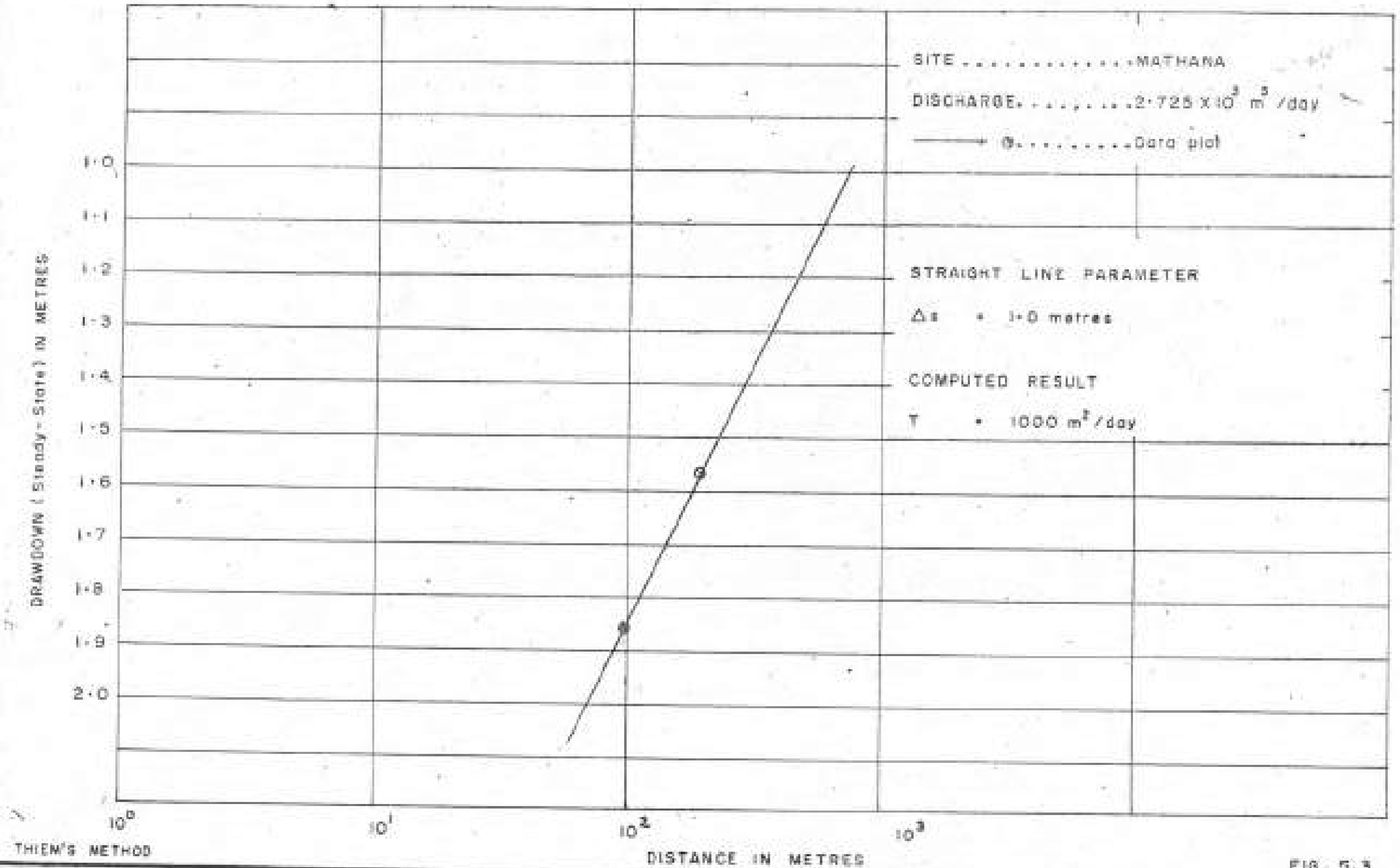


FIG. 5.3

FROM - Bhatnagar N.C., Agashe R.M., Sikka V.M. (1977)

$$s = \frac{Q}{4\pi T} W(u) \qquad (5 \cdot 4)$$

Where,

$$u = \frac{r^2 S}{4Tt} \qquad (5 \cdot 5)$$

s= the drawdown in an observation well in metres (L)

r= the distance of observation well from the pumped well in metres (L)

S= the Storativity (dimensionless)

t= time in days since pumping started (T)
W(u) is read as "well function of u"

$$= -0 \cdot 5772 - \ln u + u \quad \frac{-u^2}{2 \cdot 2!} \frac{+u^3}{3 \cdot 3!} \frac{-u^4}{4 \cdot 4!}$$

The values of W (u) as u or 1/u varies are available in the form of a table (annexure-II)

Q, T as defined earlier.

5·2·1·2·1 Theis's Method:

In addition to the general assumption listed in section 5·2, the following assumptions are also to be satisfied for the application of Theis's method,

(i) The aquifer is confined

(ii) The flow to the well is in unsteady-state

(iii) The water removed from storage is discharged instantaneously with decline of head

(iv) The storage in the well can be neglected

Theis devised a convenient method of superposition for solving Eqs. (5·4) and (5·5), which can be re-arranged as:

$$\log s = \left[\log \frac{Q}{4\pi T}\right] + \log W(u) \qquad (5 \cdot 6)$$

$$\log \frac{r^2}{t} = \left[\log \frac{4T}{S}\right] + \log u \qquad (5 \cdot 7)$$

If the discharge, Q, is held constant, the bracketed parts of Eqs. (5·6) and (5·7) are constant for a given pumping test, and w(u) is related to u in the manner that is related to r^2/t. Therefore, if values of the drawdown, s are plotted against r^2/t/or $1/t$ if only one observation well is used, on a double logarithmic paper to the same scale as the type curve of W(u) vs u, the curve of observed data will be similar to the type curve, Usually, it is convenient to plot a type curve of W(u) Vs 1/u and in that case drawdown, s is to be plotted against t/r^2 or t. This method eliminates the necessity for computing l/t values. Theis's graphical method is outlined, hereunder—

Procedure-I ("time-drawdown" analysis)

—Prepare a Theis type curve on a double logarithmic paper by plotting values of W(u) against $^1/_u$ (Type Curve : 1)

—Plot the values of as against t/r^2 or t on another sheet of logarithmic paper of the same scale as that used for the type curve.

—Place the field data plot over the type curve, the co-ordinate axes of the two curves being held parallel [s axis parallel to W (u) axis] locate the position of best match between th data plot and the type curve.

—Select an arbitrary "match point" on the overlapping portion of the two sheets of graph paper and record the co-ordinates of "match point"— W (u), $^1/_u$, s and t/r^2 or t.

—Substitute Q and these values into Eqs. (5·4) and (5·5) and determine T and S

$$T = \frac{Q}{4\pi s} W(u) \qquad (5 \cdot 8)$$

$$S = \frac{4 \, T \, tu}{r^2} \qquad (5 \cdot 9)$$

Example:

[After Bhatnagar, N C., Agashe, R. M., and Sikka, V. M. (1977)]

The drawdown data, Table (5·2), of pumping test at "Mathana" site analysed by Theis's method is presented as an example. Fig. (5·4) shows plots of t vs s for observation wells I and II.

The "match point" co-ordinates and calculation of T and S values by use of Eqs (5·8) and (5·9) is summarised below:

$Q = 2725 \ m^3/day$

(i) Observation well - I ($r = 99 \cdot 90$ m)

"Match point" co-ordinates,

$W(u) = 10^0$,

$\frac{1}{u} = 10^2$

$s = 2 \cdot 6 x 10^{-1}$ m

$t = 3 \cdot 2 x 10^2$ min. $= 2 \cdot 22 x 10^{-1}$ days

Using Eqs (5·8) and (5·9) we get,

$$T = \frac{Q}{4\pi s} W(u)$$

$$T = \frac{2.725 x 10^3 x 10^0}{4 \times 3 \cdot 14 x 2 \cdot 6 x 10^{-1}}$$

$$= 830 \ m^2/day$$

and $S = \frac{4 \, T \, tu}{r^2}$

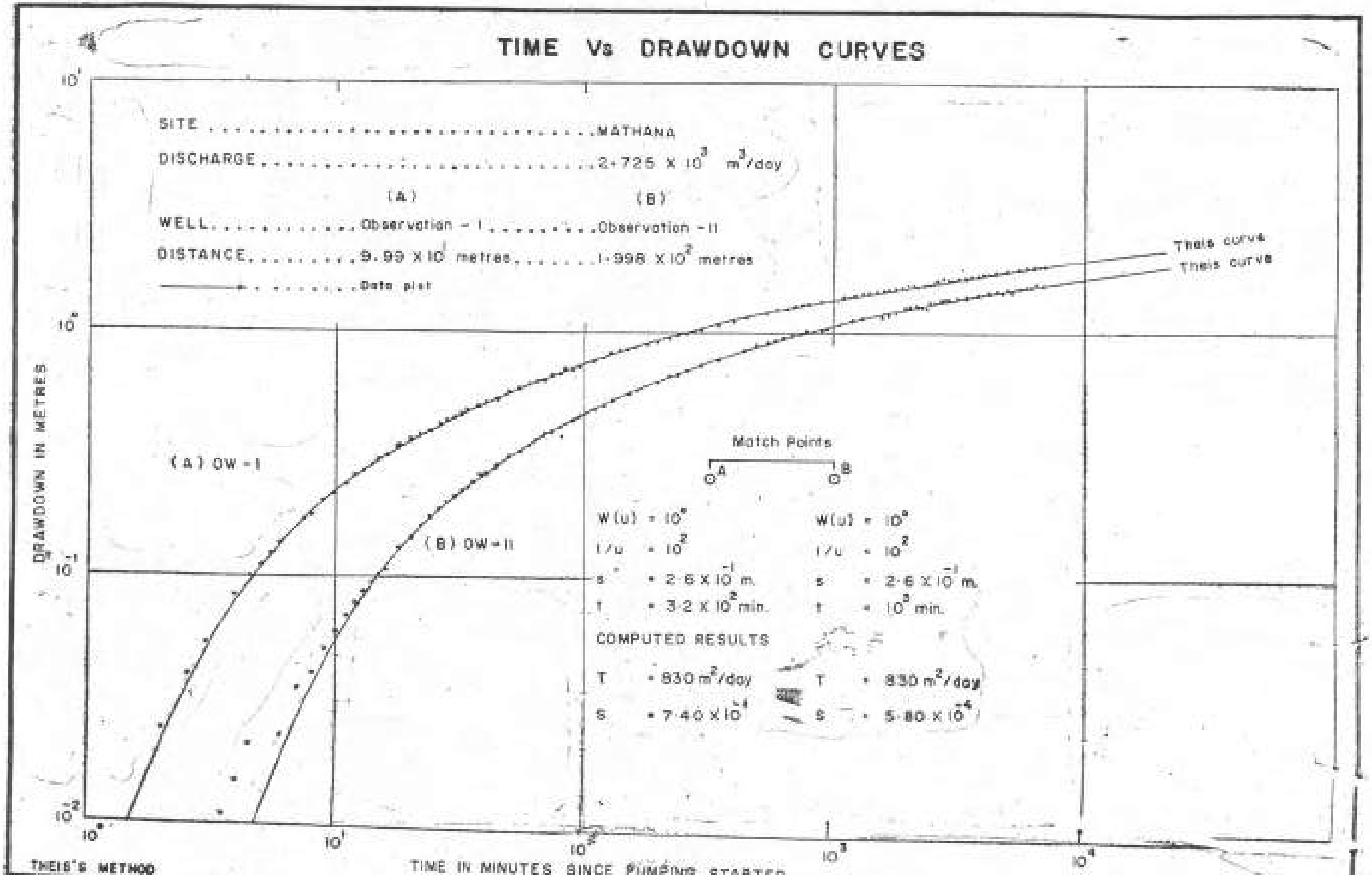


FIG. 5.4

FROM. Bhatnagar N.C., Agashe R.M., Sikka V.M. (1977)

$$S = \frac{4x8\cdot30x10^2x2\cdot22x10^{-1} \times 10^{-2}}{9\cdot98x10^0}$$

$=7\cdot40x10^{-4}$

(ii) Observation Well-II (r=199·80m) "Match point" co-ordinates being,

$W(u)=10^0$

$\frac{1}{u}=10^2$

$s=2\cdot6x10^{-1}$ m

$t=10^3$ min$=6\cdot94\times10^{-1}$ days

Using Eqs (5·8) and (5·9),

$T=830$ m²/day

$S=5\cdot80x10^{-4}$

Procedure-II ("distance-drawdown" analysis)

"distance-drawdown" data can also be analysed by Theis's type curve method for evaluation of aquifer parameters. Procedure being similar to the one outlined for "time-drawdown" analysis. The type curve used is W(u) vs u, (Type curve: 2) "distance-drawdown" field data curve is prepared by plotting the values of drawdown, s, recorded at the same time in several observation wells at various distances from the pumped well against the squares of their respective distance, r^2. In the matched position, "match point" co-ordinates W(u), u, s and r^2 are substituted into Eqs (5·8) and (5·9), for determination of T and S values. It is, however, desirable to have atleast 3 observation wells for evaluation of reliable values of aquifer parameters by use of this method.

Remarks : "time-recovery" and "distance-recovery" data may also be analysed by Theis's type curve methods for evaluation of T and S values. To determine recovery data, the water level trend during the pumping period is extrapolated through the recovery period, and differences between extrapolated states of the water level that would have been observed if the well had continued pumping and the water levels measured during the recovery period are computed (refer Fig. 5·5).

5.2.1.2.2 Jacob's Method :

Jacob's straight line method (Cooper and Jacob, 1946) is based on Theis's equation. Jacob has shown that for small value of u ($u \leq 0\cdot01$) i.e. when r is small and t is large, the Eq. (5·4) can be simplified and expressed as,

$$s = \frac{2\cdot30\ Q}{4\pi T} \log \frac{2\cdot25T\ t}{r^2S} \qquad (5\cdot10)$$

Thus a plot of drawdown (s) versus the logarithm of time (t) or distance (r) from the pumped well describes a straight line. Eq (5·10) can further be solved to give,

$$T = \frac{2\cdot30Q}{4\pi\Delta s} \qquad (5\cdot11)$$

$$S = \frac{2\cdot25\ T\ t_o}{r^2} \qquad (5\cdot12)$$

and

$$T = \frac{2\cdot30Q}{2\pi\Delta s} \qquad (5\cdot13)$$

$$S = \frac{2\cdot25\ T\ t}{r_o^2} \qquad (5\cdot14)$$

where,

t_o = the time intercept in days corresponding to interception of straight line with zero drawdown axis (T)

r_o = the distance intercept in metres corresponding to interception of straight line with zero drawdown axis (L)

Δs = slope of the straight line in metres (L)

Q, T, S, t and r as defined earlier.

For use of Jacob's methods, the following assumptions and limiting conditions need to be satisfied—

—The same conditions as for the Theis's method (Section 5·2·1·2·1)

—The values of u are small ($u \leq 0\cdot01$) i.e. r is small and t is large

As discussed, Jacob's straight line methods are based on the fact that when u becomes small ($u \leq 0\cdot01$), a plot of drawdown against the logarithm of time or distance from the pumped well describes a straight line. However, a plot of W(u) against logarithm of u shows that deviation from a straight line becomes appreciable when u exceeds about 0·02.

Hence, the time (t_{sl}) that must elapse before the straight line methods could be applied to the test data, may be determined by substituting the value of u= 0·02 in the Eq. (5·5),

$$u = \frac{r^2S}{4\ T\ t}$$

Which reduces to,

$$t_{sl} = \frac{12\cdot5\ r^2S}{T} \qquad (5\cdot15)$$

where,

t_{sl} = time in days beyond which a plot of drawdown against logarithm of time describes a straight line (T).

Procedure—I :

("Time-drawdown" analysis)

— Plot for one of the observation wells (r=constant the values of drawdown, s against the corresponding time, t on a semi-logarithmic paper, t on logarithmic scale)

DRAWDOWN / RECOVERY Vs TIME PLOT

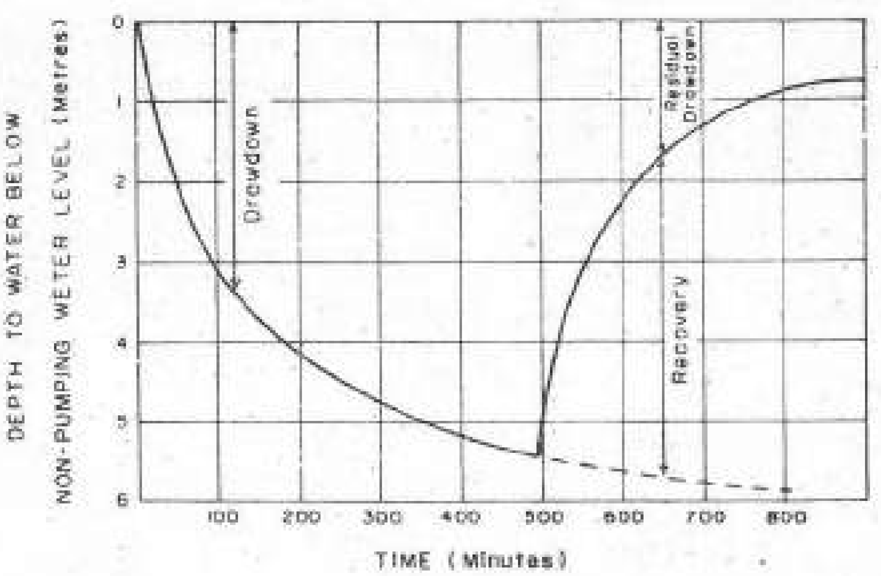


FIG. 5.5

— Draw a straight line through the plotted points leaving the initial scattered drawdown data which falls on a gentle curve.

— Extend the straight line till it intercepts the time axis where $s = 0$, and record the value of t_o.

— Determine the slope of the straight line, $\triangle s$, the drawdown difference over one log cycle of time.

— Substitute the values of Q and $\triangle s$ into Eq. (5·11) and solve for T.

— Substitute the values of computed T, t_o and r into Eq. (5·12) and determine the value of s.

Procedure—II :

(*"distance-drawdown" analysis*)

— On a semi-logarithmic paper, plot the values of drawdown, s against the corresponding values of distance, r (r on logarithmic scale) for a particular time.

— Draw a straight line through the plotted points.

— Extend the straight line till it intercepts the distance axis where $s = 0$, and record the value of r_o.

— Determine the slope of the straight line $\triangle s$, the drawdown difference over one log cycle of distance.

— Substitute the values of Q and $\triangle s$ into Eq. (5·13) and solve for T.

— Substitute the values of computed T, r_o and t into Eq. (5·14) and determine S.

Remarks:—It is desirable to have data of at least three observation wells for getting reliable results by use of Jacob's "distance-draw-down" method.

Example :

[After Bhatnagar, N.C. Agashe, R.M., and Sikka, V.M. (1977)]

I—"time-drawndown" analysis.

The drawdown data (Table 5·2) of pumping test at "Mathana" site is analysed by Jacob's methods. Fig. (56) shows plots of drawdown, s, against logarithm of time, t, for observation wells I and II. Calculation of T and S values by use of Eqs. (5. 11) and (5·12) is given hereunder—

$Q = 2725\ m^3/day$

(i) Observation Well-I ($r = 99·90$ m)

Straight line parameters being,

$\triangle s = 6·1 \times 10^{-1} m$

$t_o = 6·5$ min. $= 4·51 \times 10^{-3}$ days

$$T = \frac{2·30\ Q}{4\pi \triangle s}$$

$$= \frac{2·30 \times 2·725 \times 10^3}{4 \times 3·14 \times 6·1 \times 10^{-1}}$$

$= 820 m^2/day$

$$= \frac{2·25\ T\ t_o}{r^2}$$

$$= \frac{2·25 \times 8·20 \times 10^2 \times 4·51 \times 10^{-3}}{9·98 \times 10^3}$$

$= 8·30 \times 10^{-4}$

(ii) Observation well—II ($r = 199·80$ m)

Straight line parameters being,

$\triangle s = 6·1 \times 10^{-1} m$

$t_o = 19·5$ min $= 1·35 \times 10^{-2}$ days

$T = 820\ m^2/day$

$S = 6·20 \times 10^{-4}$

II—"distance-drawdown" analysis.

Although data of at least 3 observation wells is required for use of Jacob's "distance-drawdown" method, an attempt is made with data of 2 observation Wells only of test at "Mathana" site. Fig. (5·7) shows a plot of drawdown, s against the logarithm of distance r. Calculation of T & S values by use of Eq. (5·13) and (5·14) is given below,

$Q = 2725 m^3/day$

straight line parameters being,

$\triangle s = 1·0 m$.

$r_o = 7·3 \times 10^3 m$

$t = 7·0 \times 10^3$ min. $= 4·86$ days

$$T = \frac{2·30\ Q}{2\pi \triangle s}$$

$$= \frac{2·30 \times 2·725 \times 10^3}{2 \times 3·14 \times 1·0}$$

$= 1000 m^2/day$

$$S = \frac{2·25\ T\ t}{r^2}$$

$$= \frac{2·25 \times 1·0 \times 10^3 \times 4·86}{5·329 \times 10^7}$$

$= 2·0 \times 10^{-4}$

Remarks: "time-recovery" and "distance-recovery" data may also be analysed by Jacob's straight line methods. The recovery data is to be computed in the manner explained earlier under section 5·2·1·2·1.

TIME Vs DRAWDOWN CURVES

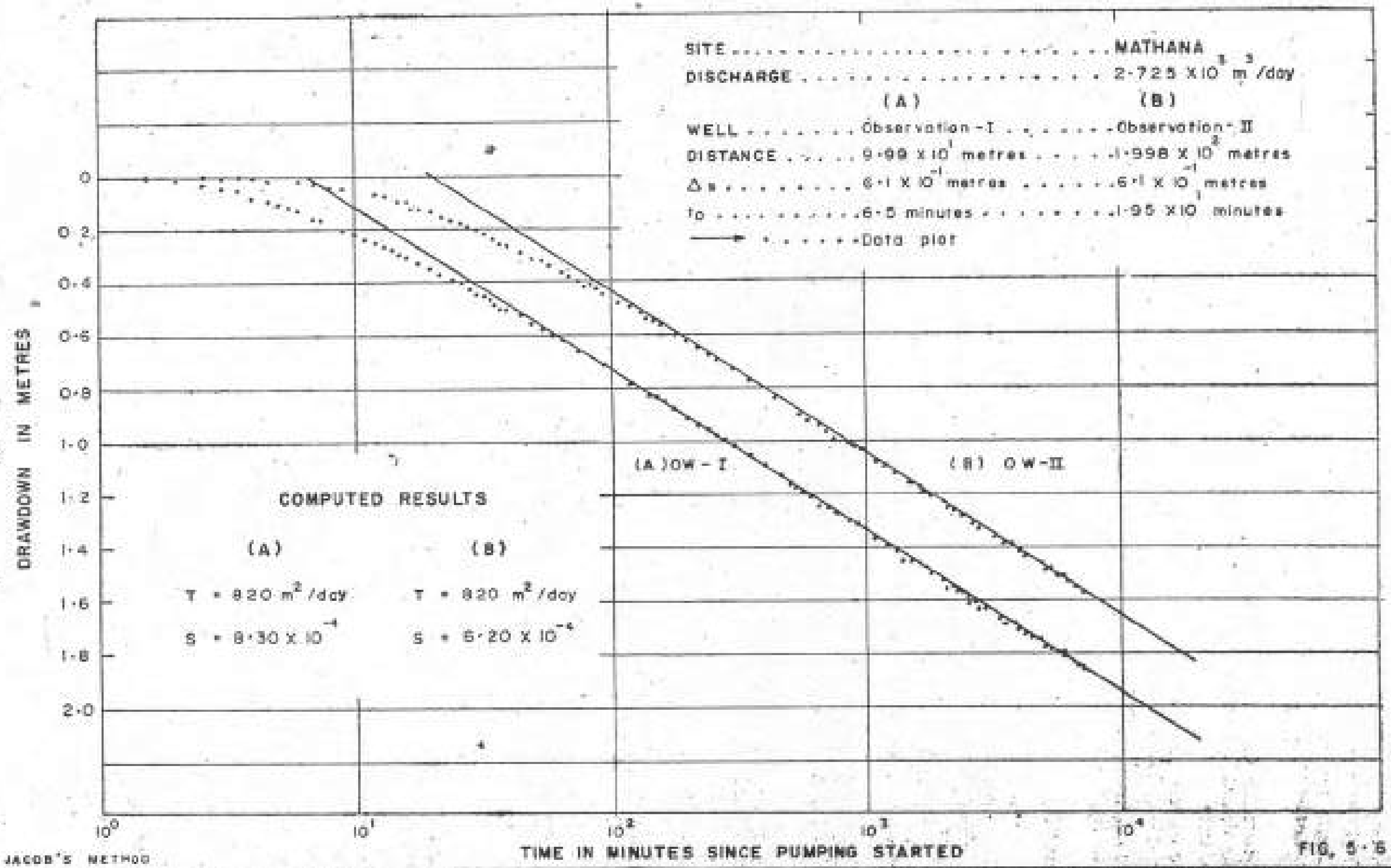
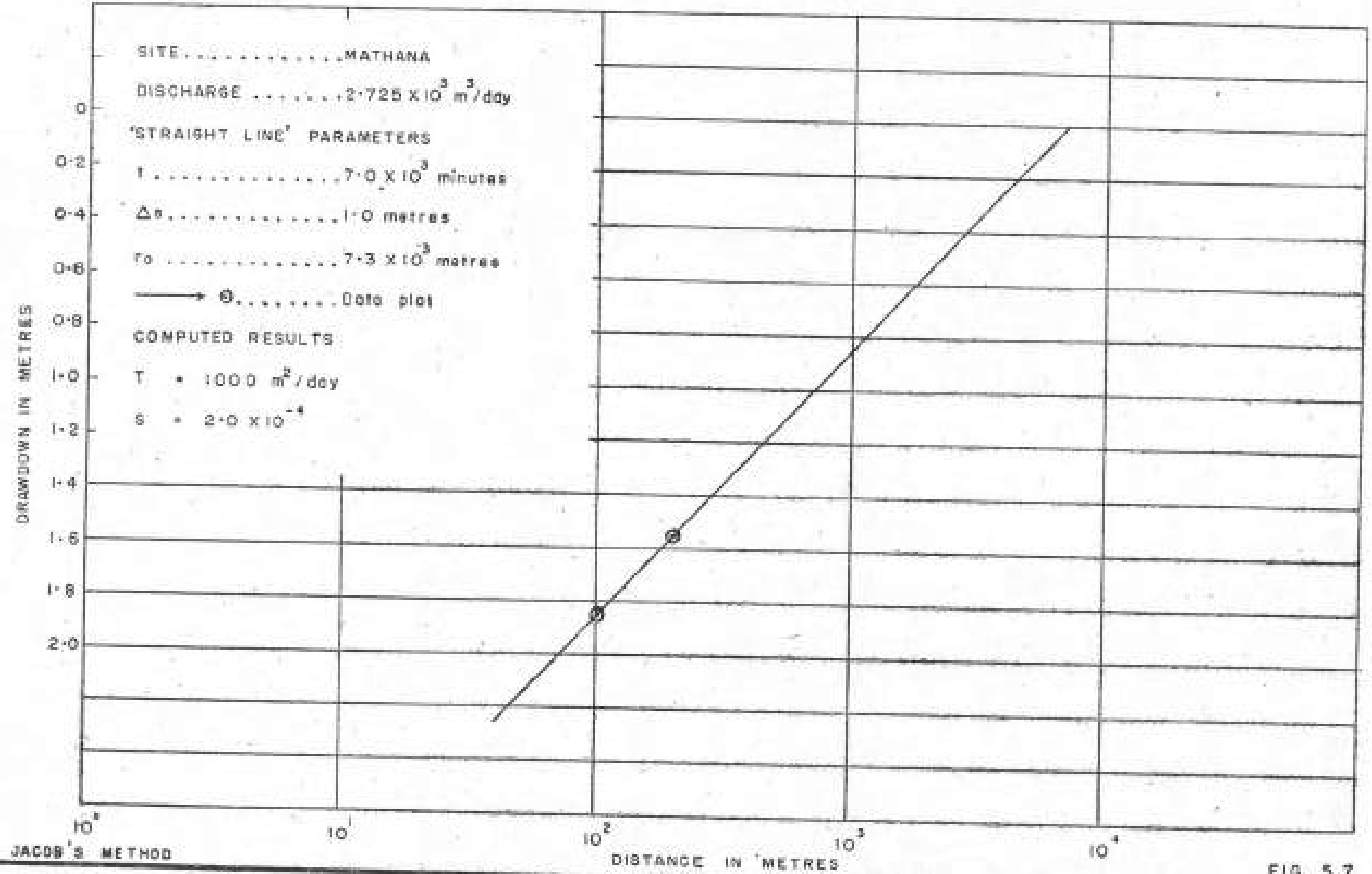


FIG. 5·6

FROM - Bhatnagar, N.C., Agashe R.M., Sikka V.M. (1977)

DISTANCE Vs DRAWDOWN PLOT



JACOB'S METHOD

FIG. 5.7

FROM - Bhatnagar N.C., Agashe R.M., Sikka V.M. (1977)

5·2·1·2·3 Chow's Method :

Chow (1952) developed a method which like Jacob's straight line method is also based on Theis equation, but has the advantage of not being restricted to small values of r and large values of t, as is the Jacob's method.

For use of Chow's method the same assumptions and conditions should be satisfied as required for Theis's method. Theis equation being,

$$s = \frac{Q}{4\pi T} W(u)$$

To find the values of W(u) and u corresponding with the drawdown, s measured at a certain moment, t, Chow (1952) introduced the function,

$$F(u) = \frac{W(u)e^{u}}{2 \cdot 30} \qquad (5 \cdot 16)$$

The relationship between F(u), W(u) and (u) is available in the form of a nomogram and tables given in Annexure -III
Also,

$$F(u) = \frac{s_A}{\triangle s^A} \qquad (5 \cdot 17)$$

Where,

s_A = drawdown in metres at point 'A' selected arbitrarily on the "time-drawdown" curve. (L)

$\triangle s_A$ = Slope of the tangent line to the curve at point 'A' i.e. drawdown difference over one log cycle of time, in metres (L)

Procedu e

- —Plot for one Observation well, the drawdown, s, against the corresponding time, t on a semi-logarithmic paper (t on logarthmic scale)
- —Select an arbitrary point 'A' on the curve and draw a tangent to the curve through the point 'A'. Record the values of drawdown and time for the point 'A' i.e. s_A and t_A.
- —Calculate the slope of the tangent, $\triangle s$ i.e. drawdown difference per log cycle of time.
- —Substitute the values of s_A and $\triangle s_A$ into Eq. (5·17) and determine the value of F(u)
- —Knowing the value of F(u), find out the corresponding values of W(u) and u from the tables, (Annexure-III).
- —Calculate the values of T and S by substituting the required values into Eqs (5·8) and (5·9) which may be written as,

$$T = \frac{Q}{4\pi s_A} W(u)_A$$

and,

$$S = \frac{4 T t_A u_A}{r^2}$$

Example :

[After Bhatnagar, N.C., Agashe, R.M. and Sikka, V.M. (1977)]

The initial 100 minutes drawdown data of observation well-II (r=199·80) of pumping test at "Mathana" site (Table 5·2) has been analysed by Chow's method to illustrate its applicability Fig. (5·8) shows the "time-drawdown" curve of observation well-II. The calculation of T and S values by Chow's method is given below :

$Q = 2725 m^3/day$

$s_A = 2 \cdot 0 \times 10^{-1} m$

$t_A = 28$ min. $= 1 \cdot 94 \times 10^{-2}$ day

$\triangle s_A = 3.9 \times 10^{-1} m$

Using the Eq. (5·17)

$$F(u) = \frac{S_A}{\triangle S_A}$$

$$= \frac{2 \cdot 0 \times 10^{-1}}{3 \cdot 9 \times 10^{-1}} = 5 \cdot 1 \times 10^{-1}$$

From the tables of F(u) W(u) and (u), Annexure III, for the determined value of F(u), we get,

$W(u)_A = 8 \cdot 0 \times 10^{-1}$

$u_A = 3 \cdot 45 \times 10^{-1}$

Using the Eqs (5·8) and (5·9),

$$T = \frac{Q}{4\pi s_A} W(u)_A$$

$$= \frac{2 \cdot 725 \times \ 0^3 \times 8 \cdot 0 \times 10^{-1}}{4 \times 3 \cdot 14 \times 2 \cdot 0 \times 10^{-1}}$$

$= 868 m^2/day$

say, $870 m^2/day$

$$S = \frac{4T\ t\ u_A}{r^2}$$

$$= \frac{4 \times 8 \cdot 68 \times 10^2 \times 1 \cdot 94 \times 10^{-2} \times 3 \cdot 45 \times 10^{-2}}{3 \cdot 992 \times 10^4}$$

$= 5 \cdot 82 \times 10^{-4}$

say, $= 5 \cdot 80 \times 1 0^{-4}$

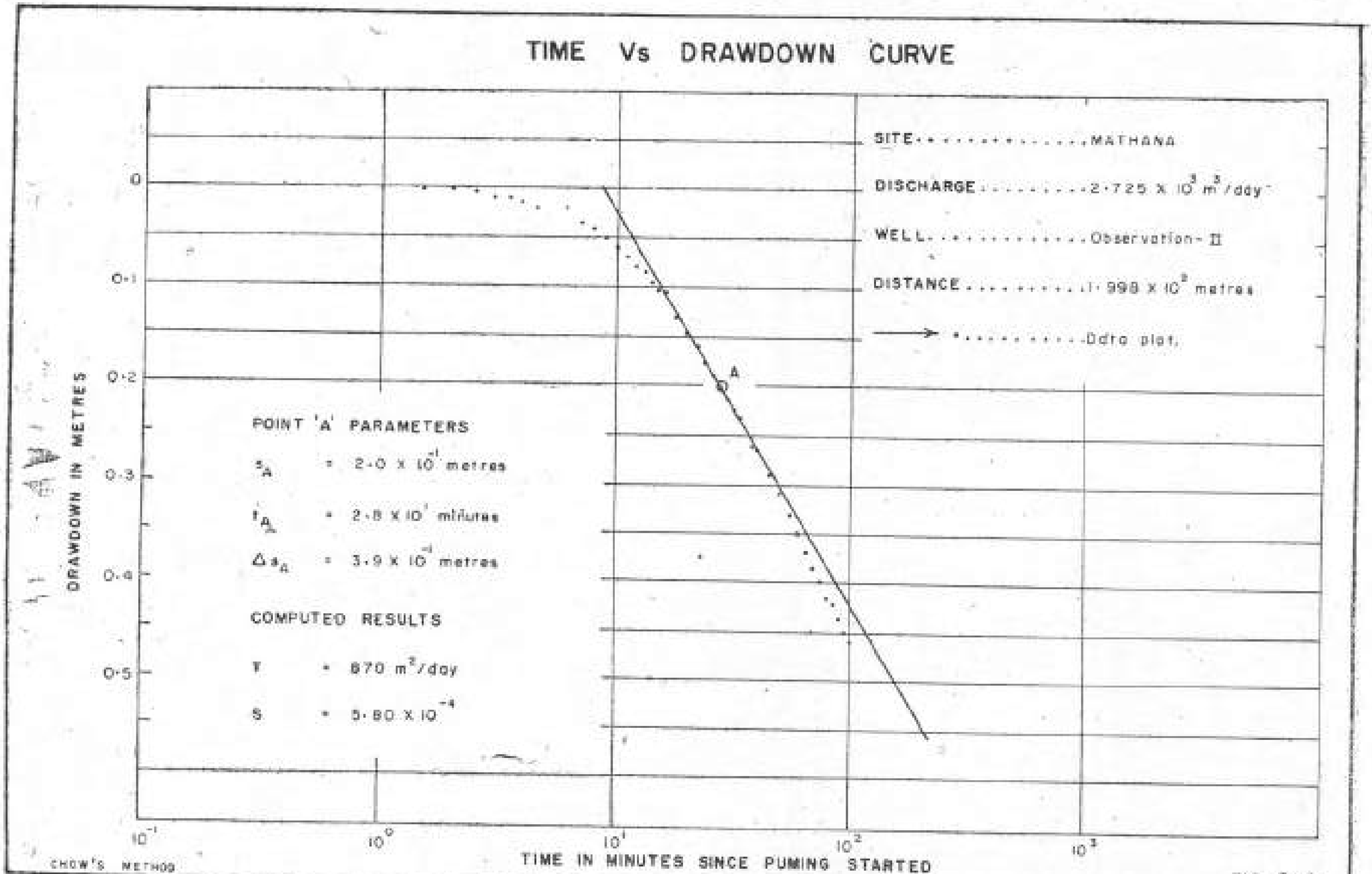


FIG. 5·8

AFTER _ Bhatnagar N.C , Agashe R.M , Sikka V.M (1977)

5·2·1·2·4 Thesis's Recovery Method :

Theis's recovery method is also based on Theis's non-equillibrium equation and therefore, for its application, the assumptions and conditions of Theis's method, should be satisfied.

If a well is pumped for a given period of time, t and then shut down the residual drawdown, s^1 (Fig. 5·5) can be approximated as the numerical difference between the drawdown in the well if the discharge had continued and the recovery of the well in response to an imaginary recharge well, of the same discharge, superimposed on the discharging well at the time it is shut down. Thus residual drawdown, s′ may be expressed as—

$$s' = \frac{Q}{4\pi T}\left[\int_{\frac{r^2S}{4Tt}}^{\infty}\frac{e^{-u}}{u}\,du - \int_{\frac{r^2S'}{4Tt'}}^{\infty}\frac{e^{-u}}{u}\,du\right] \qquad (5\cdot18)$$

Where,

s′ = residual drawdown in metres (L)

S′ = Storativity during recovery

t′ = time in days since pumping stopped (T)
Q,T,S, t and r as defined earlier.

Employing the Cooper—Jacob assumptions of the modified method, Eq (5·18) becomes

$$s' = \frac{2\cdot30Q}{4\pi T}\left\{\log\frac{2\cdot25Tt}{r^2S} - \log\frac{2\cdot25Tt'}{r^2S'}\right\} \qquad (5\cdot19)$$

Now, if S and S′ are assumed to be equal, the Eq (5·19) reduces to,

$$s' = \frac{2\cdot30\,Q}{4\,\pi\,T}\log\frac{t}{t'} \qquad (5\cdot20)$$

Thus, a plot of residual drawdown, s′ versus the logarithm of t/t′ describes a straight line, the slope of which equals 2·30 Q/4 π T

$$\therefore\ \Delta s' = \frac{2\cdot30\,Q}{4\,\pi\,T}$$

$$\text{or } T = \frac{2\cdot30\,Q}{4\pi\Delta s'} \qquad (5\cdot21)$$

Where,

Δs′ = residual drawdown difference per log cycle of t/t′in metres (L).

Remarks :

The Storativity can not be determined by this method.

Procedure :

— Plot the values of residual drawdown, s′ against the corresponding values of t/t′ (t/t′ on logarithmic scale.)

— Draw a straight line through the plotted points

— Determine the slope of the straight line, Δs′ i.e. residual drawdown difference per log cycle of t/t′.

— Substitute the values of Q and Δs′ into Eq (5·21) and solve for T.

Example :

[After Bhatnagar, N. C., Agashe, R. M., and Sikka V. M. (1977)]

The recovery data (Table 5·2) of pumping test at "Mathana" site, analysed by Theis's recovery method, is presented here as an example.

Figs. (5·9), (5·10) and (5·11) exhibit the residual drawdown, s′ versus logarithm of t/t′ plots for Observation Wells, I, II and the pumped well respectively. The calculations of T by the said method are summarised hereunder —

$Q = 2725\ m^3/day$

(i) Observation Well — I (r = 99·00 m)

$\Delta s' = 6\cdot5 \times 10^{-1} m$

Using Eq (5·29),

$$T = \frac{2\cdot30\ Q}{4\pi\ \Delta s'}$$

$$= \frac{2\cdot30 \times 2\cdot725 \times 10^3}{4 \times 3\cdot14 \times 6\cdot5 \times 10^{-1}}$$

$= 770\ m^2/day$

(ii) Observation Well — II (r = 199·80 m)

$\Delta s' = 6\cdot5 \times 10^{-1}$ m

$T = 770\ m^2/day$

(iii) Pumped well :

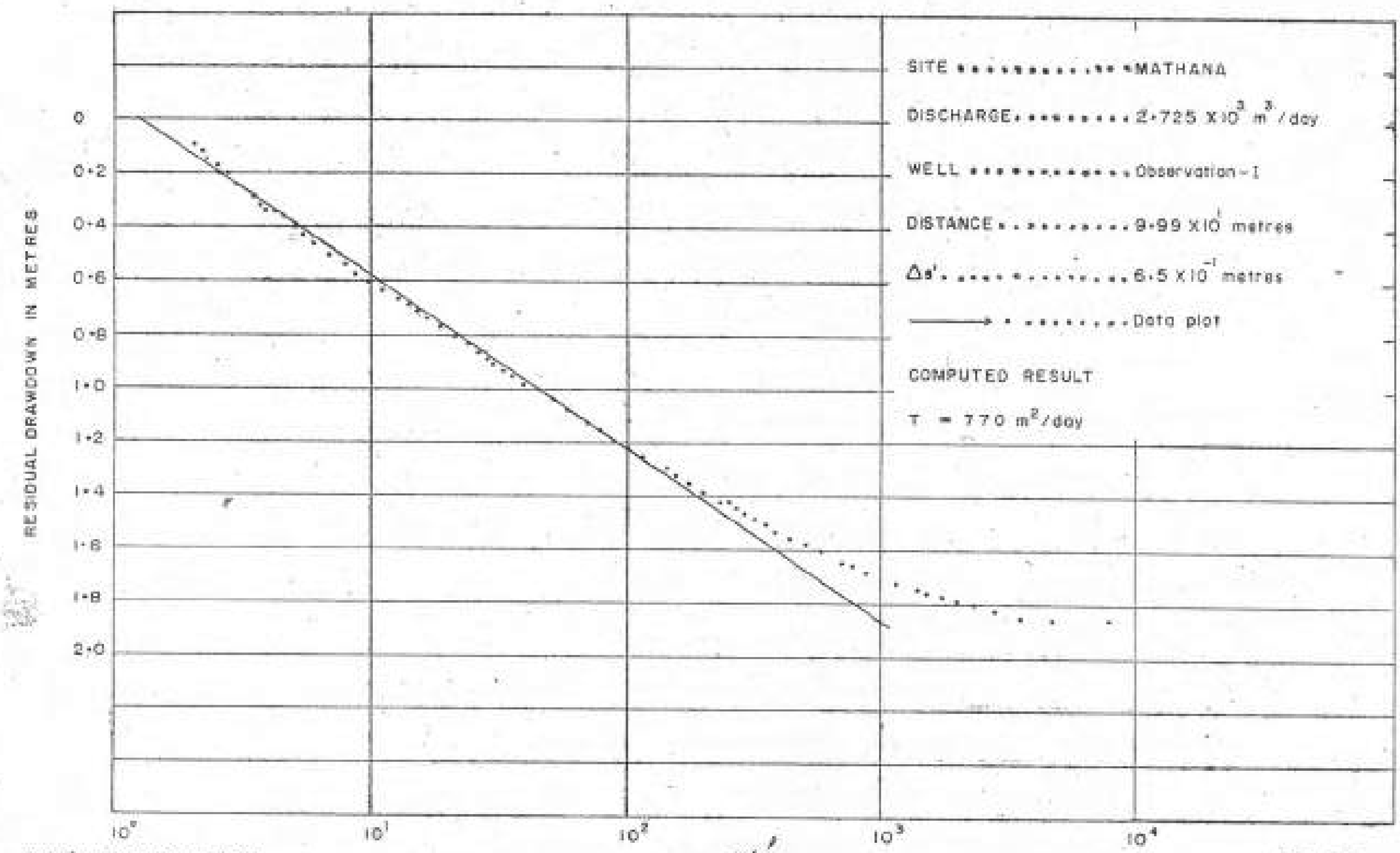
$\Delta s' = 1\cdot07$ m

$T = 465 m^2/day$

5.2.1.3 Evaluation of Methods :

The results of pumping test at 'Mathana' are given in Table (5·3). As could be seen from the table, the values of T and S calculated by Theis's and Jacob's "time drawdown" method match very closely. The other results except the one obtained from the analysis of recovery data of the pumped well, are also not very different. The significantly low value of T obtained from the analysis of pumped well data could be due to the increasing well losses during pumping.

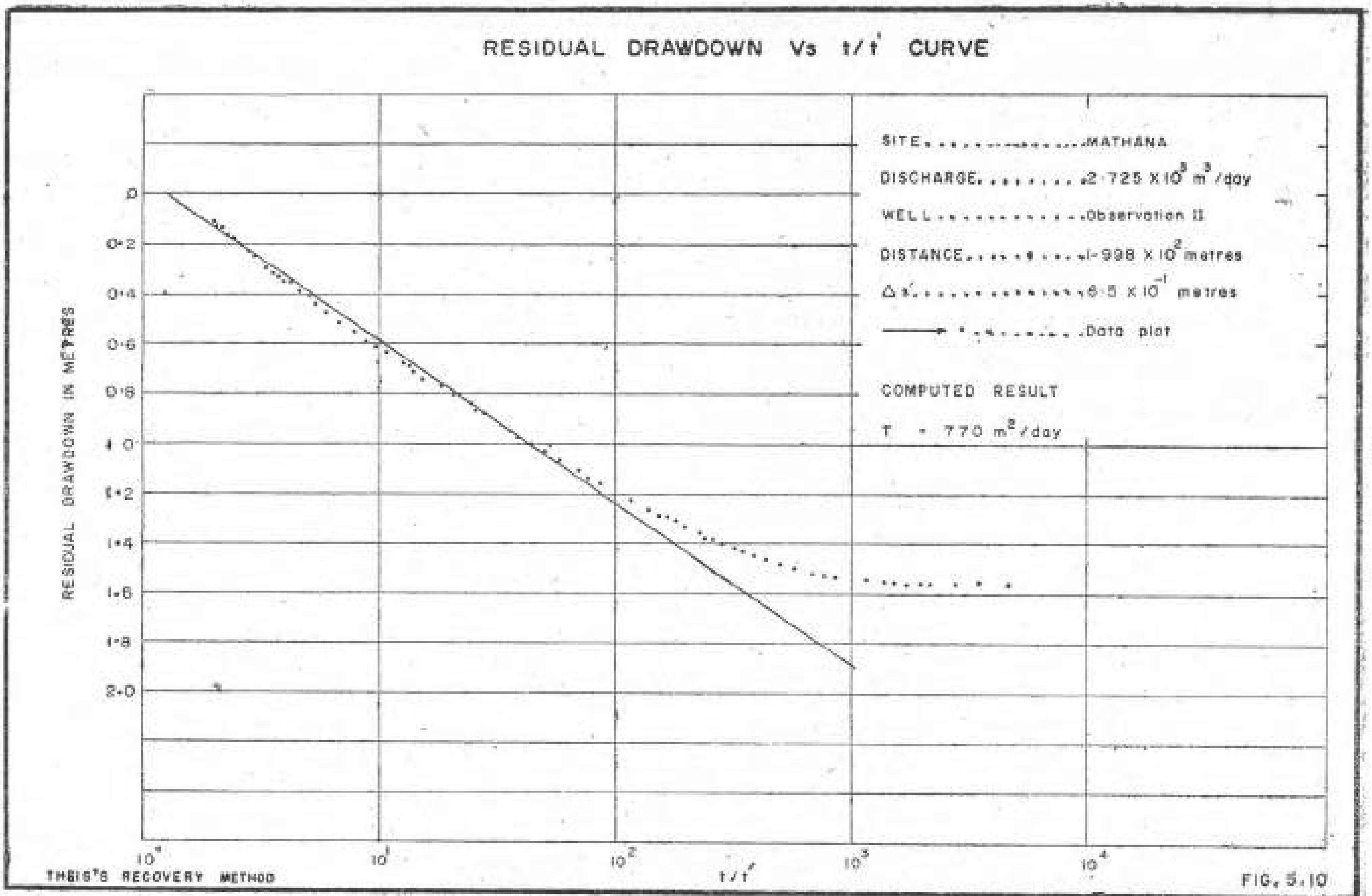
RESIDUAL DRAWDOWN Vs t/t' CURVE



THEIS'S RECOVERY METHOD

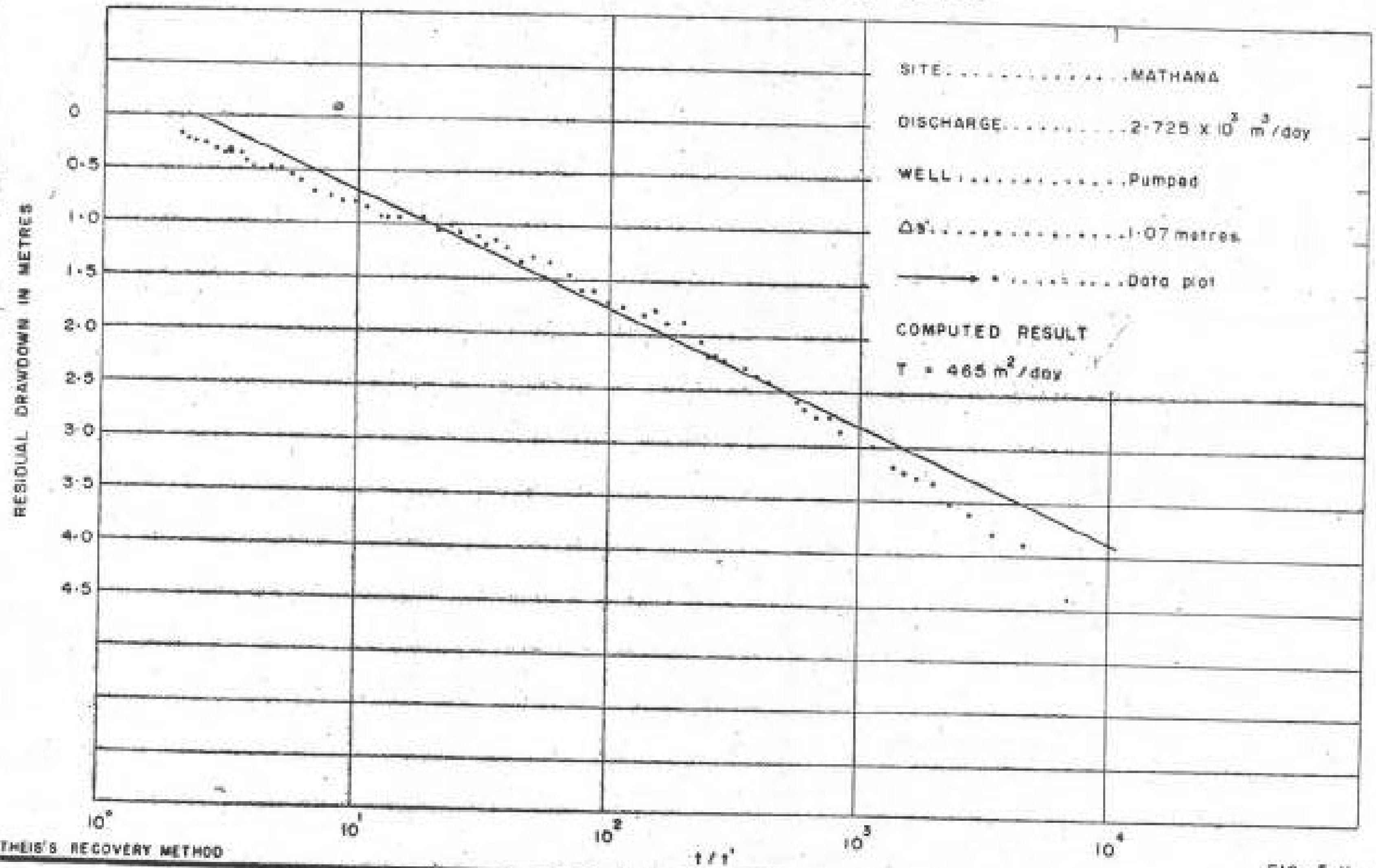
FIG. 5.9

FROM- Bhatnagar N.C., Agashe R.M., Sikka V.M.(1977)



FROM- Bhatnagar N.C., Agashe R.M., Sikka V.M (1977)

RESIDUAL DRAWDOWN Vs t/t' CURVE



THEIS'S RECOVERY METHOD

FIG. 5.11

FROM- Bhatnagar N.C., Agashe R.M., Sikka V.M. (1977)

TABLE 5·3

Summarised results of the pumping test at "Mathana" India

S. No.	Method used	Flow conditions	Data used	Plot of	T (m^2/day)	S	Remark
1.	Theim	Steady-state	OW—I & OW—II	Calculation	1035	—	
				Sm vs log r	1000	—	
2.	Theis	Unsteady-state	OW—I	log t vs s	830	$7{\cdot}40 \times 10^{-4}_{4}$	
			OW—II	log t vs s	830	$5{\cdot}80 \times 10^{-}$	
3.	Jacob	Unsteady-state	OW—I	log t vs s	820	$8{\cdot}30 \times 10^{-4}$	
			OW—II	log t vs s	820	$6{\cdot}20 \times 10^{-4}$	
4.	Jacob	Unsteady-state	OW—I and OW—II	s vs log r	1000	$2{\cdot}00 \times 10^{-4}$	
5.	Chow	Unsteady-state	OW—II	log t vs s	870	$5{\cdot}80 \times 10^{-4}$	only inital 100 min, data used
6.	Theis recovery	Unsteady-state	OW—I	R.D.D. Vs log t/t′	770	—	
			OW—II	R.D.D. Vs log t/t′	770	—	
			Pumped Well	RD.D. Vs log t/t′	465	—	

All the discussed methods are evaluated as under :

(i) Theim's Method :

Theim's method can only be applied under steady-state conditions, and since true steady-state conditions can't be attained in a confined aquifer, it can only be applied to the drawdown data recorded after sufficient duration of pumping, when the changes in drawdown with respect to time have become negligible. No value of S can be determined by this method.

(ii) Theis's Method :

Theis's type curve method being the basic method for analysis of time/distance-drawdown/recovery data for unsteady-state flow in a confined aquifer, therefore, in case of a good match of field data curve with the type curve, the values of T and S obtained by this method may be taken as best approximations. However, if the field data curve exhibits a flat curvature, several apparently good matching positions, depending on personal judgement, may be obtained. In such cases the Theis's graphical solution becomes practically indeterminate and resort to other methods must be made.

(iii) Jacob's Method :

Jacob's straight line methods are based on the fact that when u becomes small ($u \leqslant 0{\cdot}01$) a plot of drawdown against the logarithm of time since pumping started or distance from the pumped well describes a straight line. Hence, the methods can't be applied to pumping test data of short duration of pumping and/or observation wells located at great distances from the pumped well. The Jacob's methods are popular largely because of their simplicity of application and interpretation. However, as pointed out by Cooper and Jacob (1946), "the method is not applicable in some cases and it supplements rather than supersede, the type curve method".

(iv) Chow's Method :

Chow's method has the advantage of avoiding the curve matching of Theis method and not being restricted to small values of r and large values of t, as is the Jacob's method. Therefore, in cases where drawdown data is not amendable to analysis by Theis's and Jacob's methods, use of Chow's method may be resorted to.

(v) Theis's Recovery Method :

As is the case for Jacob's method, the Theis's recovery mthod is applicable for small values of r and large values of t. Value of S cannot be determined by this method. However, When observation wells data is not available. Theis's recovery method is the only effective method for analysis of pumped well data.

5·2·2 Semi-Confined (Leaky Confined) Aquifers—

All the methods applicable for confined aquifers are based on the assumption that the confining beds are incompressible, i. e. they release no water from storage, and are impervious,. However, in nature some confining beds have finite permeability and they release water from storage. Such confined aquifers where confining beds leak water either from or to the aquifer, are grouped as semi-confined aquifers. The available methods for analysis of pumping test data of semi-confined aquifers are categorised on the basis-with/without water released from storage in confining bed or beds. These methods are outlined as follows,

5·2·2·1 Semi-Confined Aquifer with Incompressible Confining Layers-Without Water Released from Storage in Aquitard :

Both steady-state and unsteady-state flow equations exist for a semi-confined aquifer which is under lain by an aquiclude and overlain by an aquitard, i. e. the lower confining bed is incompressible and impervious and the upper confining bed is incompressible but pervious to vertical passage of water through it. The methods based on these assumptions in addition to the assumptions listed in section 5·2, are outlined in the following sections.

5·2·2·1·1 Steady-state Flow :

In semi-confined aquifers, steady-state flow conditions are obtained because of the leakage from the semi-pervious confining layer. After certain time of pumping, the discharge rate equals the recharge rate of vertical leakage, resulting in steady-state flow conditions, which will be maintained as long as the phreatic level in the upper confining bed is kept constant.

5·2·2·1·1. 1 De Glee's Method :

In addition to the general assumptions listed in section 5·2, the following assumptions and conditions should be satisfied for use of De Glee's method—

- The aquifer is semi-confined without release of water from storage in confining beds.
- Flow to the well is in steady-state.
- The phreatic surface remains constant (drawdown of the phreatic surface $<5\%$ of the saturated part of the semi-pervious layer) so that leakage through the covering layer takes place in proportion to the drawdown of the piezometric level.
- $L>3$ b

The steady-state drawdown in a semi-confined aquifer may be expressed by De Glee's (1930, 1951) formula based on the assumptions listed earlier,

$$s_m = \frac{Q}{2\pi T} K_o(x) \qquad (5\cdot22)$$

where,

s_m = steady-state drawdown in metres in an observation well located at a distance, r from the pumped well (L)

$K_o(x)$ = modified Bessel function of second kind and of zero orders (Hankel function)

where,

$$x = \frac{r}{L}, \text{ as derived under}$$

$$x = \frac{\sqrt{K'/b'S} \cdot r}{\sqrt{T/S}}$$

$$= \frac{r}{\sqrt{T.b'/K'}}$$

$$= \frac{r}{\sqrt{T.C}}$$

$$= \frac{r}{L}$$

K' = Vertical Hydraulic conductivity of the upper confining layer in m/day (LT^{-1})

b' = saturated thickness of upper confining layer in metres (L)

$C = \frac{b'}{K'}$, Hydraulic resistance of the confining layer in days (T) ......(5·23)

$L = \sqrt{T.C.}$, Leakage factor in metres (L) (5·24)

Q, T and S as defined earlier

Eq. (5·22) may be written as,

$$s_m = \frac{Q}{2\pi T} Ko \left(\frac{r}{L}\right) \qquad (5\cdot25)$$

and

$$r = L\left(\frac{r}{L}\right) \qquad (5\cdot26)$$

Eqs. (5·25) and (5·26) may be expressed as

$$\log s_m \quad \left[\log \frac{Q}{2\pi T}\right] + \log K_o \left(\frac{r}{L}\right) \qquad (5\cdot27)$$

$$\log r = \left[\log L\right] + \log\left(\frac{r}{L}\right) \qquad (5\cdot28)$$

DISTANCE Vs DRAWDOWN (Steady-State) PLOT

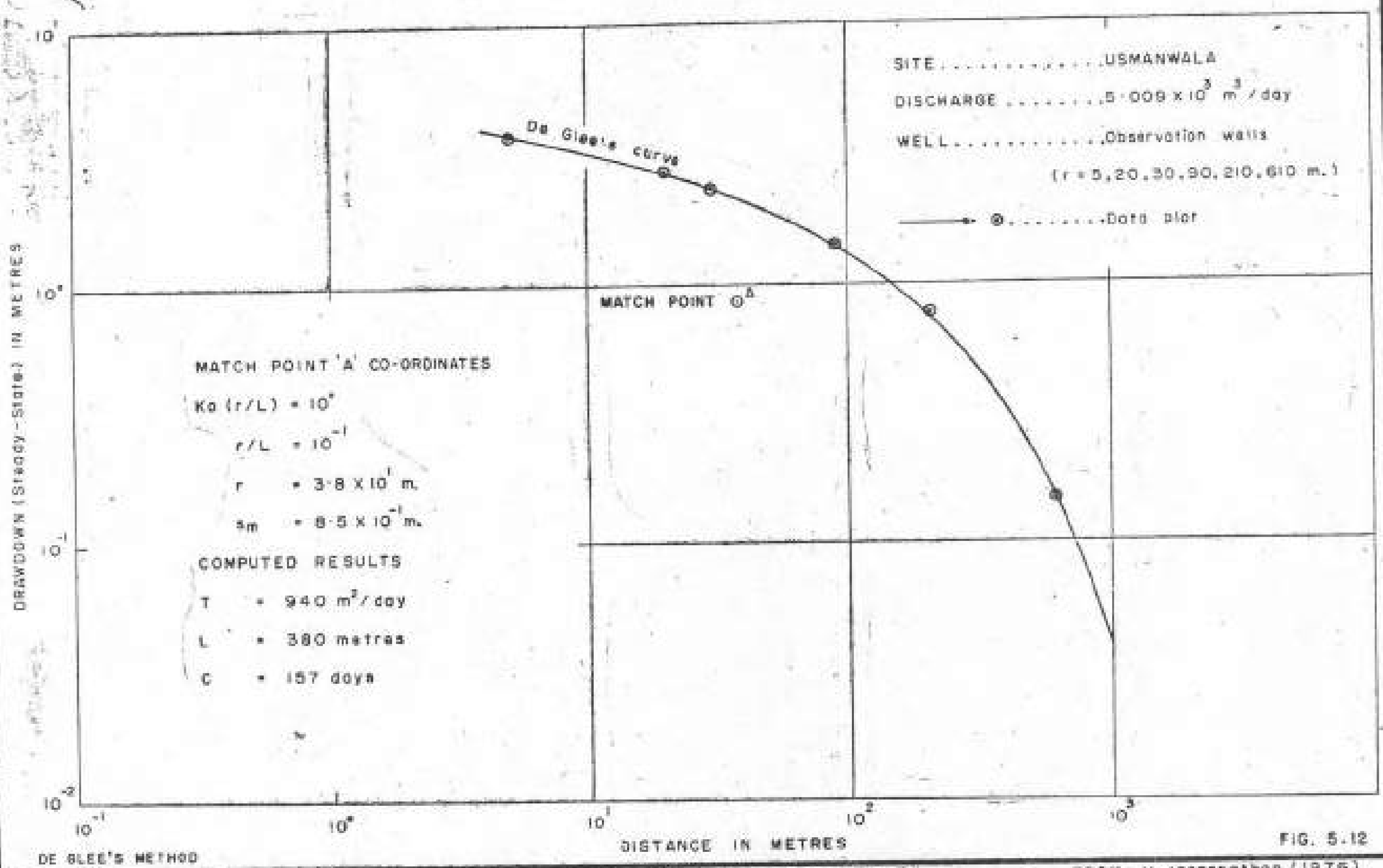


FIG. 5.12

FROM- V. Jagannathan (1976)

For a pumping test the bracketed portions are constant and the variable s_m is related to, r in the same manner that Ko (r/L) is related to (r/L). Thus, the form of Eqs. (5·27) and (5·28) once again suggests the same convenient method of graphical solution that has been described for resolving the Theis quation.

The values of Ko (x) for different values of x are available in the form of a table —Annexure IV Procedure :

— Prepare a type curve by plotting values of Ko (x) versus the values of x on a double logarithmic paper (Type curve).

— Plot on another sheet of logarithmic paper of same scale, the steady-state drawdowns, s_m of each observation well against its corresponding distance, r.

— Superimpose the field data plot on the type curve, keeping the co-ordinate axes parallel [Ko (r/L) axis parallel to s_m axis] and adjust until a position is found where most of the plotted points fall on a segment of the type curve.

— Select an arbitrary point 'A' on the overlapping portion of both sheets and note for 'A' the values of s_m, r, Ko (r/L) and r/L.

— Substitute these values into Eq. (5·22) and (5·24) and determine T, C and then by using Eq. (5·23) determine K'.

Example :

[After Jagannathan, V. (1976)]

Data of pumping test conducted in a semi-confined aquifer at 'Usmanwala' site located in the Ferozepur district, Punjab State, India, and analysed by De Glee's method is presented here. At 'Usmanwala' an alluvial semi-confined aquifer exists in the depth range of 55·5 metres to 87·5 metres—overlain and underlain by confining clayey beds which are 13·00 metres and 7·50 metres thick respectively. A fully penetrating testwell and 6 similar observation wells together with shallower observation wells located at distances of 5m, 20m, 30m, 90m, 210m, and 610m, were installed at 'Usmanwala' for testing purpose. The potentiometric head of the tested aquifer was around 2·50 metres below land surface. In April 1976, a test of 10,000 minutes duration was conducted at a constant discharge of 5·009 × 10^3 m³/day. The "time-drawdown" curves of all the observation wells indicate existence of steady-state flow conditions beyond 1000 min. of pumping. Further, drawdown in the overlying phreatic aquifer was negligible. The steady-state drawdown observed in all the observation wells at t = 1000 min. are given in the Table (5·4).

TABLE 5·4

Pumping test data, 'Usmanwala' site, India.

Well	Distance from the pumped well (m)	Steady-state drawdown at t = 1000 min (m)
1	2	3
OW—I	5	3·660
OW—II	20	2·680
OW—III	30	2·321
OW—IV	90	1·400
OW—V	210	0·754
OW—VI	610	0·142

Fig. (5·12) shows the "Distance-drawdown" (steady-state) plot analysed by De Glee's method. The calculation of aquifer parameters is given below.

$$Q = 5{\cdot}009 \times 10^3 \text{ m}^3/\text{day.}$$

Match point 'A' co-ordinates being,

$$Ko(r/L) = 10^0$$

$$r/L = 10^{-1}$$

$$r = 3{\cdot}8 \times 10^1 \text{m}$$

$$s_m = 8{\cdot}5 \times 10^{-1} \text{m}$$

Using the Eq. (5·22) we get,

$$T = \frac{Q}{2\pi s_m} \quad K_o(r/L)$$

$$= \frac{5{\cdot}009 \times 10^3 \times 10^0}{2 \times 3{\cdot}14 \times 8{\cdot}5 \times 10^{-1}}$$

$$= 938 \text{ m}^2/\text{day}$$

say, 940 m²/day

Now

r/L being 10^{-1} and r being $3{\cdot}8 \times 10^1$m

L works out to be 380 metres.

Using the Eq. (5·24),

$$C = \frac{L^2}{T}$$

$$= \frac{3{\cdot}8 \times 10^2 \times 3{\cdot}8 \times 10^{-2}}{9{\cdot}38 \times 10^2}$$

$$= 157 \text{ days}$$

b' being 13m, K' can be calculated by the use of Eq. (5·23)

$$K' = \frac{b'}{C} = \frac{1\cdot3 \times 10^1}{1\cdot57 \times 10^2} = 8\cdot28 \times 10^{-2} \text{ m/day}$$

5·2·2·1·1·2 Hantush—Jacob's Method :

Hantush and Jacob (1955) also independently derived the Eq. (5·22) which is,

$$s_m = \frac{Q}{2\pi T} \quad K_o(r/L)$$

Hantush (1956, 1964) noted that if r/L is small ($r/L \leqslant 0{\cdot}05$) Eq. (5·22) may be approximated by,

$$s_m \approx \frac{2\cdot30\,Q}{2\pi T}(\log 1\cdot12\ L/r) \qquad (5\cdot29)$$

Thus, a plot of s_m against r on a semi-logarithmic paper (r on logarithmic scale) will show a straight line relationship, for small values of r/L. For large values of r/L, the points would fall on a curve. The slope of the straight portion of the curve, i.e. drawdown difference, Δs_m per log cycle of r, is expressed by

$$\Delta s_m = \frac{2\cdot30\,Q}{2\pi T} \qquad (5\cdot30)$$

The extended straight line intercepts the r—axis at, r_o, where drawdown is zero, Eq. (5·29) reduces to,

$$O = \frac{2\cdot30\,Q}{2\pi T}\left(\log 1\cdot12 \frac{L}{r_o}\right) \qquad (5\cdot31)$$

$$\text{Or } 1.12\ \frac{L}{r_o} = 1$$

$$\text{Or } 1\cdot12 \frac{\sqrt{T.C.}}{r_o} = 1$$

$$\text{Hence, } C = \frac{(r_o/1\cdot12)^2}{T} \qquad (5\cdot32)$$

The method is applicable when the following conditions are satisfied—

— The assumptions and conditions of De Glee's method

— $r/L \leqslant 0\cdot05$

Procedure

— Plot on a semi logarithmic paper the values of s_m against the corresponding values of r (r on logarithmic scale)

— Draw a straight line through the points which appear to fall on a straight line

— Determine the slope of the straight line, Δs_m, i.e. drawdown difference per log cycle of r.

— Extend the straight line till it intercepts the r— axis where s = O, record r_o.

— Substitute the values of Q and Δs_m into Eq. (5·30) and solve for T.

— Substitute the values of T and r_o into Eq. (5·32) and determine C and consequently K', using Eq. (5·23)

Example :

[After Jagannathan, V. (1976)]

The steady-state drawdown data of pumping test at "Usmanwala" site given in Table (5·4), analysed by Hantush-Jacob's method, is presented here.

Fig. (5·13) shows the "distance-drawdown" plot for "Usmanwala" site analysed by the said method.

The calculation of aquifer parameters is given below,

$$Q = 5\cdot009 \times 10^3 \text{ m}^3\text{/day}$$

straight line parameters being,

$$\Delta s_m = 1\cdot80 \text{ m}$$
$$r_o = 5\cdot40 \times 10^2 \text{ m}$$

Using the Eq. (5·30),

$$T = \frac{2\cdot30\,Q}{2\pi\,\Delta s_m} = \frac{2\cdot30 \times 5\cdot009 \times 10^3}{2 \times 3\cdot14 \times 1\cdot80} = 1019 \text{ m}^2\text{/day}$$

say, 1020 m^2/day

Using the Eq. (5·32),

$$C = \frac{(r_o/1\cdot12)^2}{T} = \frac{(540/1\cdot12)^2}{1019} = 228 \text{ days.}$$

Using the Eq, (5·24)

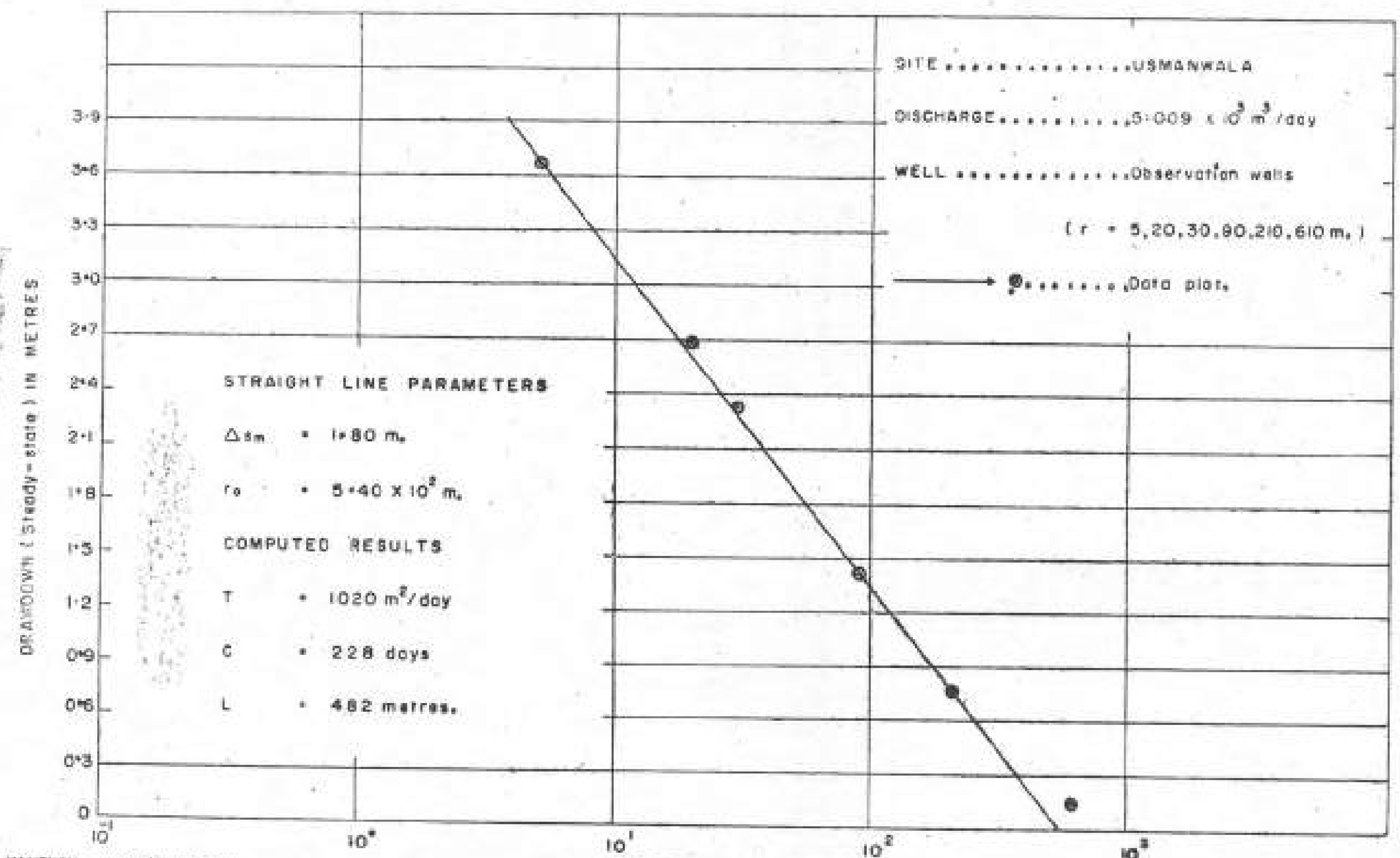
$$L = \sqrt{T.C.} = \sqrt{1019 \times 228} = 482 \text{ metres}$$

Also, b^1 = 13 m, using Eq. (5·23),

$$K^1 = \frac{b'}{C} = \frac{1\cdot3 \times 10^1}{2.28 \times 10^2} = 5\cdot70 \times 10^{-2} \text{ m/day}$$

Table (5·5) gives the summarised results of pumping test at 'Usmanwala' India.

DISTANCE Vs DRAWDOWN (Steady - State) PLOT



HANTUSH - JACOB'S METHOD

FIG. 5.13

FROM_ V. Jagannathan (1976)

TABLE 5·5

Summarised results of pumping test at 'Usmanwala' site, India.

S. No.	Method Used	Flow conditions	Data used	Plot of	T (m^2/day)	C (days)	L (metres)	Remarks
1.	De Glee	Steady-state	OW—I OW—II OW—III OW—IV OW—V and OW—VI	log r vs s_m	940	157	380	t=1000 min
2.	Hantush-Jacob	Steady-state	OW—I OW—II OW—III OW—IV OW—V and OW—VI	log r vs s_m	1020	228	482	t= 1000 min

5·2·2·1·2 Unsteady-state Flow

Hantush and Jacob (1955) derived the following equation for unsteady-state flow in a semi-confined aquifer :

$$s = \frac{Q}{4\pi T}\int_u^\infty \frac{1}{u}\exp.\left(u-\frac{r^2}{4L^2_u}\right) du. \quad (5\cdot33)$$

$$= \frac{Q}{4\pi T} W(u,\ r/L) \quad (5\cdot34)$$

where,

$$u = \frac{r^2 S}{4Tt} \quad (5\cdot35)$$

The short time drawdowns for semi-confined aquifers are described by Theis's non-equilibrium equation; whereas, long term drawdowns are described by the well function W (u, r/L). Values of W(u, r/L) for certain values of r/L, as u varies are given in tabulated form in Annexure V (after Hantush, 1956). Equations 5·34) and (5.35) derived by Hantush and Jacob (1955) have been used by Walton and Hantush to obtain aquifer parameters from the field data.

5·2·2·1·2·1 Walton's Method :

For application of Walton's method, following assumptions and limiting conditions should be satisfied :

— The assumptions listed in section 5·2

— The aquifer is semi-confined without release of water from storage in confining bed

— The flow to the well is in unsteady-state

— The water released from storage is discharged instantaneously with decline of head.

— The storage in the well can be neglected.

Procedure :

— Plot on a double logarithmic paper, values of W(u, r/L) versus 1/u for different values of r/L. This gives a family of type curves (Type curve : 4)

— Plot on an another sheet of double logarithmic graph paper of the same scale, values of s versus t/r^2 or t

— Superimpose this "time-drawdown" field data curve on the family of type curves and adjust while keeping the co-ordinate axes parallel, until a match position is found where the field data curve falls on one of the type curves (The initial part of the field data curve would generally fall on Theis curve).

— Select a match point 'A' on overlapping portion of two sheets and note for 'A' the values of W(u, r/L), 1/u, s, t/r^2 or t and r/L.

— Substitute these values and the value of Q into Eqs.(5·34) and (5·35) and solve for T and S.

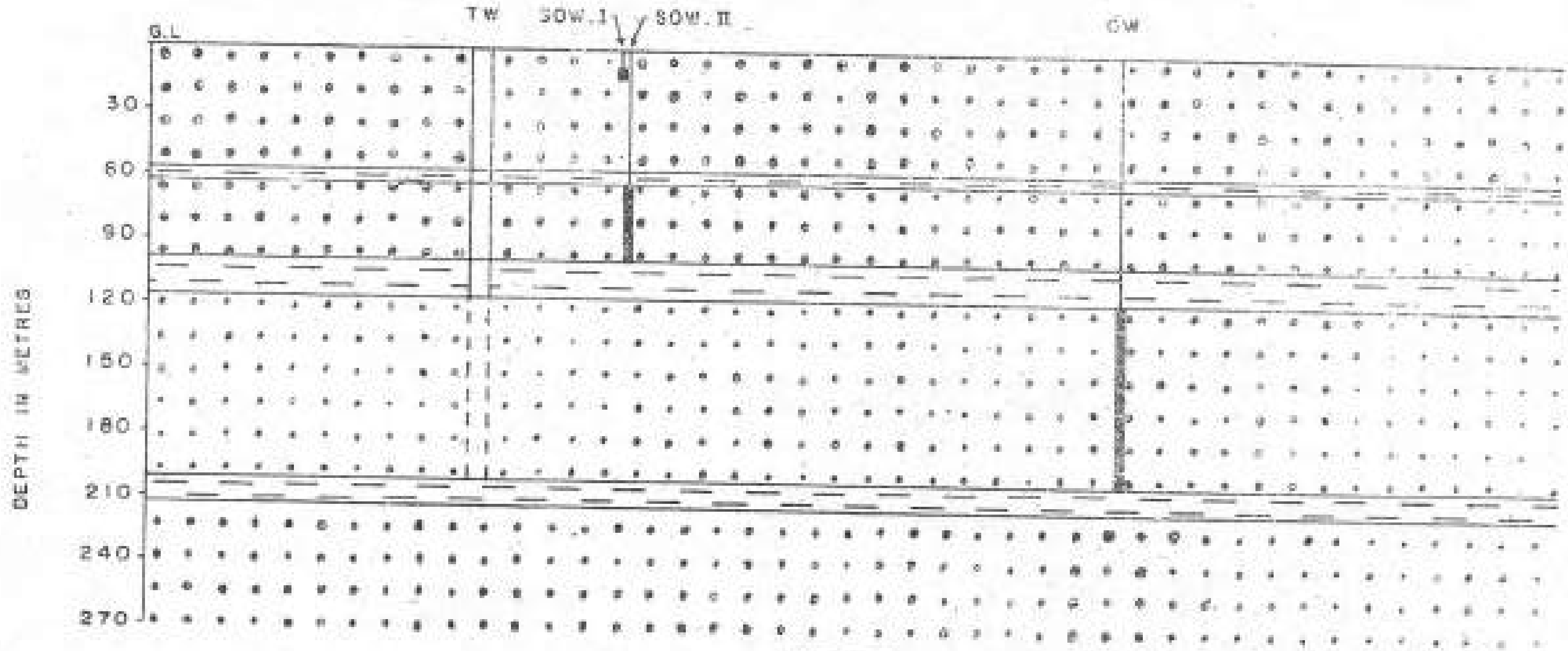
— Calculate the value of L from the r/L value of the matched type curve, and the known value of r.

— From Eq. (5·24), compute the value of C,

$$C = \frac{L^2}{T}$$

LITHOLOGICAL CROSS-SECTION OF THE PUMPING TEST SITE AT DAKOHA

SCALE (HOR.):- 1:2000



INDEX

Aquifer
(Sand with clay lenses) ....

Aquiclude / Aquitard
(Clay / Clay with silt) ....

Test Well .... T.W.

Observation Well .... O.W

.... Pumping Screen.

.... Observation Filter.

SOW. .... Shallow Observation Well.

FIG. 5.4.

FROM- Saini D.S. And Chadha D.K. (1973)

— Knowing the value of b', use Eq. (5·23) to determine the value of $K' = b'/C$

All symbols as defined earlier.

Example :

[After Saini, D. S. and Chadha, D. K. (1978)]

Data of pumping test conducted on a semi-confined aquifer at "Dakoha" site located in Upper Bari Doab basin, Punjab State, India, analysed by Walton's method, is presented here to illustrate a typical field case history. At 'Dakoha', an alluvial semi-confined aquifer exists in the depth range of 116·00 metres to 202·00 metres—underlain and overlain by confining clayey beds which are 11·00 metres and 17·00 metres thick respectively. (Fig. 5·14). In Dec. 1978, a test of 3600 minutes pumping duration was conducted on a fully penetrating testwell at a constant discharge rate of 5077 m³/day. The summarised water level data of all the wells and drawdown data collected from an observation well located at a distance of 200 metres from the pumped well, is given in table (5·6).

TABLE 5·6

Pumping test data, "Dakoha" site

Well	Distance from the pumped well (m)	Non-pumping water level (m.B.M.P.)	Maximum drawdown (m)	Remarks
Pumped	—	7·795	6·645	
OW	200·00	7·442	0·794	
SOW—I	22·00	6·850	0·089	Taps shallow aquifer
SOW-II	21·75	6·544	0·026	Taps near water table zones.

(B) Drawdown Data : (OW)

t (min) (1)	s (m) (2)
2	0·003
5	0·005
8	0·012
10	0·015
15	0·036
20	0·064
30	0·115
40	0·161
50	0·196
60	0·229
70	0·254
80	0·280
90	0·298
100	0·317
120	0·345
140	0·371
160	0·393
180	0·413
210	0·448
240	0·457
280	0·479
300	0·492
340	0·516
380	0·532
460	0·558
500	0·575
600	0·608
700	0·616
800	0·645
900	0·654
1000	0·671
1200	0·694
1400	0·710
1600	0·720
1800	0·732
2000	0·751
2200	0·756
2400	0·765
2600	0·768
2800	0·780
3000	0·782
3200	0·783
3400	0·781
3600	0·794

Fig. (5·15) exhibits "time-drawdown" curve of the observation well matched with one of the type curves of Walton. The calculation of aquifer parameters by use of Walton's method are given below,

$$Q = 5{\cdot}077 \times 10^3 \text{ m}^3/\text{day}$$

Observation well ($r = 2{\cdot}0 \times 10^2$ m)

"Match point" co-ordinates being,

$$W(u, r/L) = 10^0$$
$$1/u = 10^1$$
$$s = 2{\cdot}1 \times 10^{-1} \text{m}$$
$$t = 1{\cdot}5 \times 10^2 \text{ min}$$
$$= 1{\cdot}0416 \times 10^{-1} \text{ day}$$
$$r/L = 0{\cdot}15$$

Using Eqs. (5·34) and (5·35)

$$T = \frac{Q}{4\pi s} W(u, r/L)$$

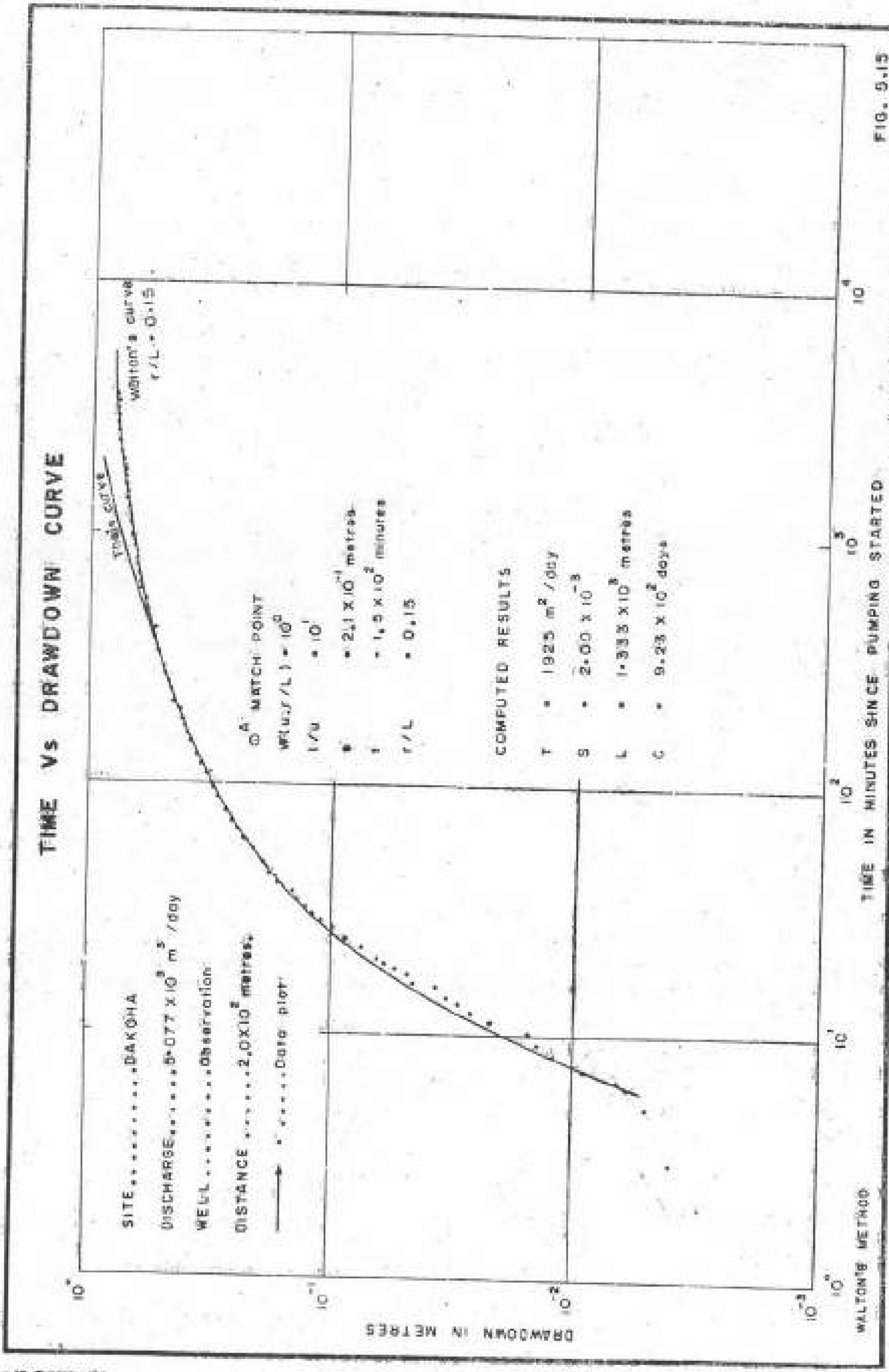

TIME Vs DRAWDOWN CURVE
SITE..........DAKOHA
DISCHARGE........8.077×10^{3} m^{3}/day
WELL..........Observation
DISTANCE........2.0×10^{2} metres.
........Data plot
Theis curve
Walton's curve
r/L = 0.15
A MATCH POINT
W(u,r/L) = 10^{0}
1/u = 10^{1}
s = 2.1×10^{-1} metres
t = 1.5×10^{2} minutes
r/L = 0.15
COMPUTED RESULTS
T = 1925 m^{2}/day
S = 2.00×10^{-3}
L = 1.333×10^{3} metres
C = 9.23×10^{2} days
TIME IN MINUTES SINCE PUMPING STARTED
DRAWDOWN IN METRES
WALTON'S METHOD
FROM - Saini D.S. and Chadha D.K. (1978)

FIG. 5.15

$$T = \frac{5 \cdot 077 \times 10^3 \times 10^0}{4 \times 3 \cdot 14 \times 2 \cdot 1 \times 10^{-1}}$$

$$= 1925 \text{ m}^2/\text{day}$$

$$S = \frac{4 \text{ T t u}}{r^2}$$

$$= \frac{4 \times 1 \cdot 925 \times 10^3 \times 1 \cdot 0416 \times 10^{-1} \times 10^{-1}}{4 \times 10^4}$$

$$= 2 \cdot 00 \times 10^{-3}$$

Because,

$r = 200$ m and $r/L = 0 \cdot 15$

$\therefore L = 1333$ m

Using Eq. (5·24)

$$C = \frac{L^2}{T}$$

$$= \frac{1 \cdot 333 \times 10^3 \times 1 \cdot 333 \times 10^3}{1 \cdot 925 \times 10^3}$$

$$= 9 \cdot 23 \times 10^2 \text{ days}$$

b' being 17·00 metres, K' can be computed by using Eq (5·23),

$$K' = \frac{b'}{C}$$

$$= \frac{17 \cdot 00}{9 \cdot 23 \times 10^2} = 1 \cdot 84 \times 10^{-2} \text{ m/day}$$

5·2·2·1·2·2 Hantush's Method :

Hantush (1956) inflection point method is based on Eq (5·33) and can be used if steady-state drawdown is approximately known. For its use following assumptions and limiting conditions should be satisfied —

— The assumptions and limiting conditions listed for Walton's method (Section 5·2·2·1·2·1).

— The steady state drawdown should be approximately known.

For an observation well, the time, t, versus drawdown, s curve on a semi-logarithmic paper (t on logarithmic scale) has an inflection point "p" where following relations hold :

(i) $s_p = 1/2 \; s_m = \frac{Q}{4 \pi T} \text{Ko}(\frac{r}{L})$ (5·36)

(ii) $u_p = \frac{r^2 S}{4 T t_p} = \frac{r}{2 L}$ (5·37)

(iii) $\Delta s_p = \frac{2 \cdot 30 \; Q}{4 \pi \; T} \; e^{-r/L}$ (5·38)

(iv) $2 \cdot 30 \frac{s_p}{\Delta s_p} = e^{r/L} \; K_o(\frac{t}{L})$ (5·29)

Where,

The index 'p' means at the inflection point. All other symbols, as defined earlier.

Values of function e^x e^{-x} Ko(x) and e^xKo (x) after Hantush (1956) are given in tabulated form in Annexure—IV.

Procedure :

— Plot for a single observation well, the values of drawdown, s against the corresponding values of time t on a semi-logarithmic paper (t on logarithmic scale).

— Record or extrapolate the value of steady-state drawdown, s_m.

— Calculate the value of s_p from Eq. (5·36) and locate the inflection point, 'p' on "time-drawdown" curve at $s = s_p$.

— Read the value of t_p, from time axis.

— Draw a tangent to the curve through the inflection point, and determine its slope, Δs_p i.e. drawdown difference over one log cycle of time.

— Substitute the values of s_p and Δs_p into Eq. (5·39) and determine the value of r/L with the help of Hantush's tables given in annexure-IV.

— Knowing r/L and r, calculate the value of L.

— Knowing r/L, read the value of $e^{-\frac{r}{L}}$ from the tables.

— Substitute the values of Q, Δs_p and $e^{-\frac{r}{L}}$ into Eq. (5·38) and solve for T.

— Substitute the value of T, t_p, r, and L into Eq. (5·37) and determine S.

— Knowing T and L, calculate the value of C from the relationship $C = \frac{L^2}{T}$ and consequently, K' being equal to $\frac{b'}{C}$

TIME Vs DRAWDOWN CURVE

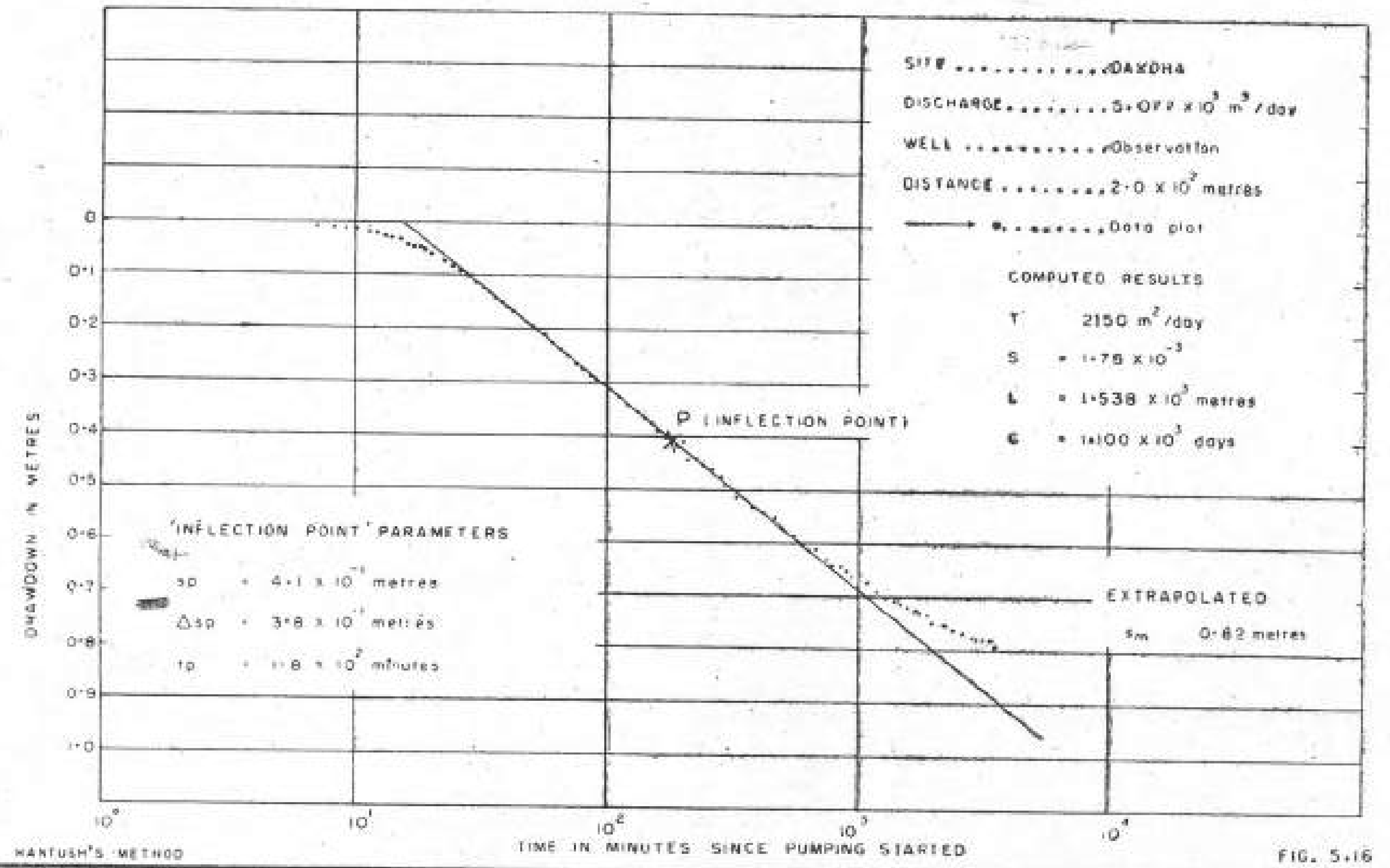


FIG. 5.16

FROM - Saini O.S. and Chadha D.K. (1978)

Example :

[After Saini, D. S. and Chadha, D. K. (1978)]

The drawdown data Table (5·6) of pumping test at 'Dakoha' site, analysed by Hantush's inflection point method is presented here as a field case history. Fig. (5·16) exhibits "time-drawdown" curve of observation well analysed by Hantush's method. The calculation of aquifer parameters are as under :

$Q = 5{\cdot}077 \times 10^3$ m³/day

Observation well ($r = 200$ m)

Extrapolated $s_m = 0{\cdot}82$ m

$s_p = 1/2\, s_m = 4{\cdot}1 \times 10^{-1}$ m

$\Delta s_p = 3{\cdot}8 \times 10^{-1}$ m

$t_p = 1{\cdot}8 \times 10^2$ min $= 1{\cdot}25 \times 10^{-1}$ days.

Substituting the values of s_p and Δs_p into Eq. (5·39)

$$2{\cdot}30 \frac{s_p}{\Delta s_p} = 2{\cdot}30 \times \frac{4{\cdot}1 \times 10^{-1}}{3{\cdot}8 \times 10^{-1}}$$

$$= 2{\cdot}48 = e^{r/L}\, K_0\,(r/L)$$

Using the tables of Hantush given in Annexure-IV, we get,

$r/L = 0{\cdot}13$

and

$\therefore \quad L = 1{\cdot}538 \times 10^3$ metres

for r/L value of 0·13, $e^{-r/L}$ is read from the Hantush's table as 0·878.

Substituting the values of Q, Δs_p and $e^{-r/L}$, into Eq. (5·38),

$$T = \frac{2{\cdot}30\, Q}{4\pi \Delta s_p}\, e^{-r/L}$$

$$= \frac{2{\cdot}30 \times 5{\cdot}077 \times 10^3 \times 8{\cdot}78 \times 10^{-1}}{4 \times 3{\cdot}14 \times 3{\cdot}8 \times 10^{-1}}$$

$= 2150$ m²/day

Introduction of known values of T, tp, r and L into Eq. (5·37) gives.

$$S = \frac{r\, 4\, T\, tp}{2\, L\, r^2} = \frac{2\, T\, tp}{L\, r}$$

$$= \frac{2 \times 2{\cdot}15 \times 10^3 \times 1{\cdot}25 \times 10^{-1}}{1{\cdot}538 \times 10^3 \times 2{\cdot}0 \times 10^2}$$

$$= 1{\cdot}75 \times 10^{-3}$$

Also,

$$C = \frac{L^2}{T}$$

$$= \frac{1{\cdot}538 \times 10^3 \times 1{\cdot}538 \times 10^3}{2{\cdot}15 \times 10^3}$$

$= 1.100 \times 10^3$ days

Now, b' being 17·00 m, K' works out to be,

$$K' = \frac{b'}{C}$$

$$K' = \frac{1{\cdot}7 \times 10^1}{1{\cdot}100 \times 10^3}$$

$= 1{\cdot}55 \times 10^{-2}$ m/day

Table (5·7) gives the summarised results of the test at "Dakoha" India.

TABLE 5·7

Summarised results of pumping test at 'Dakoha' site, India.

S. No.	Method used	Flow conditions	Data used	Plot of	T (m²/day)	S	C (days)	L (meters)
1.	Walton	Unsteady-state	OW	log t vss	1925	$2{\cdot}00 \times 10^{-3}$	923	1323
2.	Hantush	Unsteady-state	OW	log t vss	2150	$1{\cdot}75 \times 10^{-3}$	1100	1538

5·2·2·2 Semi-Confined Aquifer with Compressible Confining Layers—with Water Released From Storage in Aquitards

Hantush (1960) presented flow equations for semi-confined aquifers in which the storage of water in confining beds is taken into account.
His main equations being—

$$T = \frac{Q}{4\pi s} H(u, \beta) \qquad (5\cdot40)$$

where,

$$H(u, \beta) = \int_u^{\infty} \frac{e^{-y}}{y} \operatorname{erfc}\left(\frac{\beta/\sqrt{u}}{\sqrt{y(y-u)}}\right) dy \qquad (5\cdot41)$$

$$u = \frac{r^2 S}{4Tt} \qquad (5\cdot42)$$

and

$$\beta = \frac{r}{4b}\left[\sqrt{\frac{(K'Ss'}{K\,Ss}} + \sqrt{\frac{K''\,Ss''}{K\,Ss}}\right] \qquad (5\cdot43)$$

Where,

K = hydraulic conductivity of main aquifer

K′, K″ = hydraulic conductivities of semi-pervious confining beds.

$S = b\,Ss$, $S' = b'Ss'$, $S'' = b''Ss''$ — Storativities of the main aquifer and the semi-pervious beds respectively.

Ss, Ss′ & Ss″ = Specific storage of the main aquifer and confining beds of b, b′ and b″ thickness, respectively.

All other symbols, as defined earlier.

Eqs. (5·40) to (5·43) are the general solutions for the drawdown distribution in all confined aquifers whether they are leaky or non-leaky. Thus, if S′, S″, K′ and K″ tend to zero, the parameter β approaches zero and the Eq. (5·40) becomes Theis equation i.e. H (u, β) = W (u).

If K′, S′ and S″ approach zero, the solutions become equal to Eq (5·34) of Hantush and Jacob (1955) — the equation for unsteady state flow in a semi-confined equifer without release of water from storage of confining beds. Values of H (u,β) for certain values of β, as u varies are available in table form (Hantush 1964) — Annexure = VI.

5·2·2·2·1 Hantush's Modified Method :

For use of Hantush's modified method, following assumptions and limiting conditions should be satisfied —

- The assumptions listed in section 5·2
- The aquifer is semi-confined with water released from storage in confining beds
- The flow to the well- is in unsteady state.
- The water released from storage is discharged instantaneously with decline of head.
- The storage in the well can be neglected.

Procedure :

- Plot on a double logarithmic paper, values of H (u, β) versus 1/u for different values of β-gives a family of type curves (type curve : 5).
- Plot on an another sheet of logarithmic graph paper of the same scale, values of s versus t/r^2 or t.
- Superpose this "time-drawrown" field data curve on the family of type curve 5, keeping the H (u, β) axis parallel with the s axis and 1/u axis parallel with the t axis. Adjust until a match position is found where the field data curve falls on one of the type curves.
- Select a match point 'A' on the overlapping portion of two sheets and note for 'A' the values of H (u, β), 1/u, s, t and β.
- Substitute these values and the values of Q and r into Eqs. (5·40) and (5·42) and solve for T and S

$$T = \frac{Q}{4\pi s} H(u, \beta)$$

and

$$S = \frac{4Ttu}{r^2}$$

Example :

[Taken from Ground Water Hydraulics by S. W. Lohman (1972); data of F.S. RiLey, U.S. Geol, Survey, Calif.]

A field case history of the pumping test carried out at Pixley, Calif given in Ground-Water Hydraulics, by S. W. Lohman, Geological Survey professional paper 708, is reproduced here as an example of a semi-confined aquifer with release of water from storage in

TIME Vs DRAWDOWN CURVE

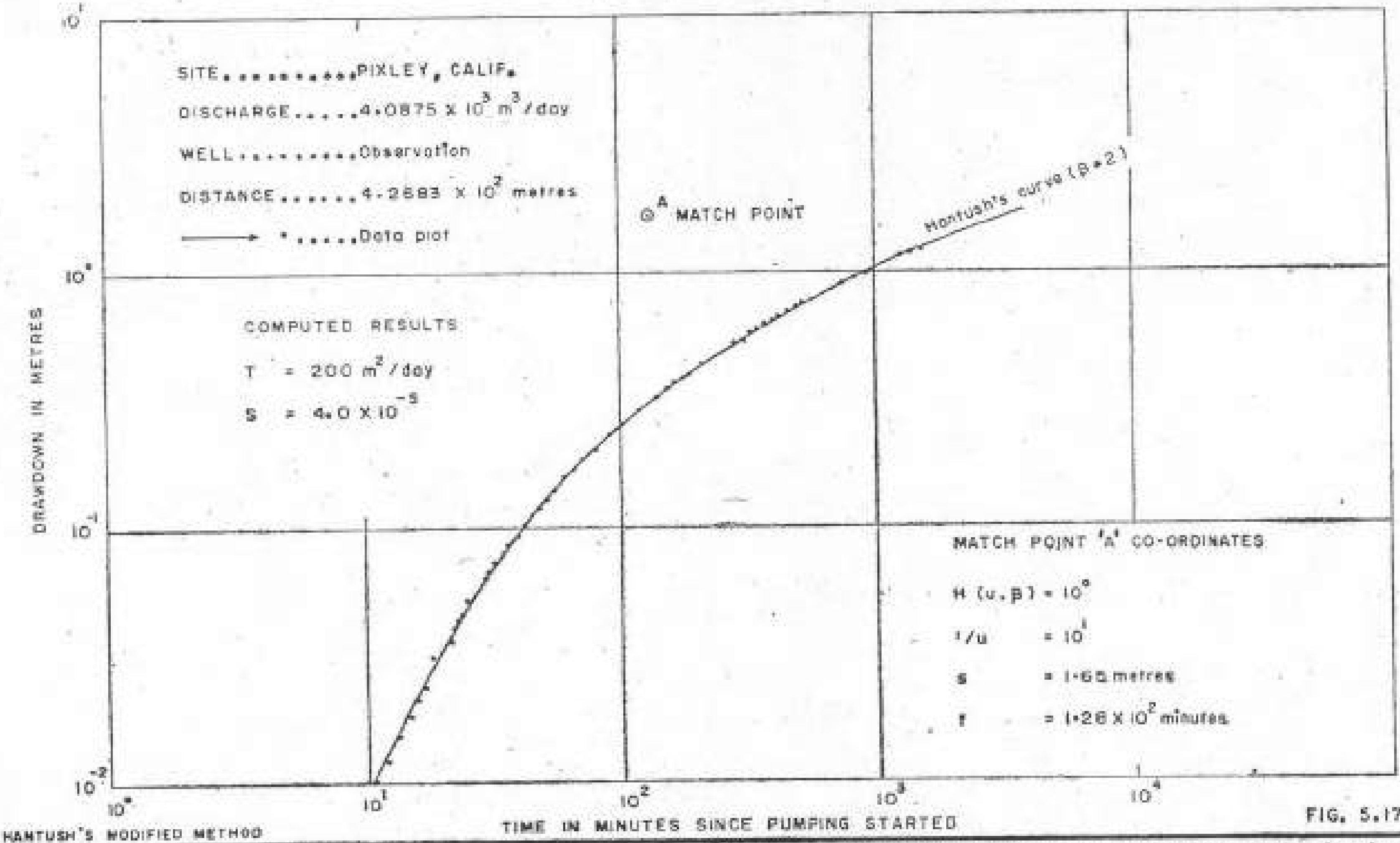


FIG. 5.17

(Data from F.S. Riley, U.S. Geol. Survey, Calif. given in - Ground water Hydraulics, S.W. Lohman) aquitard and analysed by **Hantush's** modified method. Table (5·8) gives the drawdown data of an observation well located at a distance of 1400 feet (426·83 m) from the pumped well. The pumped well is reported to be 600 feet (183 m) deep, constant discharge being 750 gpm (4087·5 m³/day) Fig. (5·17) exhibits the "time-drawdown" field data curve matched with one of the Hantush's curves,

TABLE 5·8

[Taken from Groundwater Hydraulics by S. W. Lohman (1972) Drawdown of water level in observation well 23S/25 E 17 Q2 Pixley, Calif. March 13, 1963].

[(Drawdown corrected for pre-test trend. Data from Francis S. Riley (written commun. March 5, 1968)]

Time since pumping began, (t) (min)	Drawdown (s) (ft)	(m)	Time since pumping began, (t) (min)	Drawdown (s) (ft)	(m)
6·37	0·01	0·003	70	0·60	0·183
8·58	0·02	0·006	80	0·65	0·198
10·23	0·03	0·009	90	0·75	0·229
11·90	0·04	0·012	100	0·82	0·250
12·95	0·05	0·015	137	1·04	0·317
14·42	0·06	0·018	150	1·12	0·342
15·10	0·07	0·021	160	1·17	0·357
16·88	0·08	0·024	173	1·24	0·378
17·92	0·10	0·031	184	1·27	0·387
21·35	0·12	0·036	200	1·35	0·412
21·70	0·13	0·040	210	1·40	0·427
22·70	0·14	0·043	278	1·68	0·512
23·58	0·15	0·046	300	1·76	0·531
24·65	0·17	0·052	315	1·83	0·558
29	0·21	0·064	335	1·87	0·570
30	0·22	0·067	365	1·99	0·607
32	0·24	0·073	390	2·10	0·641
34	0·26	0·079	410	2·13	0·650
36	0·28	0·085	430	2·20	0·671
38	0·30	0·092	450	2·23	0·680
41	0·33	0·101	470	2·29	0·698
44	0·36	0·110	490	2·32	0·708
47	0·38	0·116	510	2·39	0·729
50	0·42	0·128	560	2·48	0·756
54	0·46	0·140	740	2·92	0·891
60	0·52	0·159	810	3·05	0·930
65	0·56	0·171	890	3·19	0·973
			1,255	3·66	1·11
			1,400	3·81	1·162
			1,440	3·86	1·177
			1,485	3·90	1·190

The aquifer parameters calculations are given below:

$Q = 4087\cdot5$ m³/day

$r = 426\cdot83$ m

"Match Point" Co-ordinates being,

$H(u, \beta) = 10^0$

$$\frac{1}{u} = 10^2$$

$s = 1\cdot65$ m

$t = 128$ min $= 8\cdot8 \times 10^{-2}$ days

$\beta = 2$

Using Eqs. (5.40) and (5·42), we get,

$$T = \frac{Q}{4\pi s} H(u, \beta)$$

$$= \frac{4\cdot0875 \times 10^3 \times 10^0}{4 \times 3\cdot14 \times 1\cdot65}$$

$= 197$ m²/day

say, 200 m²/day

$$S = \frac{4\,T\,t\,u}{r^2}$$

$$= \frac{4 \times 1 \cdot 97 \times 10^{2} \times 8 \cdot 88 \times 10^{-2} \times 10^{-1}}{1 \cdot 8218 \times 10^{5}}$$
$$3 \cdot 84 \times 10^{-5}$$
$$\text{say} \quad 4 \cdot 0 \times 10^{-5}$$

Remarks :

'time-recovery' and "distance-recovery" data may also be analysed by the methods detailed in section 5·2·2. The recovery data to be computed in the manner explained in section 5·2·1·2·1 (Theis's method). Under steady-state conditionsl, recovery values at any particular time since pumping stopped, (t') can be computed as the difference between the steady-state water level and the water level at time, t'.

5·2·2·3 Evaluation of Methods :

(i) *De Glee's Method* :

Since true steady-state flow conditions can be attained in semi-confined aquifers without release of water from storage of confining beds, the De Glee's "distance-drawdown/recovery" steady-state type curve method can effectively be applied for determination of aquifer parameters, provided at least 3 observation well data is available. S value can't be determined by this method.

(ii) *Hantush—Jacob's Method* :

This is a "distance-drawdown/recovery" steady state straight line method and for its application besides the assumptions of De Glee's method, another assumption that $r/L < 0 \cdot 05$ should be satisfied. Therefore, steady-state drawdown data of at least 2 observation wells, and preferably of 3 observation wells, located very near to the pumped well, is needed for its use. For example, if the value of leakage factor, L is 2000 m, the observation wells should be located within 100 m distance from the pumped well.

(iii) *Walton's Method* :

Walton's type curve method is used for analysis of "time-drawdown/recovery" data for unsteady—state flow conditions in a semi-confined aquifer without release of water from storage in confining beds. Hence, shorter duration pumping tests can be analysed by this method. However, at times, it is difficult to obtain a unique match of the field data curve with one of the family of type curves of Walton, specially when sufficient number of initial data points don't fall on Theis's nonleaky type curve.

(iv) *Hantush's Method* :

Hantush's 'inflection point' method is used for analysis of "time-drawdown/recovery" data and for its application, the assumptions underlying Waltons's method should be satisfied. Moreover, extrapolated value of steady-state drawdown should be appoximately known. Therefore, short duration pumping tests wherein steady-state flow conditions are not attained, can be analysed by this method provided the steady-state drawdowns could be extrapolated accurately—upon which would depend the accuracy of the calculated values of the aquifer parameters.

Remarks :

All the methods reviewed above (De Glee's, Hantush —Jacob's, Walton's and Hantush's methods) are based on the assumption that the lower confining bed is incompressible and impervious and the upper confining bed is incompressible but pervious to vertical passage of water through it. However, many times, the lower confining bed is also observed to be leaky. In such cases, the values of parameters determined by these methods are of doubtful nature.

(v) *Hantush's Modified Method* :

This type curves method is used for analysis of "time-drawdown/recovery" data for unsteady-state flow conditions in a semi-confined aquifer with water released from storage in confining beds. Based on general flow conditions developed by Hantush (1960) its application requires thorough knowledge of the nature of confining beds.

5·2·3 Unconfined Aquifers with Delayed Yield :

Flow to wells in unconfined aquifers (water-table aquifers) is related to anisotropy resulting in delayed drainage from storage, vertical components of flow, well storage capacity, degree of well penetration, and changes in aquifer saturated thickness. Generally, all the unconfined aquifers exhibit the delayed yield phenomenon, i.e. the gravity drainage is not immediate. According to Walton (1960 a)—'Three' distinct segments of "time-drawdown" curve may be recognised under water-table conditions Unconfined stratified aquifers react initially in the same way as does a confined aquifer. Gravity drainage is not immediate but water is released instantaneously from storage by the compaction of the aquifer and by expansion of the water itself-describes the 'first' segment of "time-drawdown" curve which conforms to the Theis curve. The 'second' segment of the "time-drawdown" curve shows a decrease in slope because of the replenishment by gravity drainage from the interstices left above the cone of depression, and there is a marked discrepancy between "second"segment curve and the Theis type curve. The 'third' segment which may start from several minutes to several days after pumping has begun, represents the period during which the "time-drawdown" curve again conforms closely to the Theis type curve. In 'first' and 'third' segments, the flow is substantially radial, whereas, during intermediate 'second' segment the drawdown is controlled by vertical components of flow. Theis equation '(Theis's method or Jacob's method) could be applied to the 'first' segment of "time-drawdown" curve for determination of transmissivity and early time storativity, indicative of artesian conditions. Transmissivity and specific yield values could be determined by applying Theis equation (Jacob's method only, Theis's type curve method not applicable due to limited data curve) to the 'third' segment of "time-drawdown" curve. In case, 'third' segment drawdown data is available for at least 3 observation wells, "distance-drawdown" analysis based on Theis's equation, yields values of transmissivity and specific yield.

Boulton (1954 b, 1963, 1964) derived an equation which takes into account the delayed yield from storage. Boulton (1963) delayed yield type curves method used for analysis of "time-drawdown" data, is discussed hereunder—

5·2·3·1 **Boulton's Method**

For use of Boulton's method, following assumptions and limiting conditions may be satisfied.

— The assumption listed in section 5·2

— The aquifer is unconfined but shows delayed yield phenomenon.

— The flow to the well is in unsteady—state

— The storage in the well can be neglected. Boulton (1963) assumes that the effective co-efficient of storage of an unconfined aquifer is

$$S_A+S_Y = N\,S_A \quad (44)$$

$$\text{or } N = 1+\frac{S^Y}{S_A} \quad (5\cdot45)$$

Where,

N=a factor

$_A$ =early time co-efficient of storage

S_Y=specific yield,

The general flow equation for an unconfined aquifer with delayed yield, in analogy to the Theis equation is expressed as,

$$s=\frac{Q}{4\pi T}W\,(u_{AY}, r/B) \quad (5\cdot46)$$

Where,

$W(u_{AY}, r/B)$ may be called "well function of Boulton."

Under early—time conditions, Eq. (5·46) reduces to,

$$s=\frac{Q}{4\pi T}(u_A, r/B) \quad (5\cdot47)$$

Where,

$$u^A=\frac{r^2S_A}{4\,Tt} \quad (5\cdot48)$$

Under later—time conditions, Eq. (5·46) reduces to,

$$s=\frac{Q}{4\pi T}\,W\,(u_Y, r/B) \quad (5\cdot49)$$

Where,

$$U_Y=\frac{r^2S_Y}{4\,Tt} \quad (5\cdot50)$$

Values of $W(u_{AY}, r/B)$ for different values of $\frac{1}{u_A}$ and $\frac{1}{u_Y}$ for a practical range of values of r/B are given in Annexure VII.

The above mentioned formulae are valid when N tends to infinity. In practice this means that N>100 Boulton's method is not applicable then N<100, however, it may be applied when 10<N<100.

The element B, is called the drainage factor and is defined as,

$$B=\sqrt{\frac{T}{\alpha S^Y}} \quad (5\cdot51)$$

and is expressed in metres (L)

$\frac{1}{\alpha}$ is called the "Boulton delay index." It is expressed in days (T) and is used in combination with the "Boulton's delay index curve" Fig. (5·18) to determine the time, twt beyond which the delayed yield ceases to affect the drawdown.

Procedure :

— Construct the family of "Boulton type curves" by plotting $W(u_A, r/B)$ versus $\frac{1}{u_A}$ and $\frac{1}{u_Y}$ for a practical range of values of $\frac{r}{B}$ on a double logarithmic paper using Annexure-VII. The left hand portion of Boulton's type curves shows type—'A' curves $\left[W(u_A, r/B) \text{ vers} \qquad\right]$ and the right hand portion shows type—'Y' curves $\left[W(u_Y, r/B) \text{ versus } \frac{1}{u_Y}\right]$(Type Curve: 6)

— Plot on an another sheet of logarithmic paper of the same scale, the values of drawdown, s versus time, t—describes the "time-drawdown" field data curve.

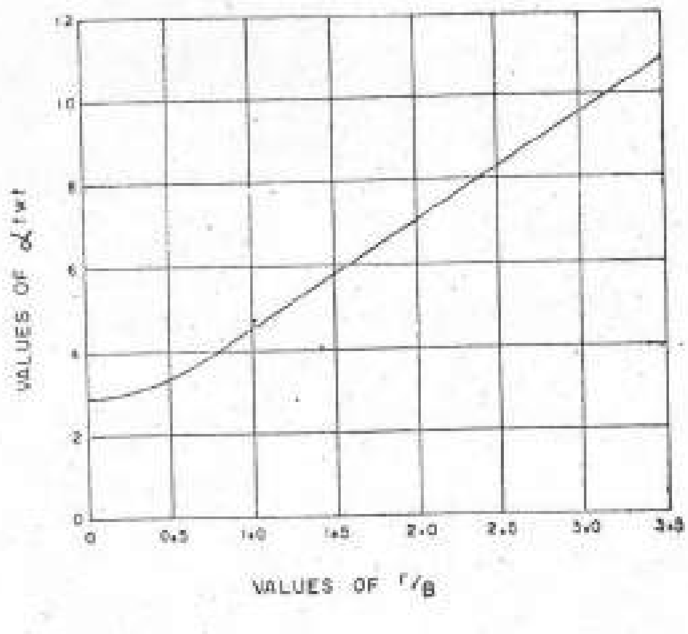
— Superpose this field data curve on the type 'A' curves, keeping the $W(u_A, r/B)$ axis parallel with s axis. Adjust until a match position is found where as much as possible of the early time curve falls on one of the 'type A' curves.

— Note the value of $\frac{r}{B}$ of the matched type 'A' curve.

— Select a match point 'A' on the overlapping portion of two sheets and note for 'A' the values of $W(u_A, r/B)$ $\frac{1}{u_A}$, s and t

FIG. 5.18

BOULTON'S DELAY-INDEX CURVE



FROM - Prickett (1965)

— Substitute these values and the known values of Q and r into Eqs. (5·47) and (5.48) and solve for T and S_A

— Move the field data curve until as much as possible of the later time curve falls on the type 'Y' curve with the same value of $\frac{r}{B}$ as for the matched type 'A' curve.

— Similarly as before, select a match point 'Y' and note the values of $W(u_Y, r/B)$, $\frac{1}{u_Y}$, s and t

— Substitute these values and the values of Q and r into Eqs. (5·49) and (5·50) and for solve T and S_Y.

— Check that the two values of T calculated by the early and the late "match point" co-ordinates, are approximately equal.

— Compute the value of N, by substituting the values of S_A and S_Y into Eq. (5·45) and ensure Boulton methods applicability.

— Calculate the value of drainage factor, B from the known values of r/B and r.

— Calculate the value of α, reciprocal of delay index, by substituting the values of B, T and S_Y into Eq. (5·51).

— From Boulton's delay index curve Fig. (5·18) read the value of αt_{wt} corresponding to the matched value of r/B

— Calculate twt, the time in days after which delayed yield ceases to affect the drawdown, from the known values of α and α twt.

— Check that at time t=twt, the "time-draw down" field data curve merges with the right hand Theis curve.

Example :

[After Bhatnagar, N. C., Agashe, R. M., and Sikka, V.M. (1977)]

Data of pumping test conducted in an unconfined aquifer at "Raipur" site located in Upper Yamuna river basin, Haryana state, India and analysed by Boulton's method is presented here. At "Raipur" an alluvial unconfined aquifer exists in the depth range of 7 metres (water-table) to 117 metres below land surface-undertain by an impervious basis clayey bed which is 14·00 metres thick. Fig (5·19) shows a lithological cross-section of the pumping test site. In January 1977, a test of 12,500 minutes pumping duration was conducted on a fully penetrating test-well at a constant discharge of 6540 m³/day. The summarised water level data of all the wells and the drawdown data of two observation wells, I and II, located at distances of 10·60 m and 31·10 m from the pumped well, is given in Table (5·9.)

TABLE 5·9

Pumping test data, "Raipur" site

(A) Water Level/Drawdown Data :

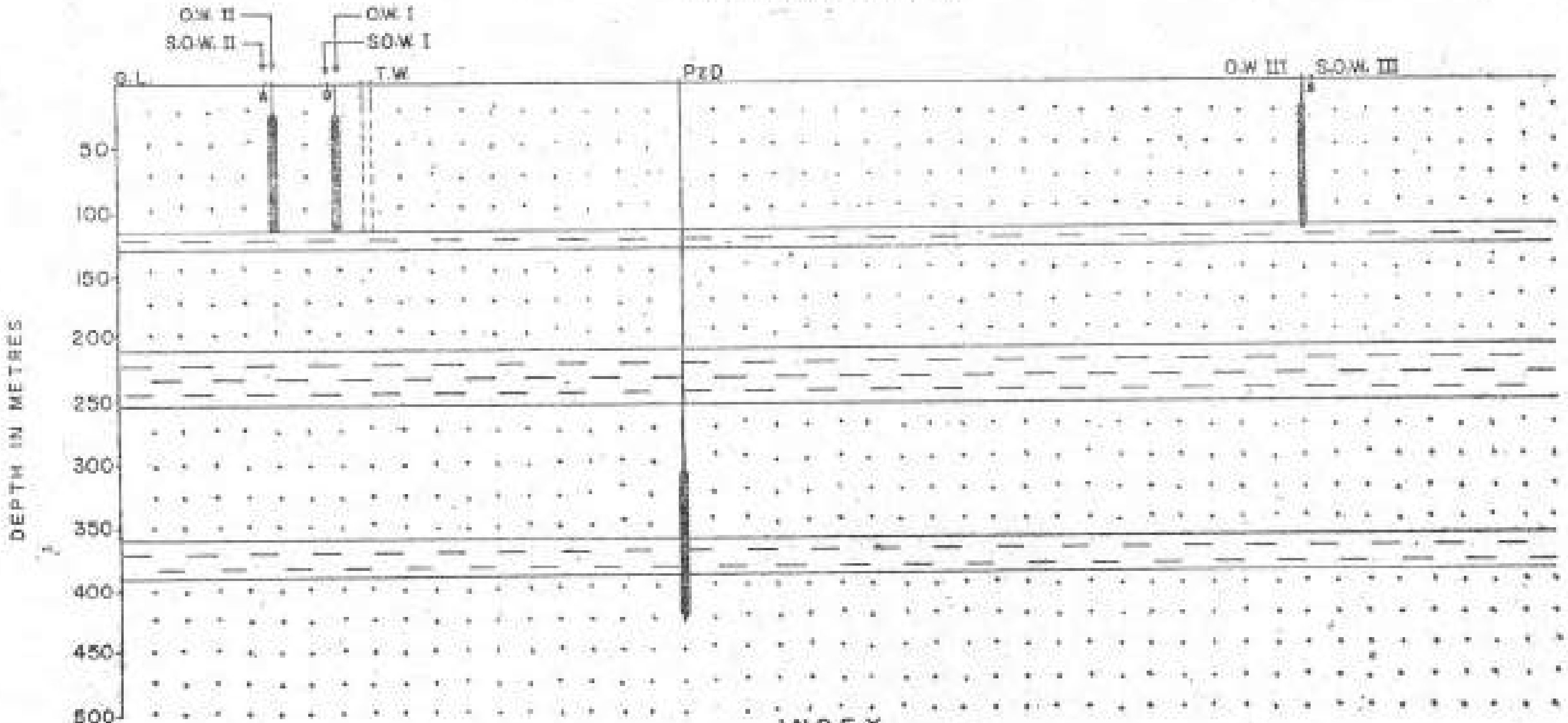
Well	Distance from the pumped well (m)	Non-pumping water level (m.B.M.P.)	Maximum Drawdown (m)	Remarks
Pumped	—	7·235	4·733	—
OW—I	10·60	7·939	2·658	—
SOW—I	10·50	7·866	0·221	Taps near water table zones.
OW—II	30·10	7·946	1·837	—
SOW—II	30·30	7·977	1·769	Taps near water table zones.
OW—III	300·70	8·418	0·429	Data erratic, not analysed by any method.
SOW-III	300·70	8·265	0·381	Taps near water table zones.
PzD	100·30	6·820	Nil	Taps deeper aquifer.

(B) Drawdown Data :

t	OW—I	OW—II
(min)	s (m)	s (m)
1	0·853	0·064
3	1·187	0·244
5	1·357	0·399
7	1·464	0·506
10	1·580	0·619
12	1·639	0·674
15	1·724	0·743
20	1·806	0·827
25	1·878	0·883
30	1·932	0·927
35	1·981	0·969
40	2·003	0·991
50	2·070	1·038
60	2·106	1·074
70	2·136	1·100
80	2·156	1·114
90	2·171	1·134
100	2·181	1·144
120	2·214	1·169
140	2·235	1·184
160	2·275	1·209
180	2·276	1·214
200	2·282	1·217
240	2·290	1·232
300	2·301	1·239
340	2·316	1·250
400	2·328	1·266
440	2·333	1·268
500	2·335	1·273

LITHOLOGICAL CROSS-SECTION OF THE PUMPING TEST SITE AT RAIPUR

SCALE (HOR) :- 1:2000



INDEX

Aquifer (Sand with clay lenses)..............

Aquiclude / Aquitard (Clay / Clay with silt)..............

Test Well .... T.W.

Observation Well .... O.W.

S.O.W. ....Shallow Observation Well.

PzD ....Deep Piezometer.

..............Pumping Screen

..............Observation Filter

FIG. 5.19

FROM. Bhatnagar N.C., Agashe R.M., Sikka V.M. (1977)

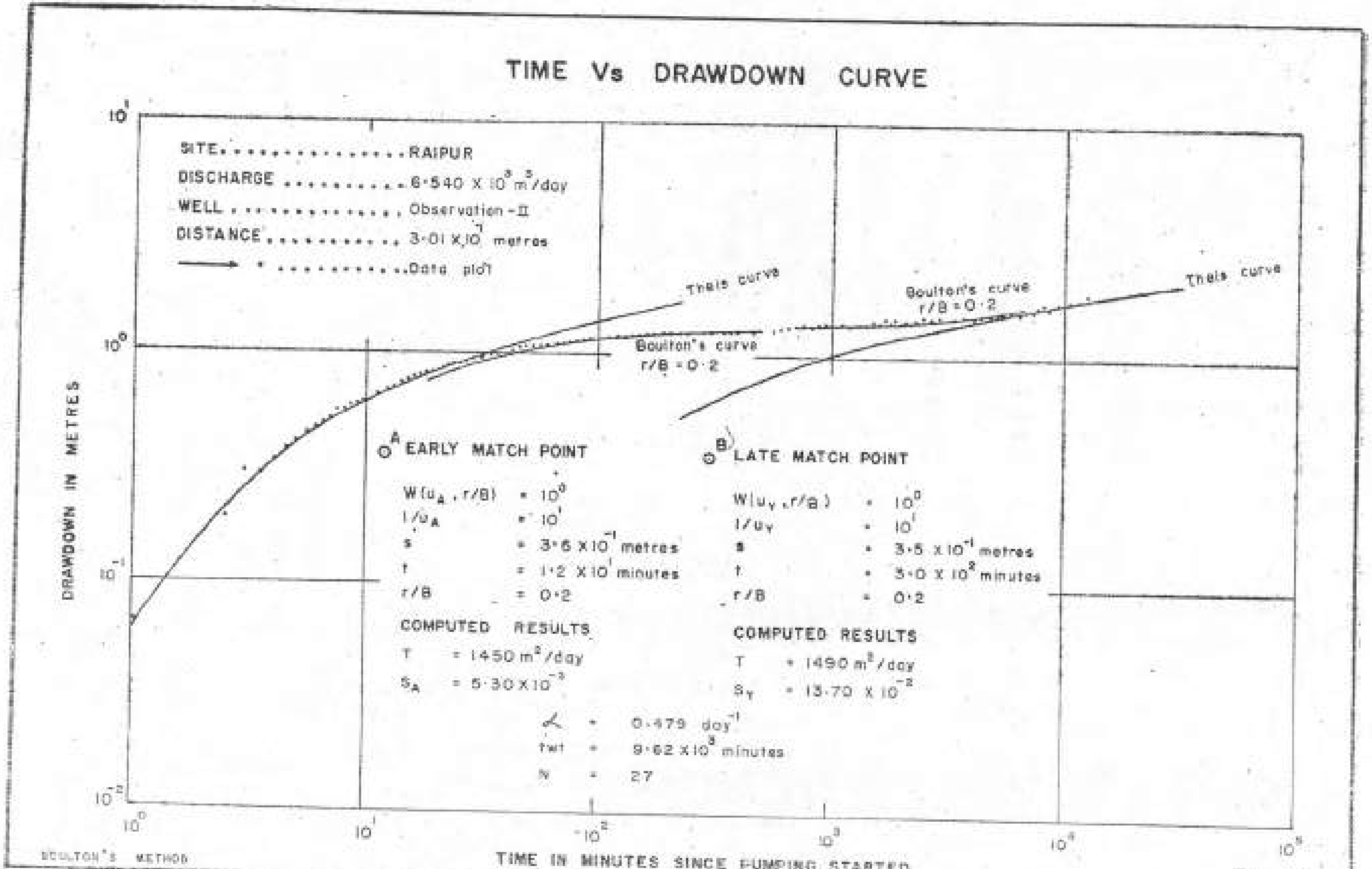


FIG. 5·20

FROM - Bhatnagar N.C., Agashe R.M., Sikka V.M. (1977)

t	OW—I	OW—II
	s	s
600	2·369	1·304
700	2·401	1·329
800	2·420	1·349
900	2·436	1·370
1000	2·451	1·384
1200	2·429	1·382
1400	2·421	1·373
1600	2·433	1·400
1800	2·436	1·413
2000	2·470	1·444
2200	2·485	1·458
2500	2·494	1·474
2700	2·490	1·459
3000	2·444	1·449
3400	2·426	1·458
4000	2·481	1·494
4400	2·418	1·498
5000	2·444	1·501
5500	2·460	1·520
6000	2·468	1·537
6500	2·496	1·569
7000	2·515	1·591
7500	2·493	1·594
8000	2·561	1·662
8500	2·511	1·648
9000	2·514	1·664
9500	2·584	1·706
10,000	2·550	1·714
10,500	2,511	1·710
11,000	2,629	1·777
11,500	2,599	1,774
1,2000	2·572	1·782
12,500	2·659	1·837

Fig. (5·20) shows the "time-drawdown" field data curve of observation well-II analysed by Boulton's method. The calculation of aquifer parameters by Boulton's method is given below,

$Q=6\cdot540\times10^3$/day

Observation Well—II ($r=30\cdot10$ m)

Early "match point" co-ordination being,

$$W(u\ ,r/B)\quad=10^{\circ}$$

$$\frac{1}{u}\quad=\ 10^1$$

$$s\quad=\ 3\cdot6\times10^{-1}\text{m}$$

$$t\quad=\ 1\cdot2\times10^1\text{min.}=8\cdot3\times10^{-3}\ \text{days}$$

$$r/B\quad=\quad0\cdot2$$

Using Eqs. (5·47) and (5·48)

$$T\ \ \frac{Q}{4\ \pi\ s}\ W(u_A\ r/B)$$

$$=\ \frac{6\cdot540\times10^3\times10^{\circ}}{4\times3\cdot14\times3\cdot6\times10^{-1}}$$

$$=\ 1450\ \text{m}^2/\text{day}$$

$$\text{and } S\ =\ \frac{4Tt\ u}{r^2}$$

$$=\ \frac{4\times1\cdot450\times\cdot10^3\times8\cdot3\quad\times-10^{-3}\times\cdot10^{-1}}{9\cdot06\times10^2}$$

$$5\cdot30\times10^{-1}$$

Late "match point" co-ordinates being,
$W(u_y\ ,r/B)=10^{\circ}$

$$\frac{1}{u}=10^1$$

$$s\ =\ 3\cdot5\times10^{-1}\text{m}$$

$$t\ =\ 3\cdot0\times10^2\text{min.}=2\cdot08\times10^{-1}\ \text{days}$$

$$r/B=\ 0\cdot2$$

Using Eqs. (5·49) and (5·50),

$$T=\frac{Q}{4\ \pi\ s}\ W(Y,\ r/B)$$

$$=\ \frac{6\cdot540\times10^3\times10^{\circ}}{4\times3\cdot14\times3\cdot5\times10^{-1}}$$

$$=\ 1488\ \text{m}^2/\text{day}$$

$$\text{say,}\ =\ 1490\ \text{m}^2/\text{day}$$

and,

$$S\ =\ \frac{4T\,t\,u}{r^2}$$

$$=\ \frac{4\times1\cdot488\times10^3\times2\cdot08\times10^{-1}\times10^{-1}}{9\cdot06\times10^2}$$

$$=\ 1\cdot37\times10^{-1}$$

or 13·70 %

Now r being 30·10 m. and $r/B=0\cdot2$, gives,

$$B\ =\ 150\cdot50\ \text{m}$$

Using Eq. (5·51), we get

$$\alpha\ =\ \frac{T}{B^2S_Y}$$

$$=\ \frac{1\cdot488\times10^3}{2\cdot265\times10^4\times1\cdot37\times10^{-1}}$$

$$=\ 0\cdot479\ \text{day}^{-1}$$

Because $r/B=0\cdot2$, we find from the Boulton's delay index curve that α twt$=3\cdot2$

$$\therefore\ \ twt\ =\ \frac{\alpha\ twt}{\alpha}\ =\ \frac{3\cdot2}{0\cdot479}$$

$$6\cdot68\ \text{days}$$

$$=\ 9\cdot62\times10^3\ \text{min.}$$

TIME Vs DRAWDOWN CURVE

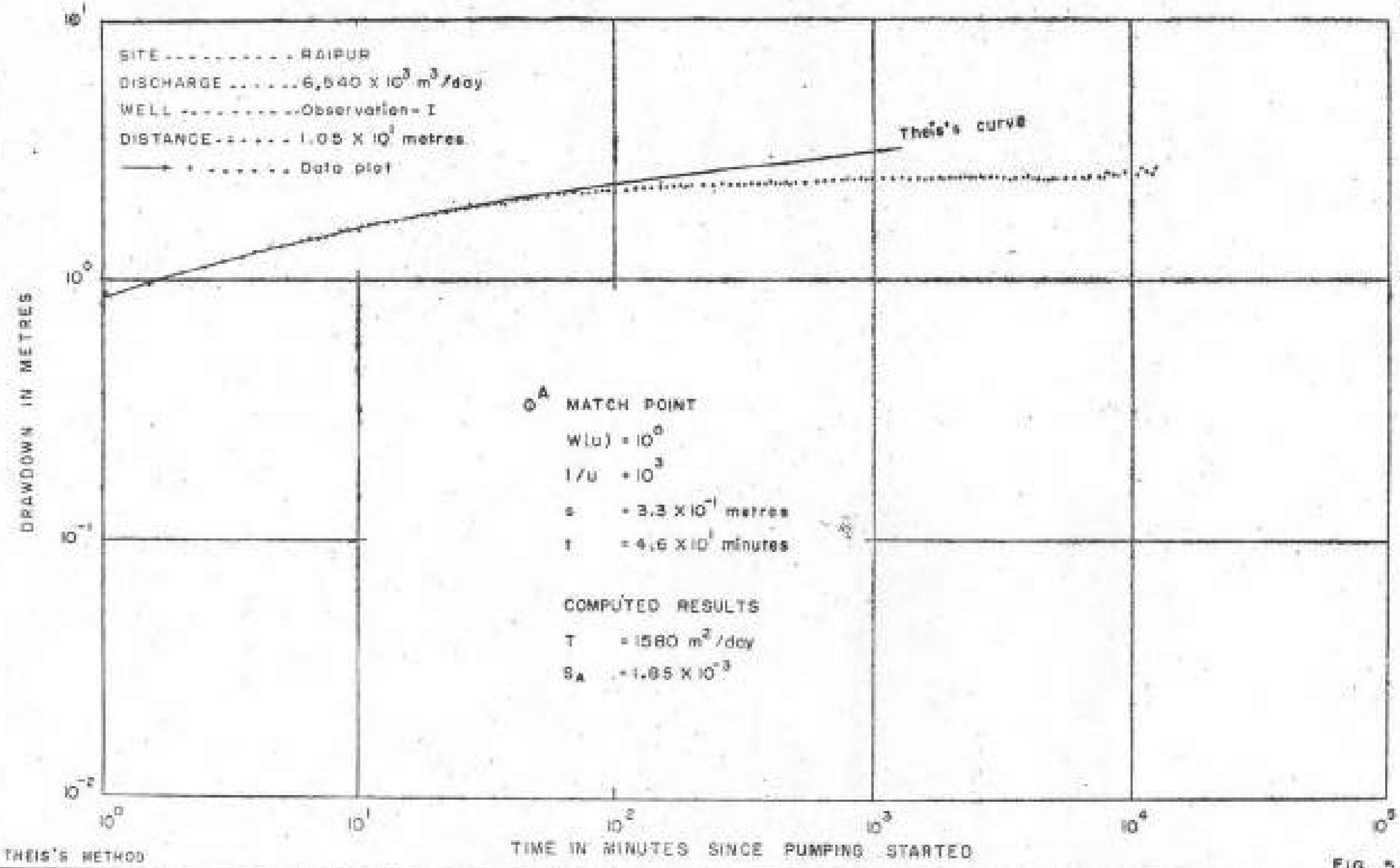


FIG. 5.21

FROM - Bhatnagar N.C., Agashe R.M., Sikka V.M. (1977)

Also from Eq. (5·45)

$$N = 1 + \frac{S}{S}$$

$$= 1 + \frac{13 \cdot 7 \times 1 \quad 2}{5 \cdot 30 \times 10^{-3}}$$

$$= 27$$

Remarks : For application of Boulton's method it is required that the "time-drawdown" curve exhibits development of 'first' and 'second' (nearly complete) segments of the curve defined earlier.

5. 2. 3. 2 Theis's Method :

Theis's type curve methods elaborated under section 5·2·1·2·1. may also be applied to the draw-down data of unconfined aquifer showing delayed yield phenomenon, under following flow conditions.

(i) *Time-drawdown analysis*

Under the flow conditions wherein Boulton's method is not applicable due to short duration of pumping or nonfulfilment of limiting conditions, the Theis's type curve method may be applied to the 'first' segment of "time-drawdown" curve for determination of T and S_A values.

(ii) *Distance-drawdown analysis*

In tests where 'third' segment drawdown data for at least 3 observation wells is available, Theis's "distance-drawdown" type curve method may be applied for determination of T and S_Y values.

(i) *Example* : (time-drawdown analysis)

[After Bhatnagar, N. C., Agashe, R. M., and Sikka V.M. (1977)]

The "time-drawdown" data of OW-I of pumping test conducted at 'Raipur' Table (5·9) which could not be analysed by Boulton's method, was analysed by Theis's method for determination of T and S_A values. Fig. (5.21) shows t vs s plot for observation well-I matched with theis curve.

The calculations are summarised below,

$$Q = 6 \cdot 540 \times 10^3 \text{ m}^3/\text{day}$$

Observation well—I (r = 10·50 m)

"Match point" co-ordinates being,

$$W(u) = 10^0$$

$$\frac{1}{u} = 10^3$$

$$s = 3 \cdot 3 \times 10^{-1} \text{ m}$$

$$t = 4 \cdot 6 \times 10^1 \text{min.} = 3 \cdot 19 \times 10^{-2} \text{ days}$$

Using the Eqs (5·8) and (5.9)

$$T = \frac{Q}{4\pi s} \quad W(u)$$

$$= \frac{6 \cdot 540 \times 10^3 \times 10^0}{4 \times 3 \cdot 14 \times 3 \cdot 3 \times 10^{-1}}$$

$$= 1580 \quad \text{m}^2/\text{day}$$

$$S_A = \frac{4T\,tu}{r^2}$$

$$\frac{4 = \times 1 \cdot 580 \times 10^3 \times 3 \cdot 19 \times 10^{-2}}{1 \cdot 10 \times 10^2} \times 10^{-3}$$

$$= 1 \cdot 85 \times 10^{-3}$$

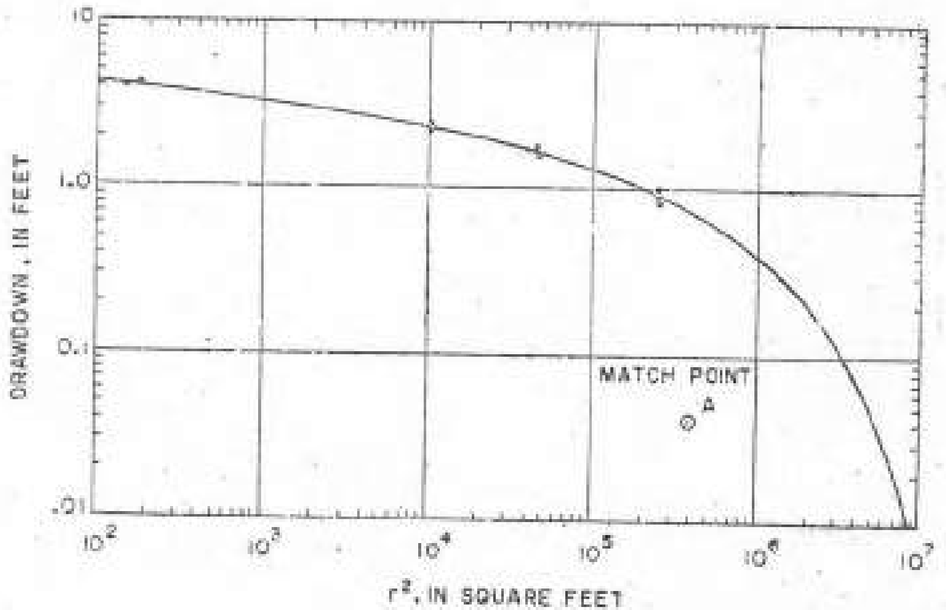
(ii) *Example* : ('Distance-drawdown' analysis)

[Taken from Ground Water Resource Evaluation by W.C. Walton (1970)]

Distance-drawdown data analysis of test near Lawrenceville, Illinois (From Prickett, 1965) is cited here as an example. The pumping test was conducted by H.F. Smith and F.X.Bushman of the Illinois State Water Survey in co-operation with the Texas company and the Heldt-Monroe Company, Water Well Contractors, on May 17 and 18,1950. The pumped well was provided with 11 observation wells, duration of pumping was 24 hrs at a constant discharge rate of 1000 g.p.m.

Corrected drawdown in observation wells, I to XI near the end of the test, when the effects of delayed gravity drainage ceased to influence the draw-downs, were plotted on a double logarithmic paper against the squares of their distances from the pumped well. This field data "distance-drawdown" curve was matched with Theis's W (u) vs u curve keeping W(u) axis parallel with s axis. Fig. (5·22 The "match point" co-ordinates were noted and were substituted into Theis Eqs, for calculation of T and

DISTANCE Vs DRAWDOWN CURVE



SITE . . . . LAWRENCEVILLE, ILLINOIS

DISCHARGE . . . . . 1000 USGPM.

WELL . . . . Observation I to XI

TIME . . . . 1400 minutes

——→ • . . . . . . Data plot

MATCH POINT 'A' CO-ORDINATES

$W(u) = 10^{-1}$

$1/u = 10^{1}$

$s = 4.4 \times 10^{-2}$ ft.

$r^2 = 4.1 \times 10^{3}$ ft^2

COMPUTED RESULTS

$T = 261{,}000$ USGPD/ft.

$S_y = 3.51 \times 10^{-2}$

THEIS'S METHOD

FIG. 5.22

FROM— Prickett (1965)

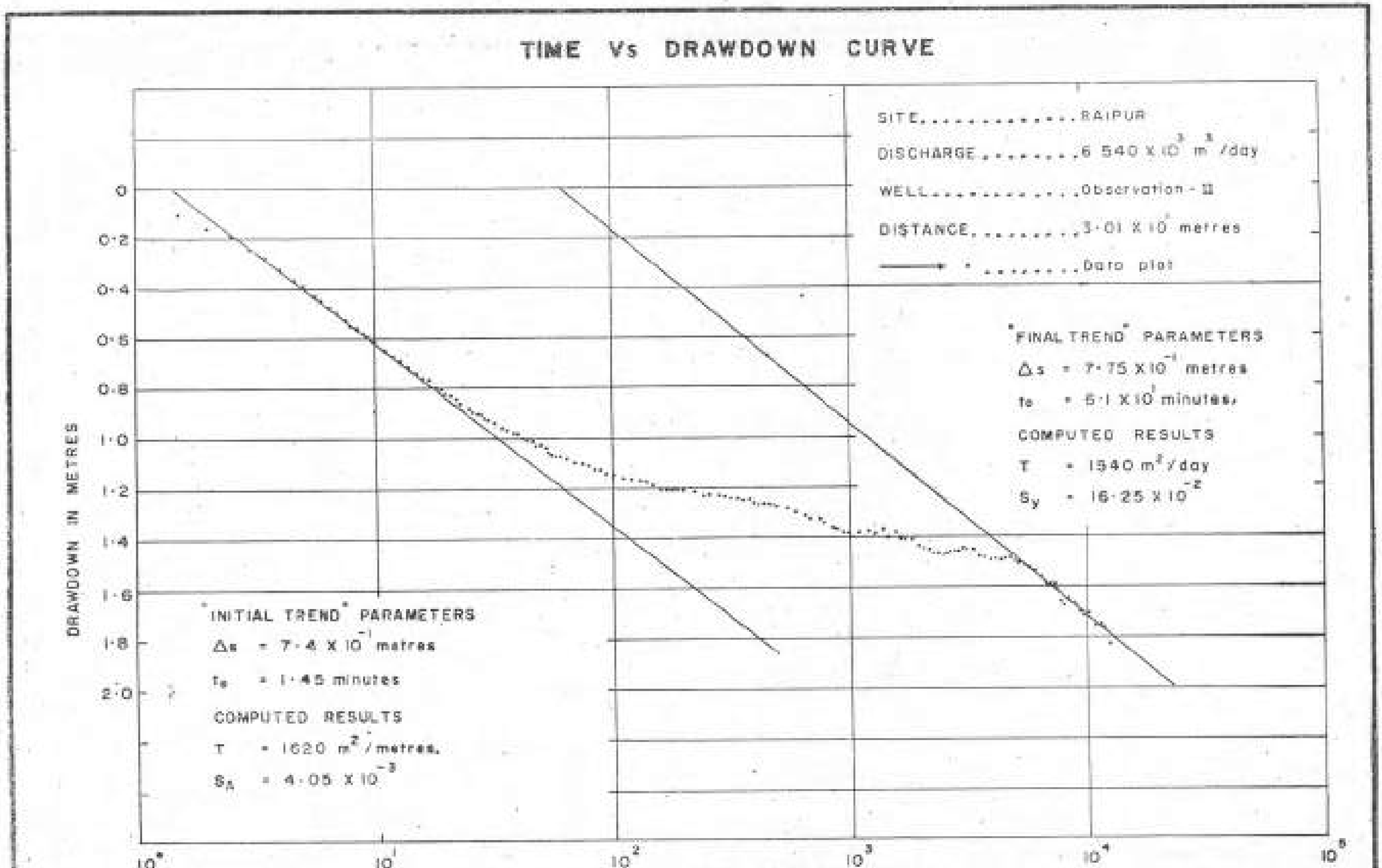


FIG. 5.23

FROM - Bhatnagar N.C., Agashe R.M., Sikka V.N. (1977)

S_Y values. The calculations made in gallon-day-foot system are reproduced here in original,

$$Q = 1000 \text{ g p m}$$
$$t = 1400 \text{ min}$$
$$b = 100 \text{ feet}$$

'Match point" Co-ordinates being,

$$W(u) = 10^{-1}$$
$$u = 10^{-1}$$
$$s = 4 \cdot 4 \times 10^{-2} \text{ ft.}$$
$$r^2 = 4 \cdot 1 \times 10^5 \text{ ft}^2$$

$$T = \frac{114 \cdot 6 \, Q \, W(u)}{s}$$

$$= \frac{1 \cdot 146 \times 10^2 \times 10^3 \times 10^{-1}}{4 \cdot 4 \times 10^{-2}}$$

$$= 261000 \text{ gpd/ft.}$$

$$S_Y = \frac{T \, t \, u}{2693 \, r^2}$$

$$= \frac{2 \cdot 61 \times 10^5 \times 1 \cdot 4 \times 10^3 \times 10^{-1}}{2 \cdot 693 \times 10^3 \times 4 \cdot 1 \times 10^{-1}}$$

$$= 3 \cdot 31 \times 10^{-2}$$

Say, $= 3 \cdot 30\%$

5·2·3·3 Jacob's Method :

Jacob's straight line "time-drawdown" method elaborated under section 5·2·1·2·2·, can be applied to the 'first' and the 'third' segments of "time-drawdown" curve of an observation well in an unconfined aquifer showing delayed yield phenomenon, for determination of T, S and S_Y values.

Jacob's "distance-drawdown" method could also be used for evaluation of T and S_Y values, provided at least 2 observation wells data (preferably 3 observation wells data) pertaining to 'third' segment of "time drawdown" is available.

Example :

[After Bhatnagar, N.C., Agashe, R.M., and Sikka, V.M. (1977)]

The "time-drawdown" data of OW-II of the pumping test conducted at 'Raipur' Table (5.9) has been analysed by Jacob's straight lin_A method for determination of T, S and S_Y values, Fig. (5.23) exhibits the "time-drawdown" curve of OW-II analysed by the said method. The calculations are summarised below,

$$Q = 6 \cdot 540 \times 10^3 \text{ m}^3/\text{day}$$

Observation well II($r = 30 \cdot 10$m). Straight lines drawn through the 'first' and the 'third' segments of the "time-drawdown" curve have the following parameters.

(i) 'Initial trend' (first segment) parameters being,

$$\triangle s = 7 \cdot 4 \times 10^{-1} \text{m}$$
$$t_o = 1 \cdot 45 \text{ min.} = 1 \cdot 007 \times 10^{-3} \text{ days}$$

Using the Eqs. (5·11) and (5·12),

$$T = \frac{2 \cdot 30 \, Q}{4\pi \triangle s}$$

$$= \frac{2 \cdot 30 \times 6 \cdot 540 \times 10^3}{4 \times 3 \cdot 14 \times 7 \cdot 4 \times 1^{-1}}$$

$$= 1620 \text{m}^2/\text{day}$$

$$S_A = \frac{2 \cdot 25 \, T \, to}{r^2}$$

$$= \frac{2 \cdot 25 \times 1 \cdot 620 \times 10^3 \times 1.007 \times 10^{-3}}{9 \cdot 06 \times 10^2}$$

$$= 4.05 \times 10^{-3}$$

(ii) 'Final trend' (third segment) parameters being,

$$\triangle s = 7 \cdot 75 \times 10^{-1} \text{m}$$
$$t_o = 6 \cdot 1 \times 10^1 \text{min.} = 4 \cdot 236 \times 10 \text{d}^{-2} \text{ays}$$

$$T = \frac{2 \cdot 30 \times 6 \cdot 540 \times 10^3}{4 \times 3 \cdot 14 \times 7 \cdot 75 \times 10^{-1}}$$

$$= 1545 \text{m}^2/\text{day}$$

$$S_Y = \frac{2 \cdot 25 \times 1 \cdot 545 \times 10^3 \times 4 \cdot 236 \times 10^{-2}}{9 \cdot 06 \times 10}$$

$$= 1 \cdot 625 \times 10^{-1}$$

or $16 \cdot 25\%$

Table (5·10) gives the summarised results of pumping test at Raipur site, India.

TABLE 5·10

Summarised results of pumping test at 'Raipur' site, India.

S. No.	Method used	Flow conditions	Data used of	Plot of	T (m^2/day)	S_A	S_Y	Remarks
1.	Boulton	Unsteady state	OW-II	log t vs s	1450	$5·30\times10^{-3}$	—	Early match
					1490	—	$1·37\times10^{-1}$	Late match
2.	Theis	Unsteady-state	OW-I	log t vs s	1580	$1·85\times10^{-3}$	—	"First" segment data used.
3.	Jacob	Unsteady-state	OW-II	log t vs s	1620	$4·05\times10^{-3}$	—	"First" segment data used.
					1445	—	$1·625\times10^{-1}$	'Third' segment data used.

5·2·3·4 Evaluation of Methods :

Use of a particular method for analysis of drawdown data of an unconfined aquifer showing delayed yield phenomenon depends upon the stages of development of "time-drawdown" curve i.e. on the development of three segments of "time-drawdown" curve defined earlier. Three types of flow situation may exist at the end of pumping which can be analysed by Boulton's Theis's and Jacob's methods, as explained hereunder:

(a) Development of 'first' and initial part of 'second' segment of "time-drawdown" curve.

Under such situations, only Theis's and Jacob's "time-drawdown" methods may be applied to the 'first' segment data for determination of transmissivity, T and early time storativity, S_A values.

(b) Development of 'first and nearly complete 'second' segments of "time-drawdown" curve.

Boulton's method may be applied for determination of T, S_A and S_Y (Specific Yield) values. However, when the drawdown data does not fit in Boulton's method due to non-fulfilment of limiting conditions, T and S_A values may be obtained by analysis of 'first' segment data by Theis's and Jacob's methods.

(c) Development of 'first', 'second' and 'third' segments of "time-drawdown" curve.

Boulton's and Jacob's "time-drawdown" methods may be used for determination of T, S_A and S_Y values Also, 'third' segment drawdown data be analysed by Theis's and Jacob's "distance-drawdown" methods.

Remarks :

Equations underlying the Boulton's, Jacob's and Theis's methods for analysis of drawdown data of unconfined aquifers, are based on the assumption that drawdown is negligible in comparison to the original saturated thickness of the aquifer. In case of appreciable drawdowns, the following equation of Jacob (1944) may be used to adjust drawdown data for decreased saturated aquifer thickness.

$$s_0 = s - \frac{s^2}{2b} \qquad (5·52)$$

where,

s_0 = corrected drawdown

s = observed drawdown

b = aquifer thickness

5.2.4 Unconfined Aquifers :

In an unconfined aquifer in which there are no delayed yield effects, the flow pattern to a pumped well is identical with the flow pattern to a pumped well in a confined aquifer. Hence, the methods described in section 5·2·1 can be used for determination of aquifer parameters of unconfined aquifers. The corrected drawdown, s_o may be used instead of observed drawdown, Eqs. (5·52) to account for dewatering of the aquifer.

However, such unconfined aquifers where the gravity drainage is immediate, rarely exist in nature. There is no available field case history to illustrate such an aquifer.

5.3. Pumping Tests under Special Conditions:

Methods discussed in earlier section 5·2 for analysing pumping test data are all based on the assumed simple natural conditions of homogeneity, isotropicity, infinite areal extent, radial flow, full penetration, constant discharge, no well storage and instantaneous yield (except for unconfined aquifers with delayed yield). Under this section, few important methods of practical field use are described where one or more of the said assumptions are not satisfied.

5·3·1 Aquifer of Limited Areal Extent :

The assumption that an aquifer is of infinite areal extent is at times invalid. Geological boundaries limit the extent of aquifers, and the pumping tests performed near the hydrogeological boundaries exhibit responses which are distorted in nature. The boundaries are of two types-barrier boundary and recharge boundry. The image well theory described by Ferris (1959) permits treatment of an aquifer limited in one or more directions. It states that "the effect of a barrier boundary on the drawdown in an observation well, as a result of pumping from a well, is the same as though the aquifer were infinite and a like discharging well were located across the real boundary on a perpendicular there to and at the same distance from the boundary as the real pumping well". For a recharge boundary the principle is the same except that the image well is assumed to be recharging the aquifer instead of pumping from it. Boundary problems are thereby simplified to consideration of an infinite aquifer in which real and image wells operate simultaneously.

5·3·1·1 Unsteady-state Flow in Confined and Unconfined (without delayed yield) Aquifers Limited by One or More Straight Barrier or Recharge Boundaries :

5·3·1·1·1 Ferris, et al. Method:

Ferris, Knowles, Brown, and Stallman (1962) described a method in which "time-drawdown" field data curve of an aquifer limited by barrier/recharge boundaries, is matched with the Theis curve in stages, that permits solving for T and S and also for the distance between the observation well and the image well. The image well distance is twice the distance to the actual boundary and if the boundary is concealed, three or more observation wells are required for its location. The method is based on 'law of times' defined by Ingersoll et al. (1948). For a given aquifer the times of occurrence of equals drawdown vary directly as the squares of the distance from an observation well to production wells of equal discharge.

The law of times is,

$$\frac{t_1}{r_1^2} = \frac{t_2}{r_2^2} = \frac{t_n}{r_n^2} \qquad (5\cdot53)$$

It follows that, if the time intercept of a given drawdown in an observation well caused by pumping a well at a given distance is known, and if the time intercept of an equal amount of divergence of the "time-drawdown" curve caused by the effect of the image well is also known, it is possible to determine the distance from the observation well to the image well using the following formula,

$$r_i = r\sqrt{t_i/t_p} \qquad (5\cdot54)$$

where,

r_i = distance from image well to observation well, in metres (L)

r = distance from pumped well to observation well, in metres (L)

t_p = time after pumping started, before the boundary becomes effective, for a particular drawdown to be observed, in days or minutes (T)

t_i = time after pumping started, after the boundary becomes effective, when the divergence of the "time-drawdown" curve caused by the influence of the image well is equal to the particular value of drawdown at t_p, in days or minutes (T)

Multiple boundary conditions can also be analysed with time-departure curves. Here, for simpler field condition, a procedure for analysis of "time-drawdown" data of an aquifer limited by two barrier boundaries, is outlined as under.

Procedure :

(For two discharging image wells)

— For an observation well, plot the values of drawdown, s, against the corresponding values of time, t, on a double logarithmic paper of the same scale as the available type curve of Theis, W(u) Vs 1/u.

— Match the initial part of the "time-drawdown" curve unaffected by the barrier boundary with the Theis curve and note the values of W(u), 1/u, t and s for the selected match point.

— Substitute the values of W(u), 1/u, t, s, Q and r into Theis Eqs. (5·8) and (5·9) and solve for T and S.

— In the matched position, extend the trace of type curve beyond early "time-drawdown" curve.

— Match the later "time-drawdown" curve affected by the closest barrier boundary with Theis type curve.

— Verify if the 'second' matched position is correct by the following relationship.—

For a particular value of the well function, W(u), the s, match point co-ordinate for the 'second' matched position of the type curve must be twice the s, match point coordinate for the 'first' matched position of the type curve.

— Match the final "time-drawdown" curve affected by both barrier boundaries, with the Theis curve.

— Verify the correctness of this 'third' match by the similar relationship, i.e. for a particular value of W(u), the s, match point co-ordinate for the 'third' matched position of the type curve must be three times, the s, match point co-ordinate for the 'first' matched position of the curve.

— Note the departures si_1 and si_2 of type curve traces 1 and 2; and 2 and 3, at convenient times ti_1 and ti_2.

— Note the values of tp_1 and tp_2 at which the first type curve trace intersects s values equal to si_1 and si_2 respectively.

— Calculate the values of the distance of both the discharging image wells from the observation well, by substituting the values of r, ti_1, ti_2, tp_1 and tp_2 into the following equations :

$$ri_1 = r\sqrt{t_{i1}/t_{p1}} \qquad (5\cdot55)$$

$$ri_2 = r\sqrt{t_{i2}/t_{p2}} \qquad (5\cdot56)$$

where,

r_{i1} = distance from observation well to first image well, in metres (L).

r_{i2} = distance from observation well to second image well, in metres (L)

r = distance from observation to pumped well, in metres (L)

t_{i1} = time after pumping started when the departure of type curve traces 1 and 2 is equal to si_1 in days or minutes (T)

t_{i2} = time after pumping started when the departure of type curve traces 2 and 3 is equal to si_2 in days or minutes (T)

t_{p1} or t_{p2} = time after pumping started for a particular value of si_1 or si_2 to be observed before either boundary becomes effective, in days or minutes (T)

Example :

[(After Mishra, A. K. (1978)]

Data of pumping test conducted at 'Naugam' site located in Baramulla district, J & K state, India, and analysed by Ferris et. al. method is presented here as an example of a confined aquifer of limited areal extent — bounded by two barrier boundaries. At 'Naugam' the aquifer exists in the depth range of 39·50 m to 153·00 m and is overlain by about 24 metres thick clayey layer. The well is terminated in a pebble/boulder bed and nothing is known about the lower confining clay layer. In Oct. 1978, a pumping test of 7000 minutes duration was conducted at a constant discharge of 5995 m^3/day.

The summarised water level data of the pumped well and the observation well and the drawdown data of the observation well is given in Table (5·11).

TABLE 5·11

Pumping test data, "Naugam", Baramulla district, J & K, India.

(A) Water Level/Drawdown Data :

Well	Distance from the pumped well (m)	Non-pumping Water level (m.B.M.P.)	Maximum drawdown (m)	Remarks
Pumped	—	1·05	20·45	—
OW	39·90	0·50	2·81	—

(B) Drawdown Data (OW)

t (min)	s (m)
1	0·02
2	0·04
3	0·065
4	0·100

TIME Vs DRAWDOWN CURVE

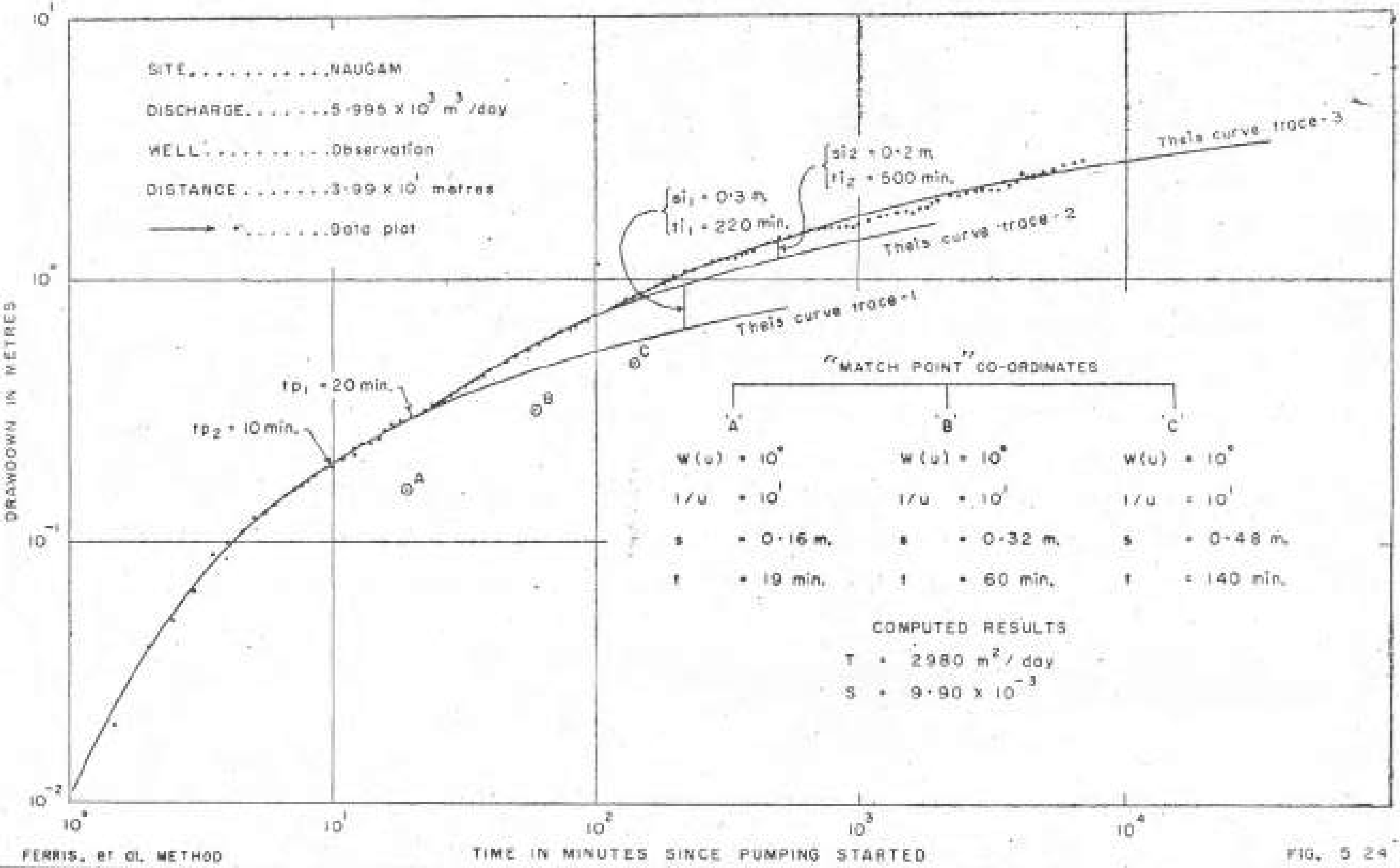


FIG. 5.24

FROM- Mishra A.K. (1978)

t (min)	s (m)
5	0·125
7	0·160
10	0·190
15	0·255
18	0·290
20	0·300
22	0·315
24	0·335
26	0·345
28	0·360
30	0·380
36	0·420
40	0·445
50	0·500
60	0·555
70	0·605
80	0·640
85	0·660
90	0·690
95	0·700
100	0·725
110	0·760
120	0·790
130	0·820
140	0·838
150	0·885
160	0·920
170	0·940
180	0·960
190	0·980
200	1·005
240	1·090
300	1·180
400	1·270
500	1·430
600	1·480
700	1·520
800	1·540
1000	1·610
1200	1·700
1500	1·777
1700	1·817
2000	1·960
2400	2·073
3000	2·157
3400	2·230
4000	2·360
4400	2·433
5000	2·520
6000	2·692
7000	2·810

Fig (5·24) exhibits the "time-drawdown" curve of the observation well analysed by Ferris et al. method. The calculation of aquifer parameters and the distance of image wells from the observation well is given below :

$$Q = 5{\cdot}995 \times 10^3 \text{m}^3/\text{day}$$

Observation well ($r = 3{\cdot}99 \times 10^1$ metres)

'Match point' co-ordinates for the initial 'first' matched position.

$$W(u) = 10^0$$

$$\frac{1}{u} = 10^1$$

$$s = 1{\cdot}6 \times 10^{-1} \text{m}$$

$$t = 19 \text{ min.} = 1{\cdot}319 \times 10^{-2} \text{ days}$$

Using the Theis Eqs. (5·8) and (5·9) we get

$$T = \frac{Q\,W(u)}{4\pi s}$$

$$= \frac{5{\cdot}995 \times 10^3 \times 10^0}{4 \times 3{\cdot}14 \times 1{\cdot}6 \times 10^{-1}}$$

$$= 2982 \text{m}^2/\text{day}$$

say, 2980m²/day

$$S = \frac{4\,T\,tu}{r^2}$$

$$= \frac{4 \times 2{\cdot}982 \times 10^{-3} \times 1{\cdot}319 \times 10^{-2} \times 10^{-1}}{1{\cdot}592 \times 10^3}$$

$$= 9{\cdot}88 \times 10^{-3}$$

say, $9{\cdot}90 \times 10^{-3}$

In the 'second' matched position, for a particular value of $W(u) = 10^0$, the 's' value of 0·32 m is twice the 's' value (0·16m) for the 'first' matched position —confirms the correctness of the 'second' match. The departure of two type curve traces 1 and 2 at a convenient time $t_{i1} = 220$ min. is noted,

$$s_{i1} = 0{\cdot}3 \text{ m}$$

For this value of s_{i1}, t_{p1} is noted, which is equal to 20 minutes.

Using the Eq. (5·55),

$$r_{i1} = r\sqrt{t_{i1}/t_{p1}}$$

$$= 39{\cdot}9\sqrt{\frac{220}{20}}$$

$$= 132 \text{ meters.}$$

In the 'third' matched position, it is seen that for a particular value of $W(u) = 10^0$'s' the y value of 0·48 m is thrice the 's' value for the 'first' matched position ($s = 0{\cdot}16$m)-confirms the correctness of the 'third' matched position.

Here again, the departure of type curve traces 2 and 3 at a convenient time $t_{i2} = 500$ min. is noted.

$$s_{i2} = 0{\cdot}2\text{m}$$

For this value of s_{i2}, t_{p2} is read which is equal to 10 min.

Using Eq. (5.55),

$$r_{i2} = r.\sqrt{t_{i2}/t_{p2}}$$

$$= 39 \cdot 9\sqrt{\frac{500}{10}}$$

$$= 282 \text{ metres.}$$

5·3·1·1·2 Stallman's Method :

A much simpler type curve method for solution of single boundary problems involving either a barrier or a recharge boundary, was devised by Stallman (1963 b).

If s is the drawdown in an observation well and s_p and s_i are the components of that drawdown caused respectively by the pumped well and by the discharging or recharging, image well, then s is the algebraic sum of s_p and s_i

i.e. $s = s_p + s_i$

For this flow condition, Theis non-equilibrium equation may be expressed as,

$$s = \frac{Q}{4\pi T}\,[(W(u_p) \pm W(u_i)]$$

$$= \frac{Q}{4\pi T}\,\Sigma\,W(u) \qquad (5\cdot57)$$

and,

$$u_p = \frac{r_p^2 S}{4Tt} \qquad (5\cdot58)$$

$$u_i \quad \frac{r_i^2 S}{4Tt} \qquad (5.59)$$

Where,

r_p = distance from the observation well to the pumped well, in metres (L)

r_i = distance from the observation well to the image well, in metres (L)

From Eqs. (5·58) and (5·59)

$$u_i = \left(\frac{r_i}{r_p}\right)^2 u_p \qquad (5\cdot60)$$

or $u_i = R^2 u_p$

Where,

$$R = \frac{r_i}{r_p} \qquad (5\cdot61)$$

R is a constant of proportionality. Stallman has given a family of logarithmic type curves of Σ W(u) versus $\frac{1}{u_p}$ for many values of his $R = \frac{r_i}{r_p}$

(Type curve : 7)

Procedure :

— For an observation well, plot the values of drawdown, s against the corresponding values of time, t on a double logarithmic paper of the same scale as the type curves of Stallman.

— Superpose the field data "time-drawdown" curve on Stallman's type curves, keeping s axis parallel with W(u) axis and t axis parallel with $\frac{1}{u_p}$ axis

— Adjust till a match position is found where the initial field data curve matches with the Theis type curve and the later falls on one of the Stallman's type curves. Note the value of R, for the matched Stallman's curve.

— Select a match point 'A' and note for 'A' the values of W(u) $\frac{1}{u_p}$, s and t

— Substitute these values alongwith the known values of Q and r_p into Eqs (5·57) and (5·58) and solve for T and S.

— Use Eq. (5·61) to calculate the value of r_i

Example :

[(After Jindal M.C., Jagannathan V., and Sikka, V.M. (1979)]

Data of pumping test conducted at 'Sector-38 Chandigarh' site, India and analysed by Stallman's method is presented here as an example of a confined aquifer of limited areal extent-bounded by a single barrier boundary. At this site, the aquifer exists in the depth range of 117 metres to 390 metres-underlain and overlain by 7 meters and 50 metres thick confining clayey beds respectively.

In April 1975, a pumping test of 1250 minutes-duration was conducted on a fully penetrating test well at a constant discharge of 1199 m³/day. The summarised water level data of the pumped well and the observation well, and the drawdown data of the observation well is given in Table (5·12)

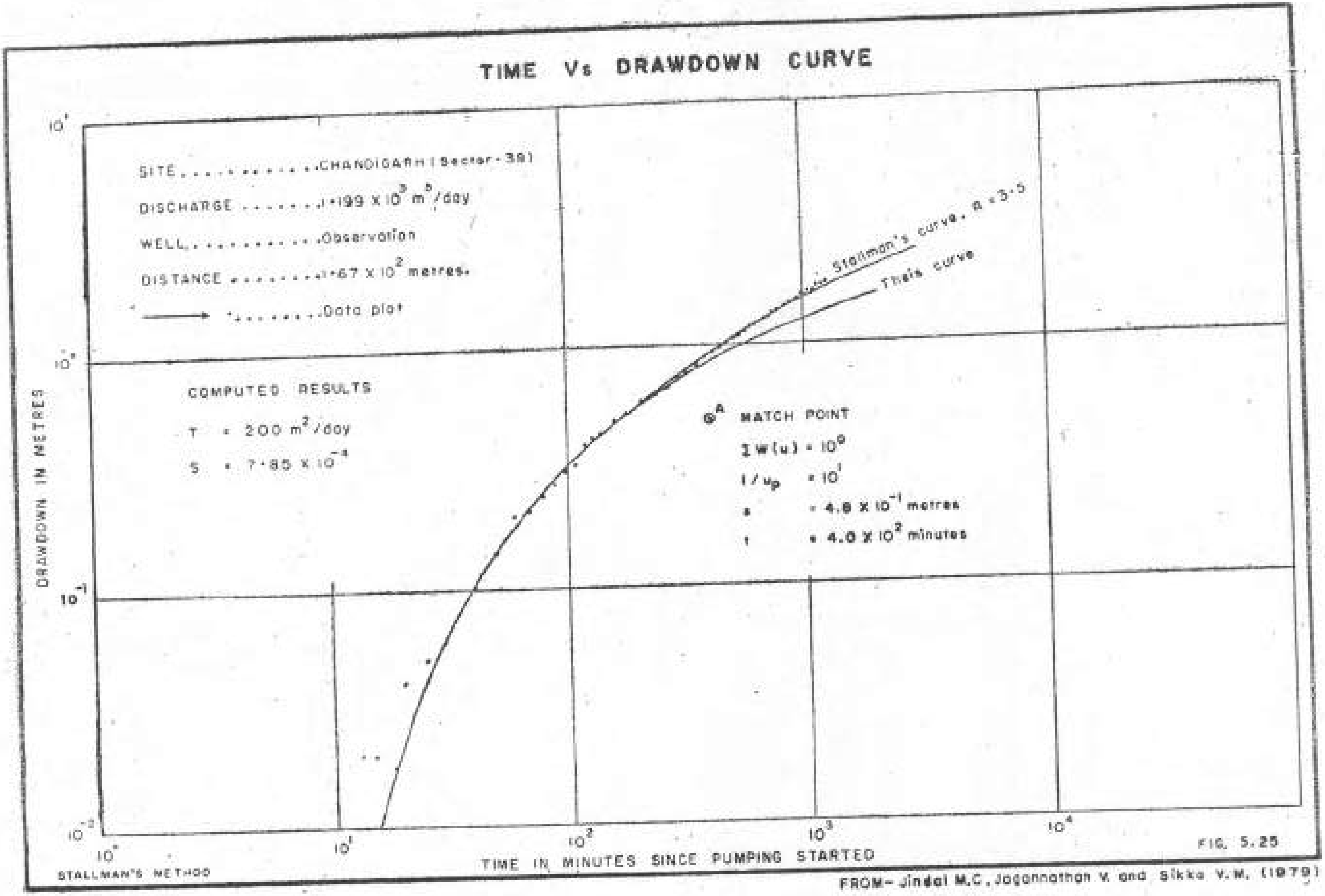


FIG. 5.25

FROM- Jindal M.C, Jagannathan V. and Sikka V.M. (1979)

TABLE 5·12

Pumping test data, "Sector 38 Chandigarh" site, India.

(A) Water Level Drawdown data :

Well	Distance from the pumped well (m)	Non pumping water level (m.B.M.P.)	Maximum drawdown	Remarks
Pumped	—	19·20	12·49	—
OW	167	16·75	1·76	—

(B) Drawdown Data (OW)

t (min)	s (m)	t (min)	s (m)
1	Nil	240	0·63
5	Nil	300	0·73
9	0·01	360	0·83
15	0·02	420	0·92
20	0·04	480	1·00
30	0·06	540	1·08
40	0·10	600	1·16
50	0·14	720	1·29
60	0·20	840	1·40
80	0·24	960	1·52
100	0·30	1020	1·58
120	0·39	1140	1·66
140	0·43	1250	1·76
180	0·52		
210	0·57		

Fig. (5·25) shows the "time-drawdown" curve of the observation well analysed by Stallman's method. the calculation of aquifer parameters and the distance to the Image well is given below,

$$Q = 1.199\times10^3 m^3/day$$

Observation well ($r_p = 1\cdot67\times10^2$ metres)

'Match point' co-ordinates being—

$$\Sigma\, W(u) = 10^0$$

$$\frac{1}{u_p} = 10^{-1}$$

$$s = 4\cdot8\times10^{-1} m$$

$$t = 4\cdot0\times10^2 \text{ min.} = 2\cdot77\times10^{-1} \text{ day}$$

$$R = 3\cdot5$$

Using Eq. (5·57) and (5·58) we get,

$$T = \frac{Q}{4\pi s}\,\Sigma\, W(u)$$

$$= \frac{1\cdot199\times10^3\times10^0}{4\times3\cdot14\times4\cdot8\times10^{-1}}$$

$$= 198 m^2/day$$

say, $200 m^2/day$

and $$S = \frac{4\,T\,t\,u_p}{r_p^2}$$

$$= \frac{4\times1\cdot98\times10^2\times2\cdot77\times10^{-1}\times10^{-1}}{2\cdot7889\times10^4}$$

$$= 7\cdot86\times10^{-4}$$

say, $7\cdot85\times10^{-4}$

Using Eq. (5·61),

$$R = \frac{r_i}{r_p}$$

$$r_i = R.r_p = 3\cdot5\times167$$

$$= 584 \text{ metres}$$

5·3·2 Tests in Flowing Artesian Wells :

If the water level in an artesian well stands above the land surface, the well is called a flowing artesian well. Analysis of test data of flowing wells in confined and semi-confined aquifers is outlined in following sections.

5·3·2·1 Flowing Wells in Confined Aquifers :

Aquifer parameters of a confined aquifer can be determined from the aquifer test data of a fully penetrating flowing well by the use of type-curve graphical method devised by Jacob and Lohman (1952) based on the equations in which the drawdown is constant and discharge varies with time. These conditions are met when a naturally flowing well in a confined aquifer is kept shut for sufficient time for the head to recover, then opened and allowed to flow for a period of 2 to 4 hours, during which the declining discharge rates are measured with corresponding time.

Jacob and Lohman's equations of constant drawdown conditions are based upon the assumptions that the aquifer is homogeneous, isotropic, infinite areal extent, fully penetrated with no well storage and the flow is in unsteady-state. It is expressed as,

$$T = \frac{Q}{2\pi W(\lambda) s_w} \qquad (5\cdot62)$$

Where

$$\lambda = \frac{T\,t}{S r_w^2} \qquad (5\cdot63)$$

and,

s_w = constant drawdown in discharging well (L)

r_w = radius of discharging well (L)

$W(\lambda)$ = well function for confined aquifers for constant drawdown conditions.

$$W(\lambda) = \frac{4\lambda}{\pi}\int_0^{\infty} xe^{-\lambda x^2}\left[\frac{\pi}{2} + \tan^{-1}\left(\frac{Yo(x)}{Jo(x)}\right)\right]dx \quad (5 \cdot 64)$$

Where, Jo (x) and Yo (x) are Bessel functions of zero order of the first and second kinds respectively. Values of $W(\lambda)$ in terms of the practical range of λ, given by Jacob and Lohman (1952) are presented in Annexure-VIII. Type curve-8, is a logarithmic plot of type curve from the data given in Annexure-VIII.

Procedure:

— Plot on a double logarithmic paper of the same scale as the type curve, the values of Q (vertical scale) against t (horizontal scale)

— Superpose the field data curve of Q vs t on the type curve keeping the $W(\lambda)$ axis parallel with Q axis and the λ axis parallel with t axis.

— Adjust till the field data curve matches with the type curve.

— Select a match point 'A' and note for 'A' the values of $W(\lambda)$, λQ, and t.

— Substitute the values of Q, $W(\lambda)$ alongwith the known value of s_w (constant drawdown) into Eq. (5·62) and solve for T.

— Substitute the values of T, r_w, t and λ into Eq (5·63) and solve for S

A much simpler straight line method has been described by Jacob and Lohman (1952) which is based on the assumption that $u \leqslant 0 \cdot 01$. Jacob and Lohman (1952) showed that for all but extremely small values of t, the function $W(\lambda)$ can be approximated very closely by 2/W (u). It was shown by Jacob that for small values of $u \leqslant 0 \cdot 01$, Theis equation [Eq 5.4)], could be simplified and W(u) expressed as,

$$W(u) = 2 \cdot 30 \log \frac{2 \cdot 25\ Tt}{r^2 S} \quad (5 \cdot 65)$$

Making the substitution of $W(\lambda)$ by 2/W(u) and adding the subscript w to s and r^2, Eq. (5·62) may be expressed as,

$$T = \frac{2 \cdot 30 Q}{4\pi s_w} \log. \frac{2 \cdot 25 Tt}{r_w^2 S} \quad (5 \cdot 66)$$

$$\text{or} \quad \frac{s_w}{Q} = \frac{2 \cdot 30}{4\pi T} \log. \frac{2 \cdot 25\ Tt}{r_w^2 S} \quad (5 \cdot 67)$$

Thus, a plot of $\frac{s_w}{Q}$ versus the logarithm of time describes a straight line. Eq (5·67) could further be solved to yield,

$$T = \frac{2 \cdot 30}{4\pi\Delta\ (s_w/Q)} \quad (5 \cdot 68)$$

$$\text{and } S = \frac{2 \cdot 25\ T\ t_o}{r_w^2} \quad (5 \cdot 69)$$

where,

$\Delta(s_w/Q)$ = Slope of the straight line in m^{-2} day ($L^{-2}T$)

t_o = the time intercept in days corresponding to the interception of straight line with time axis, where s /Q=0 (T)

Procedure:

— Plot on a semi-logarithmic paper, the values of s /Q versus t. (t on logarithmic scale)

— Fit a straight line through the plotted points and extend till it intercepts the time axis where $\frac{s_w}{Q} = 0$. Note t_o, at that point.

— Determine the value of slope of this straight line Δ (s_w/Q) i.e. the difference of (s_w/Q) per log cycle of t.

— Substitute the value of Δ (s_w/Q) into Eq (5·68) and solve for T

— Substitute the values of T, t_o and r_w into Eq. (5·69) and solve for S.

Remarks:

If the value of r_w is doubtful owing to well construction, well development or caving, the value of S may not be determined by this method.

Example :

[Taken from Ground Water Hydraulics by S.W. Lohman (1972)]

Field data for flow test on Artesia Heights well near Grand Junction Colo. conducted on September 22, 1948 [data from Lohman, 1965, tables 6 and 7, well 28, given in Ground Water Hydraulics by S.W. Lohman(1972)] has been analysed by the said methods to illustrate a field case history. Table (5·13) gives the field data, which is converted into metric units for use here.

TABLE 5·13

[Taken from Ground Water Hydraulics by S.W. Lohman (1972)]

Field data for flow test on Artesia Heights well near Grand Junction, Colo., Sept. 22, 1948 [Valve opened at 10·29 A. M. $s_w = 92·33$ ft (28·15 m) $r_w = 0·276$ ft (0·084 m) Data from Lohman (1965, table 6 and 7, well 28)]

Time of observation	Rate of flow (gpm)	Rate of flow (m3/day)	Time since flow started (min)	Sw/Q (ft gal^{-1}min.)	Sw/Q (m^{-2} day)
1	2	3	4	5	6
10·30	7·28	39·68	1	12·7	0·709
10·31	6·94	37·82	2	13·3	0·744
10·32	6·88	37·50	3	13·4	0·751
10·33	6·28	34·23	4	14·7	0·822
10·34	6·22	33·90	5	14·8	0·830
10·35	6·22	33·90	6	14·8	0·830
10·37	5·95	32·43	8	15·5	0·868
10·40	5·85	31·88	11	15·8	0·883
10·45	5·66	30·85	16	16·3	0·912
10·50	5·50	29·98	21	16·8	0·939
10·55	5·34	29·10	26	17·3	0·967
11·00	5·34	29·10	31	17·3	0·967
11.10½	5·22	28·45	41·5	17·7	0·989
11·20	5·14	28·01	51	18·0	1·005
11·30	5·11	27·85	61	18·1	1·001
11·45	5·05	27·52	76	18·3	1·023
12·00	5·00	27·25	91	18·5	1·033
12·12	4·92	26·81	103	18·8	1·050
12·22	4·88	26·60	113	18·9	1·058

The static head just prior to the test was 94·55 ft (28·83m) above the measuring point and 92·33 ft (28·15 m) above discharge point. Analyses of the data by type curve and straight line methods is summarised below,

(i) Type Curve Method :

Fig (5·26) shows a plot of Q vs t on a double logarithmic paper. The calculating of aquifer parameters are given hereunder,

$s_w \quad = 28·15\text{m}$

$r_w \quad = 8·4 \times 10^{-2}\text{m}$

'Match point' co-ordinates being,

$W(\lambda) \quad = 10^{-1}$

$\lambda \quad = 10^{-4}$

$Q \quad = 1·95 \times 10^{1}\text{m}^3/\text{day}$

$t \quad = 1·5 \text{ min.} = 1·042 \times 10^{-3} \text{ days}$

Using the Eqs. (5·62) and (5·63)

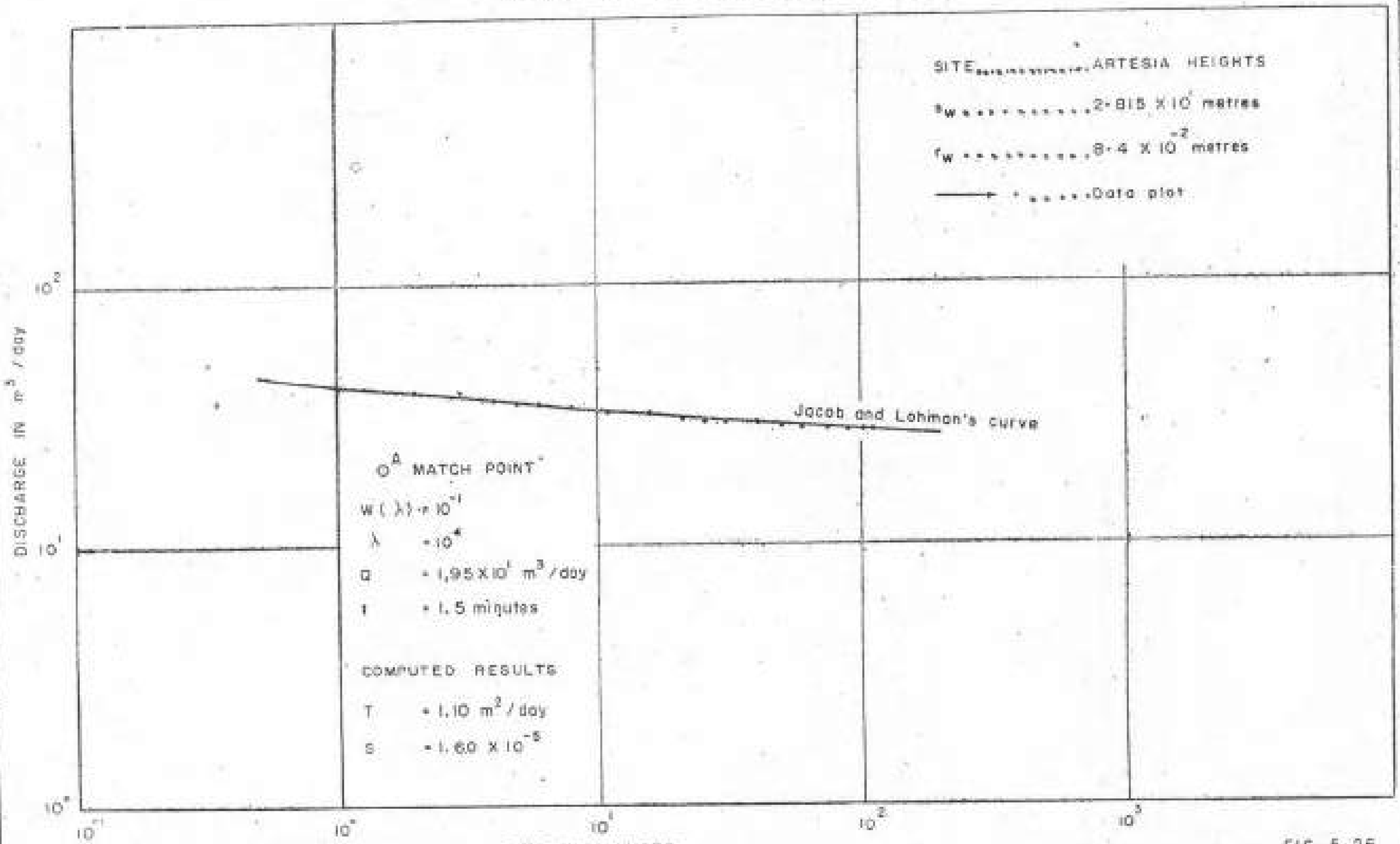
$$T = \frac{Q}{2\pi W(\lambda) s_w}$$

$$T = \frac{1·95 \times 10^{1}}{2 \times 3·14 \times 10^{-1} \times 2·815 \times 10^{1}}$$

$$= 1·10\text{m}^2/\text{day}$$

$$S = \frac{T\,t}{r_w^2}$$

TIME Vs DISCHARGE CURVE



JACOB and LOHMAN'S METHOD

FIG. 5.26

FROM- Lohman S.W. (1948)

$$= \frac{1.10 \times 1.042 \times 10^{-3}}{10^{2} \times 7.056 \times 10^{-3}}$$

$$= 1.62 \times 10^{-5}$$

say, $\quad 1.60 \times 10^{-5}$

(ii) Straight Line Method :

Fig. (5.27) shows a semi-logarithmic plot of sw/Q versus t. The calculations of T and S are given below,

$s_w \quad = 28.15$ m

$r_w \quad = 0.084$m or 8.4×10^{-2}m

$\triangle (S_w/Q) = 1.68 \times 10^{-1} m^{-2}$ day

$t_o \quad = 6.0 \times 10^{-5}$min. $= 4.166 \times 10^{-8}$ days

Using the Eqs (5.68) and (5.69) we get,

$$T = \frac{2.30}{4\pi \triangle (s_w/Q)}$$

$$= \frac{2.30}{4 \times 3.14 \times 1.68 \times 10^{-1}}$$

$$= 1.09 m^2/day$$

$$S = \frac{2.25\, T.t_o}{r_w^2}$$

$$= \frac{2.25 \times 1.09 \times 4.166 \times 10^{-8}}{7.056 \times 10^{-3}}$$

$$= 1.45 \times 10^{-5}$$

5.3.2.2 Flowing Wells in Semi-Confined Aquifers:

Hantush (1959) derived an equation for determining T and S for a well of constant drawdown that is discharging by natural flow from a semi-confined aquifer. The equations being.

$$T = \frac{Q}{2\pi s_w\, W(\lambda, r_w/L)} \qquad (5.70)$$

where

$$\lambda = \frac{T\,t}{S r_w^2} \qquad (5.71)$$

$W(\lambda, r_w/L)$ is read as the "well function for semi-confined aquifers without water released from storage in aquitard and constant drawdown conditions".

Also,

$$\frac{r_w}{L} = \frac{r_w}{\sqrt{T/(k'/b')}} \qquad (5.72)$$

$$W(\lambda, r_w/L) = \left(\frac{r_w}{L}\right)\frac{K_1(r_w/L)}{Ko(r_w/L)} + \frac{r}{\pi^2}\exp.\left[-\lambda\left(\frac{r_w}{L}\right)^2\right] \int_0^{\infty} \frac{u \exp(\lambda u^2)}{J^2o(u) + Yo^2(u)} \frac{du}{u^2 + (r_w/L)^2} \qquad (5.73)$$

where,

$K_1(x)$ = modified Bessel function of second kind, first order.

$Ko(x)$ = Modified Bessel function of second kind, zero order

All other symbols as defined earlier. Values of $W(\lambda, r_w/L)$ in terms of the practical range of λ and r_w/L values are given by Hantush (1959)-Annexure-IX. Type Curve : 9, is a logarithmic plot of type curve from the data given in Annexure IX (after Walton 1962)

Procedure ;

— Plot on a double logarithmic paper of the sam e scale as the type curves, the values of Q (vertical scale) against t (horizontal scale).

— Superpose the field data curve of Q vs t on the type curves keeping the W (λ, r_w/L) axis parallel with Q axis and λ axis parallel with t axis.

— Adjust till the field data curve falls on one of the type curves.

— Select a match point 'A' and note for 'A' the values of W (λ, r_w/L), λ, t and Q.

— Substitute the values of W (λ, r_w/L) Q and s_w into Eq. (5.70) and solve for T.

— Substitute the values of T, t, λ and r_w into Eq (5.71) and solve for S.

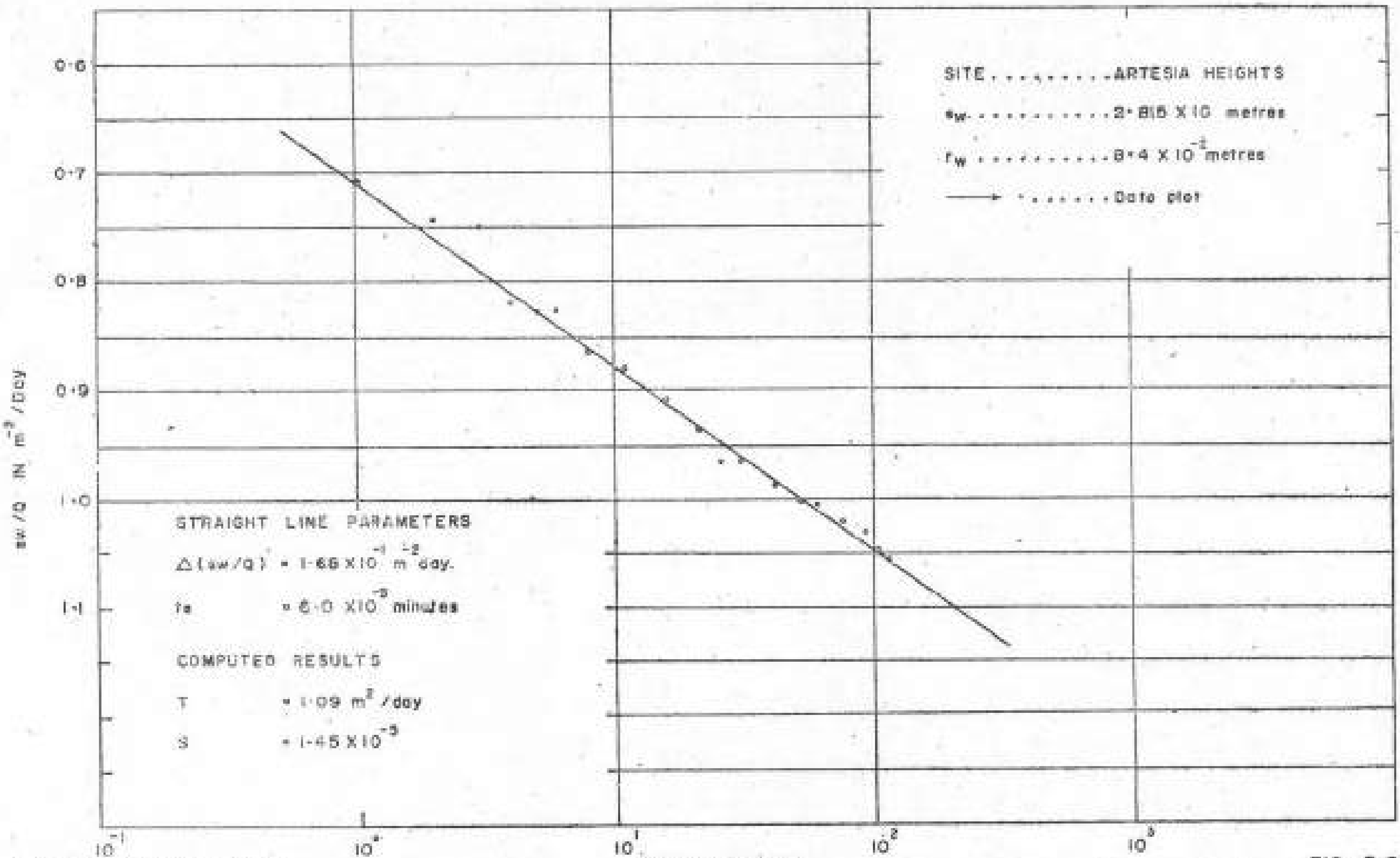
— Substitute the value of r_w/L corresponding to the matched type curve, alongwith the known values of r_w, T and b, into Eq. (5.72) and determine the value of K'.

5.3.3 Tests in Partially Penetrated Aquifers :

Around partially penetrating pumping wells, the flowlines in the aquifer are not horizontal within a radius of $r < 2b$. The drawdowns measured at distances, $r < 2b$ are influenced by vertical flow components and are to be corrected for these partial penetration effects prior to analysis of data by the methods outlined under section 5.2.

In practice, the partial penetration effects are appreciable within the distances $r < 1.5$ b. Analysis/correction of drawdown data of partially penetrating wells by some of the important available.

s_w/Q Vs t PLOT



JACOB and LOHMAN'S METHOD

FIG. 5.27

FROM- Lohman S.W (1948)

methods is discussed in the following sections

5·3·3·1 Steady-state Flow in Confined/Semi-Confined Partially Penetrated Aquifers.

5·3·3·1·1 Huisman Correction Method-I

In anonymous (1964, pp. 73 and 91) a formula developed by Huisman is given for the calculation of the correction of the steady-state drawdown in an observation well at a distance of r ($r < 2b$) from a partially penetrating pumped well in a confined aquifer, which is (s_m) partially—(s_m) fully =

$$\frac{Q}{2\pi T}\,\frac{2b}{\pi d}\sum_{n=1}^{\infty}\frac{1}{n}\left[\sin\left(\frac{n\pi b_1}{b}\right)-\sin\left(\frac{n\pi a}{b}\right)\right]\cos\left(\frac{n\pi z}{b}\right).\,k_0\left(\frac{n\pi r}{b}\right) \qquad (5·74)$$

where, (refer Fig. 5·28)

- a = distance from the bottom of the well screen to the impervious bottom layer (L)
- b_1 = distance from the top of the well screen to the impervious bottom layer (L)
- z = distance from the middle of the observation well screen to the impervious bottom layer (L)
- d = length of the well screen (L)
- (s_m) partially = observed steady-state drawdown (L)
- (s_m) fully = corrected steady-state drawdown (L)

All other symbols as defined earlier.

The angles are expressed in radians.

This formula can be used if the following assumptions and conditions are satisfied—

— The general assumptions listed in section 5·2 wherein the assumption "the pumped well penetrates the entire aquifer thickness" is replaced by "the pumped well does not penetrate the entire aquifer thickness"

— The aquifer is confined/semi-confined.

— Flow to the well is in steady-state.

— $r > r_w$, (r_w being the effective radius of the pumped well)

Procedures :

— Calculate the value of drawdown that would have occurred if the pumped well had been fully penetrating, (s_m)fully, using the Eq. (5·74) with an approximate value of T.

— Calculate a corrected value of T by using (s_m) fully in an appropriate method for steady-state conditions in a confined/semi-confined aquifer.

— If there is a large difference between the calculated T and the assumed value of T, repeat the procedure by substituting the calculated value of T and obtain better result.

Remarks :

— This correction method can not be applied to the pumped well data.

— A few terms, 4 to 5 of the series behind the Σ sign Eq. (5·74) will generally suffice.

5·3·3·1·2 Huisman's Correction Method-II

According to Huisman (Anonymous, 1964 p. 93) for steady-state flow conditions in a confined/semi-confined aquifer, the extra drawdown in a pumped well due to partial penetration can be expressed as,

(s_m) partially—(s_m) fully

$$= \frac{Q}{2\pi T}\left[\frac{1-p}{p}\right] \ln\frac{\varepsilon b}{r_w} \qquad (5·75)$$

where, (refer Fig. 5·28)

- p = $\dfrac{d}{b}$ = the penetration ratio
- l = distance between the middle of the well screen and the middle of the aquifer.
- ε = function of p and l (Annexure-X)

All other symbols as defined earlier.

For use of this formula, the assumptions and conditions of Huisman's correction method-I should be satisfied with the exception that $r = r_w$.

Procedure :

— Calculate the value of (s_m)fully by assuming an approximate value of T and substituting it alongwith the values of p, Q, ε, b, r_w and (s_m)partially into Eq. (5·75)

— Calculate the corrected value of T by using (s_m)fully value by an appropriate method for steady-state conditions in a confined/semi confined aquifer.

FIG. 5.28.

SCHEMATIC ILLUSTRATION OF THE PARAMETERS OF THE HUISMAN CORRECTION METHODS FOR PARTIAL PENETRATION

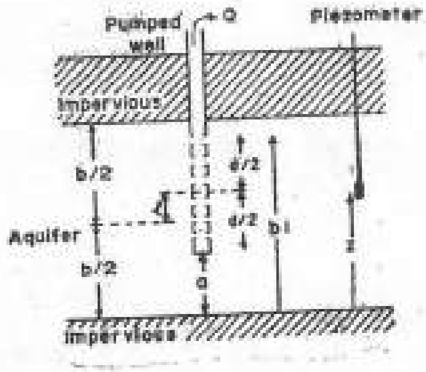
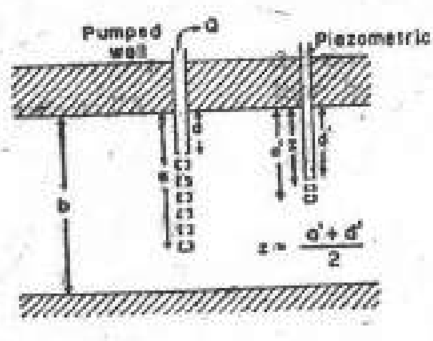


FIG. 5.29

SCHEMATIC ILLUSTRATION OF THE PARAMETERS OF THE HANTUSH MODIFICATION OF THE THEIS METHOD FOR PARTIAL PENETRATION



— If the difference between the computed value of T and the assumed value of T are large, substitute the computed value of T into Eq. (5·75) to obtain better result.

5·3·3·2 Unsteady-State Flow in Confined Partially Penetrated Aquifers :

Hantush (1962) developed modifications of Theis's and Jacob's methods applicable under unsteady-state flow conditions in partially penetrated confined aquifers-described below.

5.3.3.2.1 Hantush's Modification of the Theis Method:

For a relatively short period of pumping $\left[t < \frac{(2b-a-z)^2\,(S/b)}{20K}\right]$ the drawdown in an observation well at a distance r ($r<2b$), according to Hantush (1962) is given by,

$$s = \frac{Q}{8\pi K\,(a-d)}\ E\left[u, \frac{a}{r}, \frac{d}{r}, \frac{z}{r}\right] \qquad (5\cdot76)$$

Where (refer Fig. 5·29)

$$E\left[u, \frac{a}{r}, \frac{d}{r}, \frac{z}{r}\right] = M(u, \beta_1) - M(u, \beta_2) + M(u, \beta_3) - M(u, \beta_4) \qquad (5\cdot77)$$

$\beta_1 = (a+z)/r$
$\beta_2 = (d+z)/r$
$\beta_3 = (a-z)/r$
$\beta_4 = (d-z)/r$

M (u, β) is a function for which numerical values are given in Annexure-XI.

$$u = \frac{(r^2\,S/b)}{4Kt} \qquad (5\cdot78)$$

S/b = specific storage (L^{-1})

Hantush modification of Theis method can be used if the following assumptions and limiting conditions are satisfied.

—The general assumptions listed in section 5·2·1·2·1 wherein the assumption "the pumped wells penetrates the entire aquifer thickness" is replaced by "the pumped well does not penetrate the entire aquifer thickness"

—The time of pumping is relatively short

$$t < \frac{(2b-a-z)^2\,(S/b)}{20K}$$

Procedure :

—Determine for one of the observation wells the values of β_1, β_2, β_3 and β_4 and calculate its E-function for different values of u, from Eq. (5·77) using the tables of function M (u, β) given in Annexure-XI.

—Plot on a double logarithmic paper the values of E versus $\frac{1}{u}$ to obtain the type curve.

—On an another sheet of double logarithmic paper of the same scale as the type curve, plot the observed values of drawdown, s, against the corresponding values of time, t which gives a field data curve for the observation well under consideration.

—Superpose the field data curve on the type curve and while keeping the axes parallel, adjust till the early time field data curve falls on the type curve. (For large values of t, the field data curve would deviate from the type curve trace, which is based on the assumption that the pumping time is relatively short).

—Select a match point 'A' on the overlapping portions of the two sheets in the range where curves match, and note for 'A' the values of E, $\frac{1}{u}$ and t.

—Substitute the values of s and E and known values of Q, a and d into Eq. (5·76) and solve for K.

—Substitute the values of $\frac{1}{u}$, t and the known values of r and K into Eq. (5·78) and calculate S/b.

—If the field data curve departs from the type curve, note the value of $\frac{1}{u}$ at the point of departure, $\frac{1}{u\ \text{dep.}}$

—Calculate b, the aquifer thickness from the following equation

$$b \approx 0\cdot5\left(a+z+r\sqrt{5/u\ \text{dep}}\right) \qquad (5\cdot79)$$

—Compute the value of T, being equal to K.b.

—If the field data curve does not deviate from the type curve trace, note the value of $\frac{1}{u}$ at a point near the last field data plot. Substitute the value of this $\frac{1}{u}$ into Eq. (5·79) and calculate b. The thickness of aquifer would be greater than the value obtained having substituted the value of $\frac{1}{u}$ instead of $\frac{1}{u_{dep}}$

—Repeat this procedure for all the observation wells for $r<2b$.

5.3.3.2.2 Hantush's Modification of the Jacob Method

For a relatively long period of pumping $\left[t>\frac{b^2(s/b)}{2K}\right]$ the drawdown, according to Hantush (1962) is given by,

$$s=\frac{Q}{4\pi T}\left[(u)+fs\left(\frac{r}{b},\frac{a}{b},\frac{d}{b},\frac{z}{b}\right)\right] \qquad (5·80)$$

Where,

W(u) is the Theis well function.

and,

$$fs=\frac{4b^2}{\pi^2(a-d)(a'-d')}\sum_{n=1}^{\infty}\left(\frac{1}{n^2}\right)Ko\left(\frac{n\pi r}{b}\right)\times\left[\sin\left(\frac{n\pi a}{b}\right)-\sin\left(\frac{n\pi d}{b}\right)\right]\left[\sin\left(\frac{n\pi a'}{b}\right)-\sin\left(\frac{n\pi d'}{b}\right)\right] \qquad (5·81)$$

The angles are expressed in radians, symbols explained in Fig. (5·29).

A semi-logarithmic plot of s versus t (t on logarithmic scale) would describe a straight line for large values of t, The slope of this line, Δs, i.e. the drawdown difference per log cycle of time, is given by

$$\Delta s=\frac{2·30Q}{4\pi T} \qquad (5·82)$$

The time intercept to, of this straight line with time axis where s=0, is given by,

$$to=\frac{S\,r^2}{2·25\,T\,e^{fs}} \qquad (5·83)$$

when the difference between a' and d' is smal $[(a'-d')<0·05b]$ Eq. (5·81) may be expressed as,

$$fs=\frac{4b}{\pi(a-b)}\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)Ko\left(\frac{n\pi r}{b}\right)\left[\cos\left(\frac{n\pi z}{b}\right)\right]\left[\sin\left(\frac{n\pi a}{b}\right)-\sin\left(\frac{n\pi d}{b}\right)\right] \qquad (5·84)$$

The assumptions and limiting conditions of this method are same as those for Hantush's modification of Theis method (5·3·3·2·1) except that time is relatively large : $t>b^2(S/b)/2K$

Procedure :

—On a semi-logarithmic paper, plot for one of the observation wells, s versus t (t on logarithmic scale).

—Draw a straight line through the later data plots and extend the line till it intercepts the time axis where s=o, Note the time intercept, t_o.

—Calculate the slope of this line, Δs, i.e. drawdown difference over one log cycle of time.

—Calculate the value of T from the Eq. (5·82)

—Calculate the value of fs from Eqs. (5·81) or (5·84) whichever is applicable. A few terms of the series involved are generally sufficient.

—Calculate e^{fs} using the tables of functions x, e^x given by Hantush (1956)—Annexure IV.

—Substitute the values of r, t_o, T and e^{fs} into Eq. (5·83). and solve for S.

—Repeat this procedure for all the observation wells with $r<2b$.

Remarks ;

The advantage of Hantush's modifications of Theis and Jacob methods is that the knowledge of thickness of aquifer is not required for their application.

5.3.3.3 Unsteady-state Flow in Partially Penetrated Unconfined Aquifers with Delayed Yield.

Streltsova (1974) has given an equation for flow to partially penetrating wells in a water table aquifer exhibiting delayed yield phenomenon, which is given below.

$$s=\frac{Q}{4\pi T(l')}W(u_A,u_B,\beta,l'y') \qquad (5·85)$$

FIG. 5.30

WATER-TABLE AQUIFER WITH PARTIALLY PENETRATING WELLS

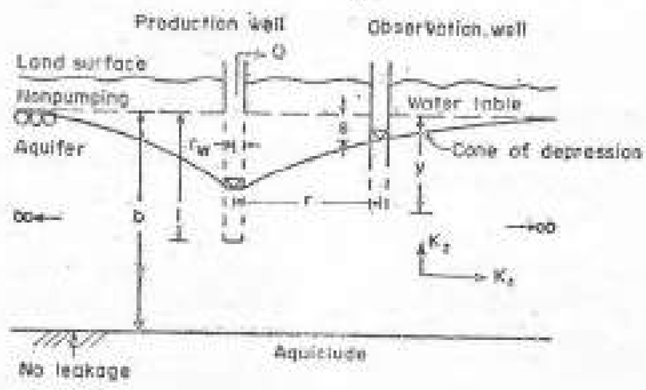
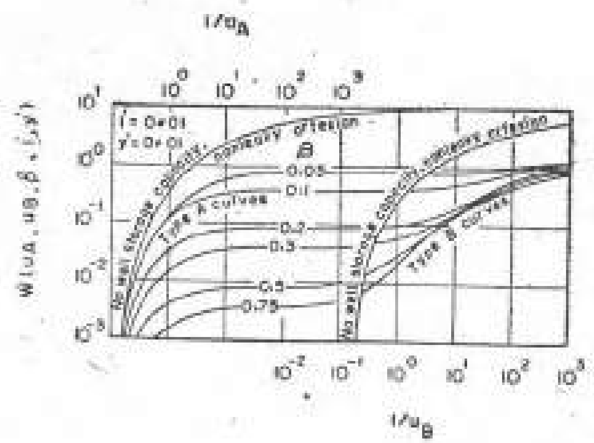


FIG. 5.31

TYPE CURVES FOR WATER-TABLE AQUIFER WITH PARTIALLY PENETRATING WELLS HAVING NO STORAGE CAPACITY



FROM – Streltsova (1974)

Where,

$W(u_A, u_B, \beta, l', y')$ = Well function for water table aquifer with partially penetrating wells having no storage capacity (dimensionless).

$l' = \frac{l}{b}$ (dimensionless)

$y' = \frac{y}{b}$ (dimensionless)

l = Vertical distance from non-pumping water table to bottom of pumped well screen in metres (L).

y = Vertical distance from non-pumping water table to bottom of observation well in metres (L).

For symbols refer Fig. (5·30) Values of $W(u_A, u_B, \beta, l', y')$ given by Streltsova (1974) in terms of selected range of $\frac{1}{u_A}, \frac{1}{u_B}, \beta, l', y'$ are presented in Annexure-XII. The numerical values in Annexure-XII for $l' = 0{\cdot}1$ are plotted as two asymptotic families of type curves in Fig. (5·31).

5.3.4. Tests in Fractured Hard Rocks :

5.3.4.1 Introduction :

Generally the properties of aquifers are determined by laboratory experiments and field tests. The properties of aquifer can be divided into two categories (i) properties related to the capacity of aquifers to store fluids (e.g. porosity) and (ii) properties related to the capabilities of the aquifer to transmit fluids (permeability). Once the properties of the aquifers are known the equations of flow can be solved to predict the quantitative behaviour of some parameters as pressure and discharges. The main condition to be accomplished in such a calculation is that flow equations will describe quite accurate the flow through the actual type of aquifer. The porous media flow system have well defined equations and flow conditions can be accurately predicted. This is not so easy to predict flow characteristics in case of fractured rock, by using conventional approaches. Moreover, there are a wide ranges of types of fractured aquifers and every type is characterised by different equations of flow.

In case of fractured rocks, the first approximation of primary geological and geophysical information is to be used in order to establish the type of fractured aquifers. Then only we can associate the aquifer with a model of aquifer characterised by a set of flow equations. The purpose of this section is only to highlight with field cases how fractured rock aquifers behave to pumping discharges and to what degree of magnitude the aquifer properties can be evaluated by conventional approaches with some modifications as the each case warrants.

5.3.4.2 Flow Through Fractured Media :

A fractured media consists of an interconnected network of fractures surrounding the rock blocks. Therefore, we can define and distinguish between the properties (e.g. permeability and storativity) of the network of fracatres and those of the blocks.

The permeability and storativity of the network of fractures will depend on :

(i) the width of the fracture,

(ii) the properties of the porous material filling the fractures,

(iii) the frequency of the fractures,

(iv) the distribution of fracture width in the system, which determine the inhomogeneity,

(v) the orientation of the fracture system and fracture width which determines the anisotropy.

The blocks may be :

(i) Practically impervious and the properties of the fractured medium are determined by the properties of the net work of fractures; One permeability—storativity system.

(ii) Pervious due to a secndary system of fractures or granular porosity; the properties of such a medium are determined by the properties of both fractures and blocks : double permeability—storativity system.

There are two basic approaches to the mathematical treatment of fractured media.

(i) the discrete approach

(ii) the continual approach

(i) The Discrete Approach :

In the discrete approach the net work of fractures is treated in a similar way as a network of pipes, i.e.

equation of flow are written separately for each fissure and the system of equation is solved with the basic condition of continuity of fluxes and pressures at the point of intersection and for the appropriate boundary and initial condition. This implies that geometry of the fracture system as a whole and the geometry and hydraulic properties of the individual fractures within the system be known. This method is of little practical utility, since it is difficult to define and evaluate fracture dimensions etc., accurately at every point specially when the rock formations are subjected to series of deformation and metamorphism. Presently (fractured rock aquifer system analysis) this approach is not adopted.

(ii) The Continuum Approach :

The continuum aproach is similar to ordinary porous media model. For one conductivity storativity system, the impervious blocks play the role of impervious grains and void spaces of the fracture system is similar to the voids of an ordinary porous medium. Therefore, at large similar parameters as for an ordinary porous medium e.g. porosity and permeability may be defined. This approach is applied in the analysis of fractured rock aquifers.

The conventional laboratory experiments on small cores for determination of porosity and permeability cannot be applied in the case of fractured rocks; these parameters are to be determined by field tests e.g. pumping tests, recover' tests, etc.

Sometimes pervious blocks due to secondary system of fractures may get filled by porous material and introduce secondary porosity and conductivity in the system. These are best to treat them as source producing under the pressure difference between the fluid in the block and surrounding fractures (personnel communication from Prof. Carol Braestee, University of Tromdheim Norwegian Technical High School).

The following types of flow phenomena can be observed in fractured rocks :

(i) at the first stages of pumping, the water is released first from the fractures of high permeabilities.

(ii) as pressure decreases a difference in pressures created between the water contained in the blocks and the water flowing through the fractures leads to release of water from blocks to the main system of fractures—more or less similar to leakage through aquitard as in the case of porous media.

Proper assumptions for different types of fracture rocks may lead in some cases to simplified forms of equations. The pumping tests analysis carried out in hard rock areas, is based on concept of aniostropic aquifer model as applicable to porous media. One has to distinguish between the hydraulic conductivity of a fracture and hydraulic-conductivity of the fractured rocks as a whole, which are parameters of practical interest and are measurable in field tests. The hydraulic conductivity of individual fracture is an apparent value. The following practical conclusions have been derived from studies in hard rock areas,

(1) In view of the small values of porosity the storage capacity of an aquifer without secondary system of porosity is very small and without practical interest. In such aquifers, instantaneous rates of flow can be obtained which will be large than ordinary porous medium, but without continuous supply from an exterior source, the rate of flow will decrease in time and the aquifer will be empty after a relatively short period.

(2) A secondary system of porosity may considerably increase the porosity of the whole system and therefore the storage capacity of the fractured rock aquifer may be high.

Darcy (1856) has shown that the rate of flow of water through porous sand is directly proportional to the hydraulic gradient in the direction of flow and to the permeability of the sand. Over a century later, Snow (1969) showed, through model studies that permeabilities of defractured or jointed rocks may be represented by anisotropic permeability tensors[1] equivalent to that of a continuous or porous medium. Where only one set of joints or fractures are well developed, the permeability tensor has the shape of an oblate spheroid (disc-shaped). Whereas, the permeability tensor of rocks with two well-developed joints sets has the shape of a prolate spheroid (rod-shaped). Where three well-developed joints sets are present, the permeability tensor is isotropic (sphere-shaped).

In granitic terrain, the permeabilities of aquifer are derived mainly from sheet fractures; the permeability tensors of such aquifers are assumed to be disc shaped ellipsoids with maximum permeability values in the plane of the aquifer. Permeability values will be lowest normal to the aquifer. Sheet fractures are hydraulically connected with each other

[1]Tensors are vector quantities, the magnitudes of which depend on their directions.

and with overlying semi-confining beds by two or more intersecting near-vertical joints sets. Permeability tensors of the joints sets are assumed to be rod-shaped having maximum values parallel to the joints sets or normal to the plane of the aquifer. The affect of such permeability distributions is to enhance vertical directions of flow (upward and downward) above and between sheet fractures and horizontal flow directions within sheet fractures or the main aquifer.

In basaltic terrain the permeabilities of the aquifers are due to formations of vesicular basalts or inter-trappean zones of weathered basalt. Those aquifers are assumed to have hydraulic characteristics equivalent to that of porous media, however, their shapes may be "lense-like" and of small extent.

Basaltic aquifers are overlain and interalain by massive dense basalts of negligible inter-granular porosity. These dense basalts have near-vertical joints sets, which are assumed to have low permeabilities and rod-shaped permeability tensors, which results in near-vertical flow.

In his classic paper of 1940, Hubbert demonstrated that the fluid potential (or piezometric) gradient is due to the presence of a mechanical, potential energy field coincident with the flow field, which can be measured at any point by the height to which water will rise above a standard datum in a piezometer tube open at that point. He further showed that the maximum fluid potential gradient is normal to the equipotential surfaces; and the direction of groundwater flow is in the negative direction of the maximum gradient. He showed as well that groundwater flow occurs within flow tubes and under natural conditions, a linear relationship exists between the fluid potential gradient and specific volume discharge along the flow tube, where the permeability is constant. Furthermore, under pumping conditions, where flow is radially inward through the aquifer towards the pumping well, the change in piezometric level at any point within the zone of influence varies directly with the logarithm of its distance from the pumping well.

Theim made use of this relationship to develop a method of determining permeability values from piezometric levels in two piezometers completed in the aquifer within the zone of influence of the pumping well. Considering the form of the Theim equation it is evident that the limitations for application of the Theim method is that the change in the piezometric gradient toward the pumping well must remain constant not that equilibrium conditions must be reached. Thus where the piezometric gradient (due to pumping) between two piezometers installed in an aquifer has become relatively constant with time and where a linear relationship exists between their drawdowns and the logarithms of their distance from the pumping well, the linear relationship indicates that converging flow conditions exist, Theim equation may be applied to solve for transmissivity T values.

Where a negative boundary is present in the aquifer, migration of the zone of influence is restricted and the slope of the "drawdowm—log distance plot" is relatively steep in the direction of the boundary. However, by substitution of this high value of slope into the Theim equation and solving for T an estimate of the "effective" transmissivity of the aquifer in the direction of the boundary may be computed.

Theim equation may be applied in the analyses of long term aquifer-test data in order to calculate real and effective transmissivity values for hard rock aquifers. Negative boundary conditions are present in one or more directions from the pumping wells at most of these sites; these negative boundaries are attributed to geological and topographical effects.

Theis (1935) showed that for particular aquifer conditions, drawdown varies exponentially with time. However drawdown versus time data from aquifer tests in hard rocks similarly show that in most cases the log of drawdown varies linearly with the log of time after the first few minutes of pumping Figs. (5·32) and (5·33). Based on those relationships the optimum pumping rate can be estimated from preliminary tests in order that the maximum available drawdown is used during the time of pumping provided the boundary conditions do not occur. The effects of those boundary conditions can be measured subsequently during the aquifer test.

The optimum pumping rate can be gicen by,

$$Q_P = \frac{SE}{n} \times \frac{T_A}{15 \cdot 81} \qquad (5 \cdot 86)$$

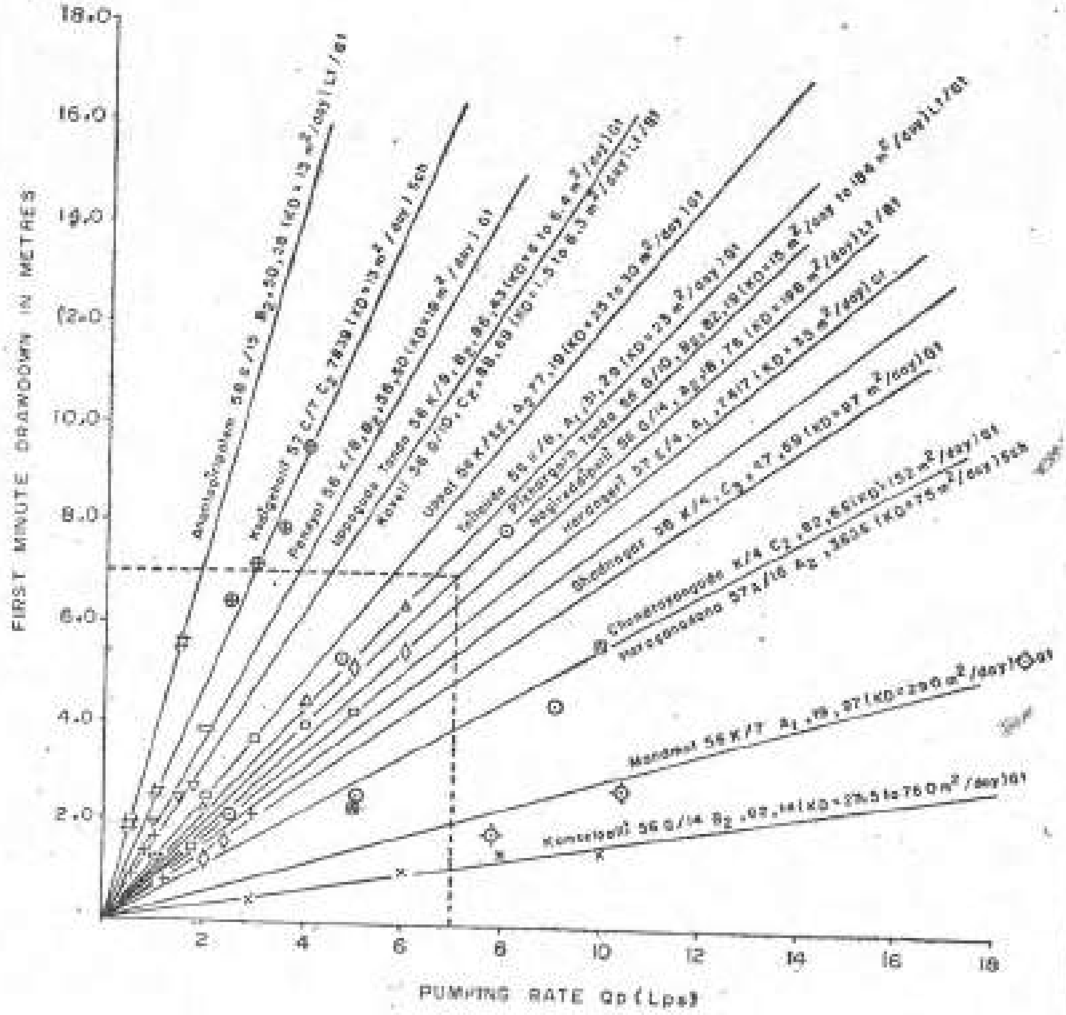
Where,

Q_P = Optimum pumping rate (L.P.S.)
SE = Effective available drawdown (m)
T_A = Apparent transmissivity (m^2/day)
n = Proposed pumping duration in log cycles.

Jacob's and Theis recovery methods elaborated under section 5·2, can be used for determination of aquifer parameters using drawdown/residual drawdown data of an observation well. These can also be applied to pumped well data and in that case the value of transmissivity alone, would be obtained. If the discharge in expressed in L.P.S. instead of

FIG. 5.32

PLOT OF PUMPING RATE 'Qp' VERSUS DRAWDOWN (S) DURING THE FIRST MINUTE OF PUMPING FOR AQUIFER TEST SITES IN GRANITIC, BASALTIC & SCHISTOSE TERRAINS



Lt = LATERITE, Bt = BASALT, Gt = GRANITE, Sch = SCHIST

FROM—Dr. Rao A.A. (1980)

ESTIMATION OF OPTIMUM PUMPING RATE FOR PRELIMINARY AQUIFER TEST

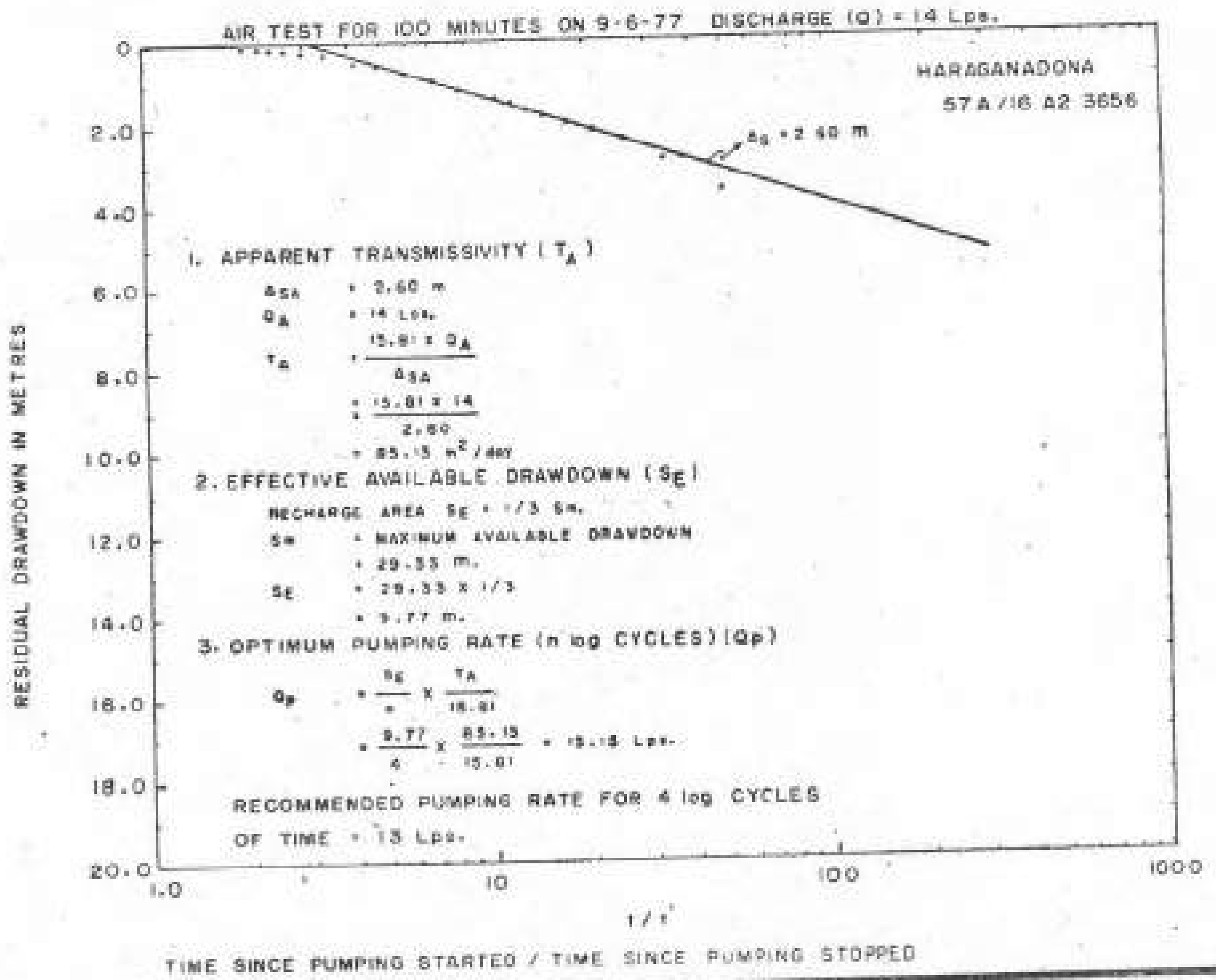


FIG. 5.33

FROM - Dr. Rao A.A. (1980)

m^3/day, Jacob's and Theis recovery equations may be expressed as,

$$T = \frac{15 \cdot 81Q}{\Delta s} \qquad (5 \cdot 87)$$

and

$$T = \frac{15 \cdot 81Q}{\Delta s'} \qquad (5 \cdot 88)$$

Where,

Q is in L.P.S.

All other symbols as defined earlier.

Therefore, where Theis conditions are apparent after the first minute of pumping, drawdown (from a preliminary test) versus the log of time form a straight line and the Cooper-Jacob method may be used to solve for transmissivity. Subsequently that transmissivity value and the ratio of the effective available drawdown to the number of log cycles of the planned pumping time of the long term aquifer test may be substituted into Eq. (5·86) and the optimum pumping rate computed.

All the preliminary and long term aquifer tests conducted in hard rock terrain show that a linear relationship exists between pumping rate and the first minute of drawdown. This initial drawdown develops the piezometric gradient in the aquifer around the borehole that is required to move the water into the borehole at the pumping rate. The effective available drawdown varies from recharge to discharge areas. In case of recharge areas it is 1/3rd of the available drawdown whereas, in discharge areas 2/3rd of the available drawdown.

5.3.4.3 Analysis of Test Data

Since the fractured and cavernous rocks are characterized by a high degree of hetereogeneity, the drawdown/recovery data of the pumping tests conducted in these rocks are commonly not subject to interpretations by general methods applicable for homogeneous porous media such as sand stones, sand and gravel. Jointing or dissolution may result in development of porosity-permeability features such as infinite permeability in one direction to absence of permeability in other directions—which would lead to large distortion of the cone of influence around the pumped well. Thus, various combinations of fracture location, fracture width, fracture content (fine-grained material) and amount of groundwater for storage in the vicinity of the well, can result in any type of "time-drawdown" curve-not amenable to analysis by the standard pumping test methods. Further, an observation well may tap an entirely different fracture system to the one encountered in the pumped well, rendering observation well data of little value. However, at times, pumping tests carried out in hard rocks where jointing or weathering has resulted in uniform development of secondary porosity-permeability system, could be analysed in the usual manner.

Pumping tests carried out so far in fractured rocks, have brought out the prevailance of aquifer conditions such as confined aquifer, Leaky confined aquifer with release of water from storage in aquitards, unconfined aquifer showing delayed yield and aquifer of limited areal extent. Some of the typical field case histories are reproduced here under.

Example-1

[After Rao, A.A. (1980)]

Pumping test conducted at "Sivane" site located in Vedavity River basin, Karnataka State, India is given here as a field case history where a fractured hard rock aquifer behaved as a confined aquifer for 500 minutes of pumping. At 'Sivane', weathered Amphibolite/Granite occur within 10 metre depth, below which upto 40 metres depth, weathered and fractured Amphibolites, Hornblende Chlorite schists and Hornblende Gneisses are present. Depth to water level is about 8·00 metre below land surface.

A pumping test of 500 minutes duration was conducted in May 1979 at a constant discharge of 129·60 m^3/day (1·5 L.P.S.) Fig. (5·34) & (5·35) depict the time Vs drawdown and Residual drawdown Vs t/t^1 plots for the pumped well analysed by Jacob's and Theis's recovery methods respectively. The values of T, obtained by the said methods is as given below :

(i) Jacob's Method :

(Time Vs drawdown plot)

$Q = 129 \cdot 60$ m^3/day (1·5 L.P.S.)

$\Delta s = 2 \cdot 40$ m

$T = 9 \cdot 88$ m^2/day

(ii) Theis's Recovery Method :

(Residual drawdown Vs t/t^1 plot)

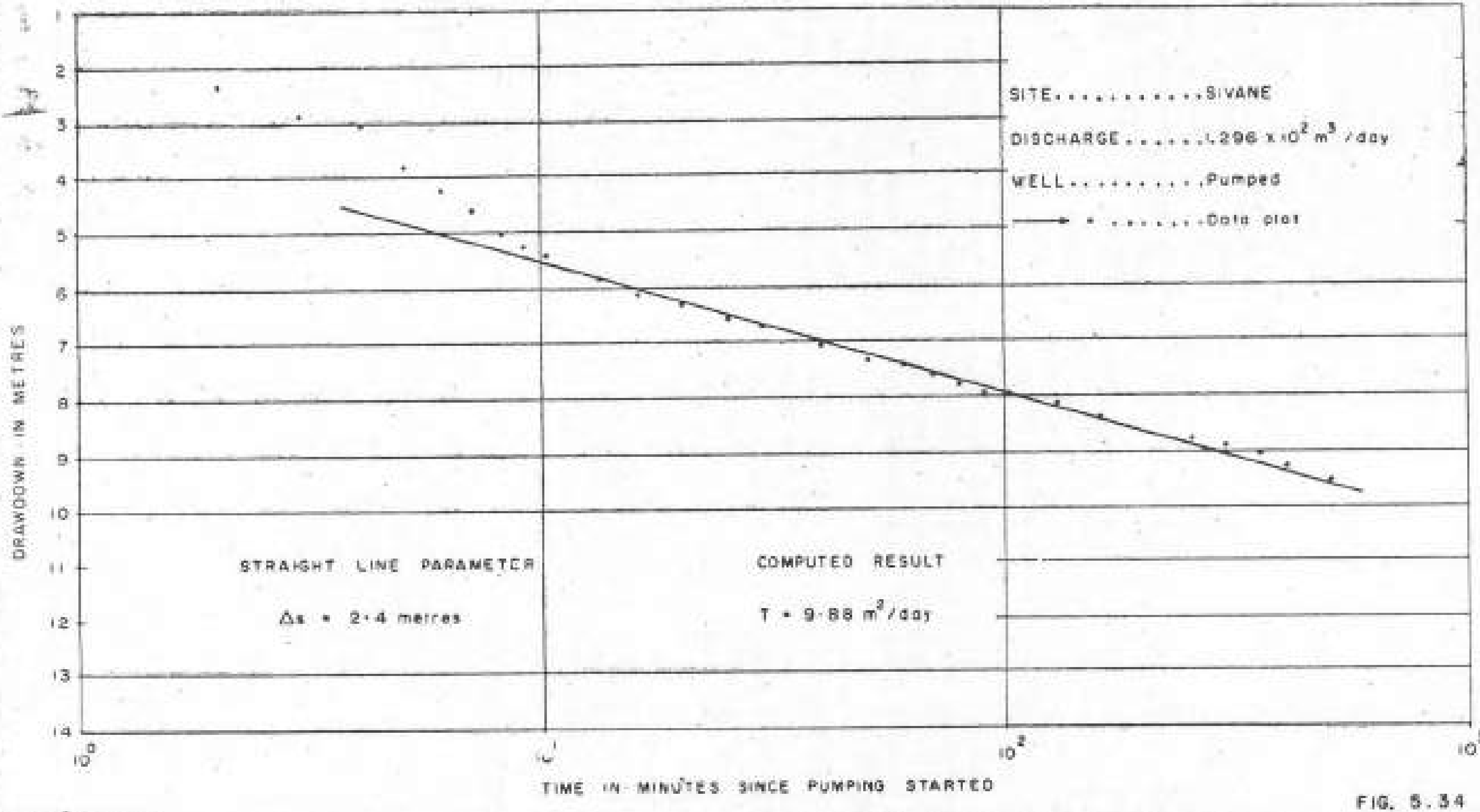
$\Delta s' = 1 \cdot 70$ m

$T = 13 \cdot 95$ m^2/day

Example-2 : [After Rao, A.A, (1980.)]

Pumping test conducted at 'Hagalwadi' site located in Vedavati River basin, Karnataka State, India is given here to illustrate a field case history where a fractured hard rock aquifer behaved as a confined aquifer bounded by two negative boundaries. At 'Hagalwadi' water bearing fractured granitic rocks

TIME Vs DRAWDOWN CURVE

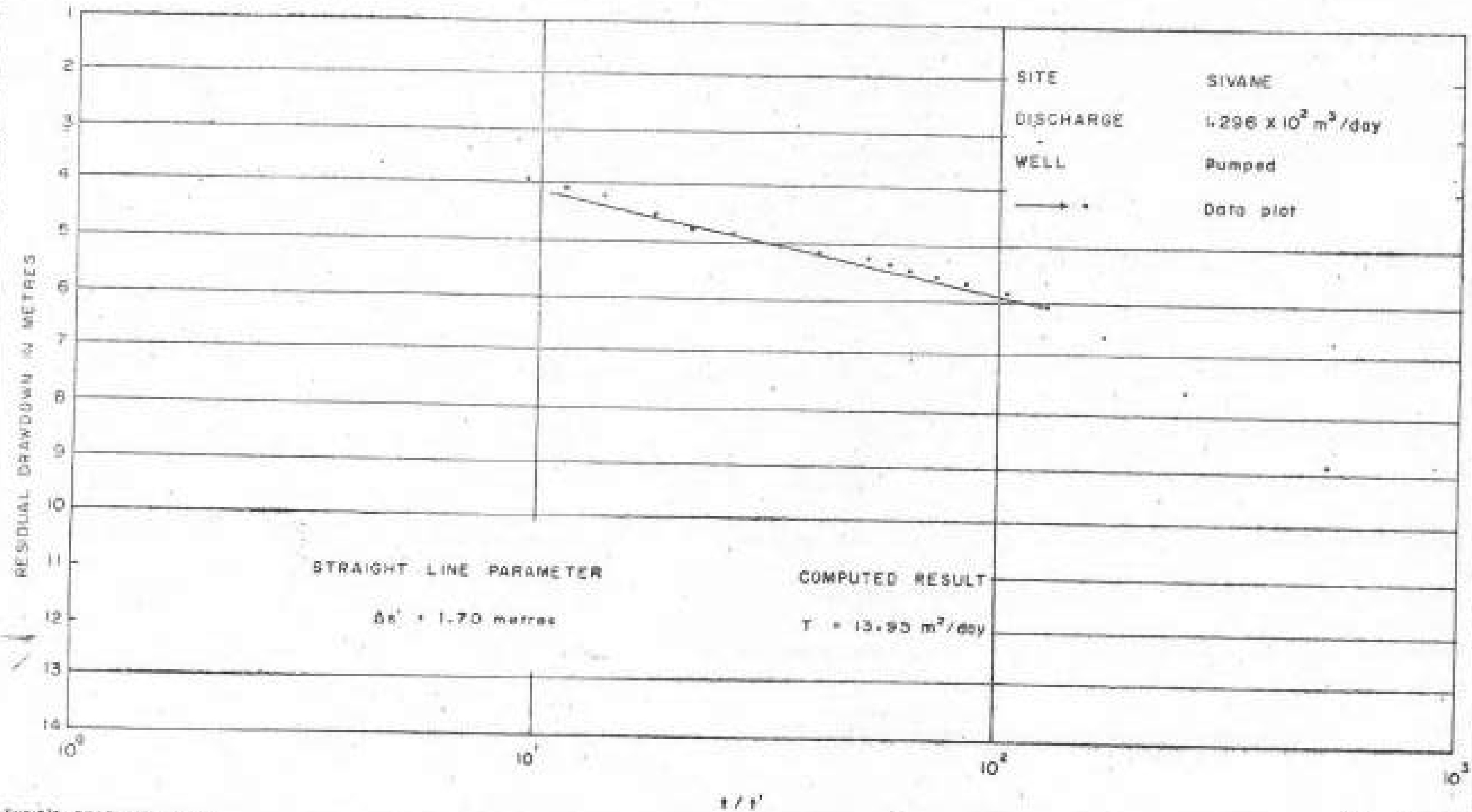


JACOB'S METHOD

FIG. 5.34

FROM- Dr. Rao A.A (1980)

RESIDUAL DRAWDOWN Vs t/t' CURVE



THEIS'S RECOVERY METHOD

FIG. 5.35

FROM - Dr. Rao A.A. (1980)

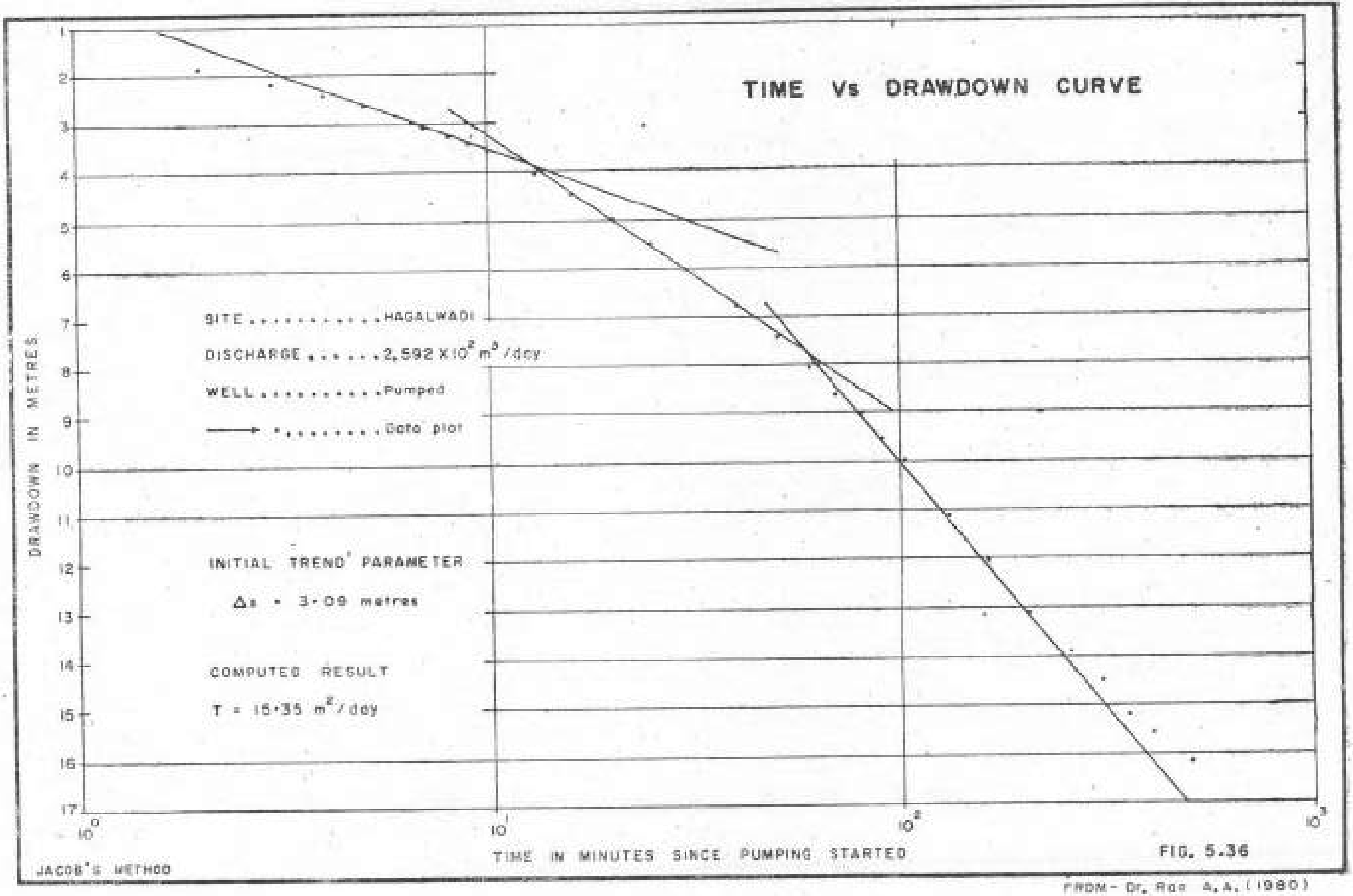


FIG. 5.36

FROM- Dr. Rao A.A. (1980)

occur in the depth range of 10 to 34 metres, overlain by weathered granite. The depth to water level being about 11 metres below land surface.

A pumping test of 500 minutes duration was conducted in April, 1979 at a constant discharge of 259·2 m^3/day (3·0 L.P.S.) Fig. (5·36) depicts the "time-drawdown curve" for the pumped well. The twice steepening trend of the curve suggests existence of two barrier boundaries. The initial trend has been analysed by Jacob's method to yield the local transmissivity value for the fractures tapped in the well. The calculations being—Jacob's Method (Time-drawdown plot)

$Q =$ 259·2 m^3/day (3.0 L.P.S.)

$\Delta s =$ 3·09 m

$T =$ 15·35 m^2/day

Example-3 :

[After Rao, A.A. (1980)]

Pumping test conducted at 'Kudatini' site located in Vedavati River basin, Karnataka State, India, is presented here to illustrate a field case history where a fractured hard rock aquifer behaved as a confined aquifer bounded by a recharge and then a barrier boundary. At 'Kudatini' water bearing fractured granitic rocks occur in the depth range of 32 to 60 metres overlain by weathered granites. The depth to water level being about 4·50 metre below land surface.

A pumping test of 1000 minutes duration was conducted in Dec. 1977 at a constant discharge rate of 172·80 m^3/day (2·0 L.P.S.) Fig. (5·37) depicts the 'time-drawdown' curve for the pumped well. The field data suggests existence of a recharge boundary effective beyond 10 minutes of pumping, followed by a barrier boundary effective beyond 200 minutes of pumping. The initial trend has been analysed by Jacob's method to yield local Transmissions value for the fractures encountered in the borehole. The calculations of T, are as given below :

Jacob's Method (Time-drawdown plot)

$Q =$ 172·80 m^3/day (2·0 L.P.S.)

$\Delta s =$ 5·38 m

$T =$ 5·88 m^2/day

Example-4 :

[Taken from report No. 1·11 of SIDA ASSISTED GROUND WATER PROJECT IN NOYIL, PONNANI AND AMRAVATI RIVER BASINS, TAMIL NADU AND KERALA (1980)]

Pumping test conducted at "Kumarapalaiyam" site located in Noyil River basin, Tamil Nadu State, India, is presented here to illustrate a field case history where a fractured hard rock aquifer behaved as an unconfined aquifer showing delayed yield phenomenon. At "Kumarapalaiyam" water bearing weathered and fractured biotite-genesis with pegmatite and quartz veins occur within a depth of 143 metres. The depth to water level is about 13 metres below land surface.

A pumping test of 3000 minutes duration was conducted in Dec. 1978, at a constant discharge rate of 144 m^3/day (1·66 L.P.S.) and drawdown/recovery data was collected from the pumped well and an observation well located at a distance of 43·25 metres from the pumped well. The "time-drawdown" curve observation well suggests existence of an unconfined aquifer showing delayed yield pheonomenon. Thus, the "time-drawdown" data was analysed by Boulton's and Jacob's methods for determination of aquifer parameters. The calculations are given below.

I. Boulton's Method :

Fig. (5·38) depicts the "time-drawdown" curve analysed by Boulton's method.

$Q =$ 144·00 m^3/day (1·66 L.P.S.)

$r =$ 43·25 m

Early 'match point' co-ordinates being,

$W(u_A, r/B) = 10$

$1/u_A = 10^3$

$s = 3{\cdot}2 \times 10^{-1}$m

$t = 3{\cdot}0 \times 10^{1}$ min $= 2{\cdot}08 \times 10^{-2}$ days

$r/B = 0{\cdot}6$

Using the Eqs (5·47) and (5·48)

$T =$ 35·80 m^2/day

$S_A = 1{\cdot}60 \times 10^{-4}$

Late 'match point' co-ordinates being,

$W(u \quad r/_B) = 10^0$

$\frac{1}{u_Y} = 10^2$

$s = 3{\cdot}2 \times 10^{-1}$ m

$t = 5{\cdot}6 \times 10^2$ min $= 3{\cdot}88 \times 10^{-1}$ days

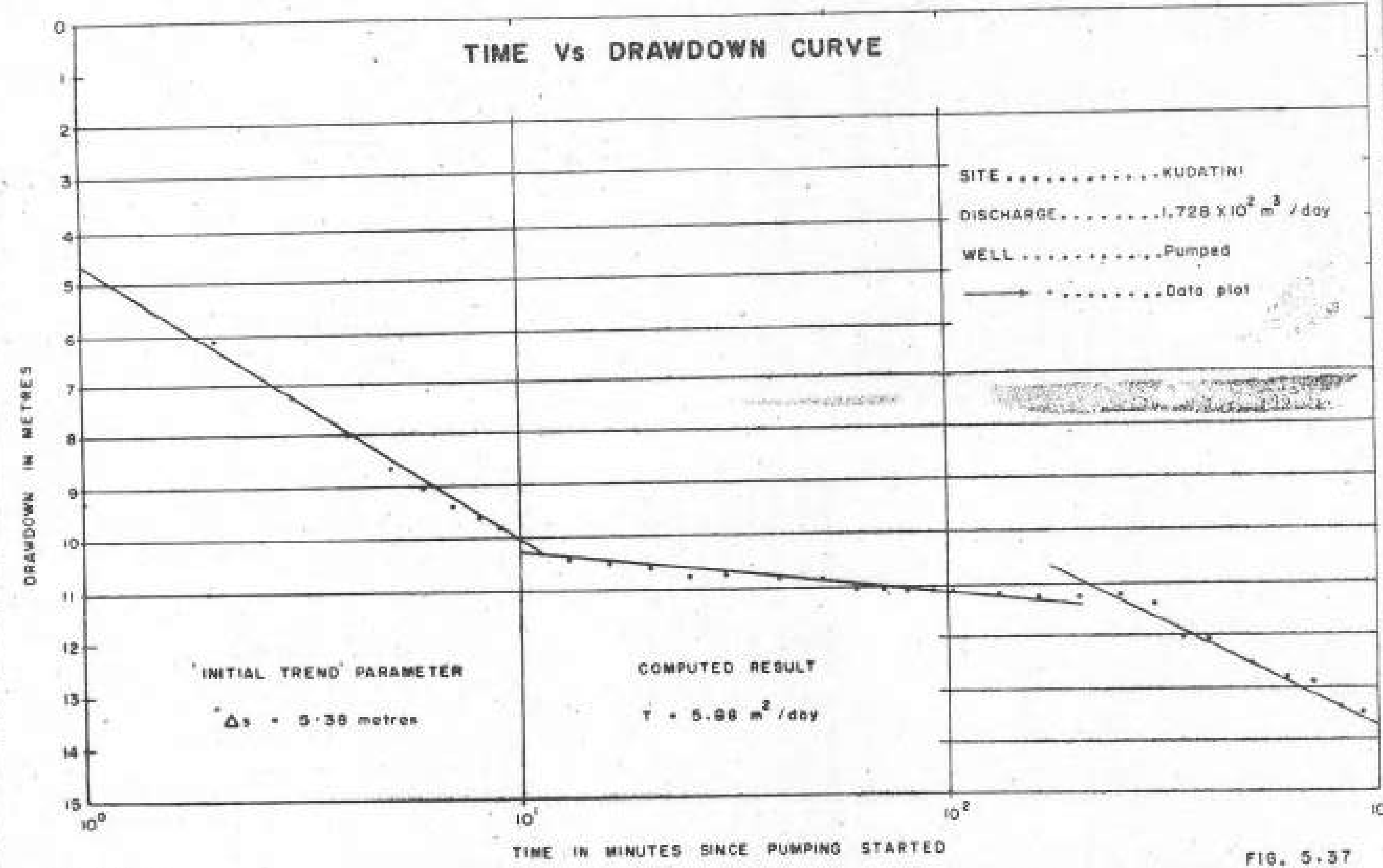
$r/_B = 0{\cdot}6$

Using the Eqs(5·49) and (5·50)

$T =$ 35·80 m^2/day

$S_Y = 2{\cdot}98 \times 10^{-3}$

say, 0·30%



JACOB'S METHOD

FIG. 5.37

FROM— Dr. Rao A.A. (1980)

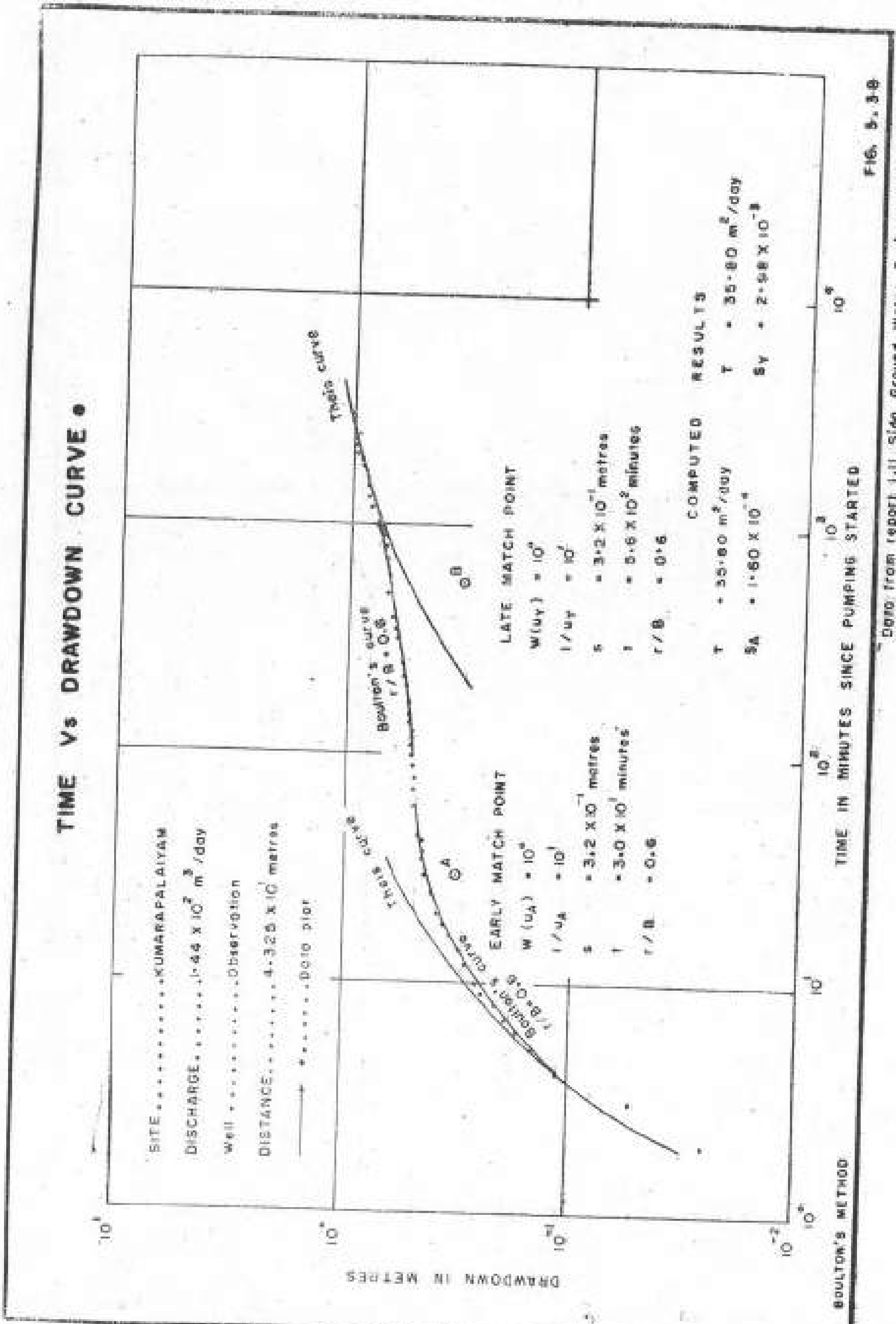
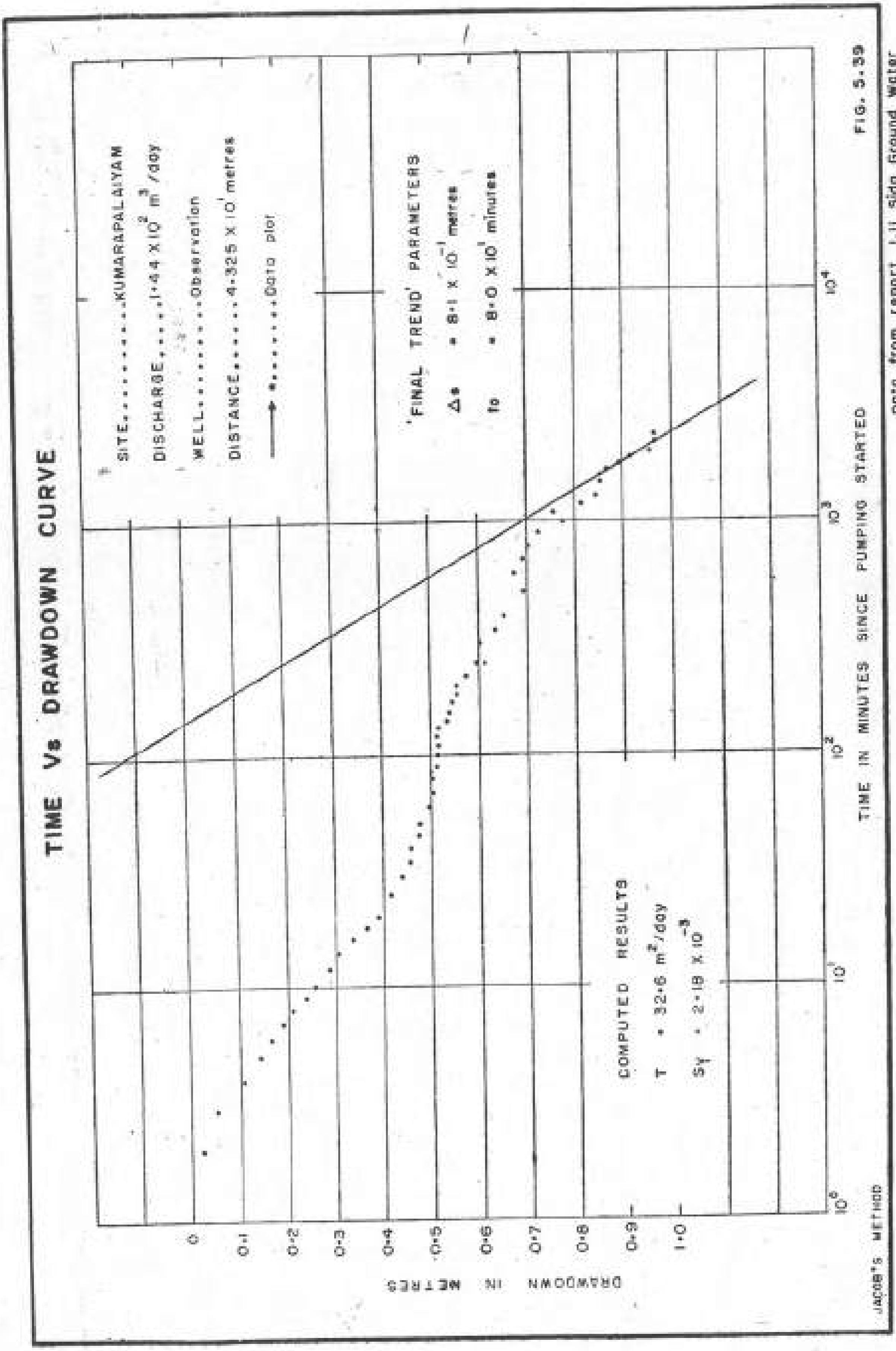


FIG. 5.3.8

Data from report 1-11, Sida Ground Water Project, INDIA (1960)



Data from report 1,11 Sida Ground Water Project, INDIA (1980)

FIG. 5.59

II Jacob's Method :

The third 'segment' data of "time-drawdown" curve has been analysed by Jacob's straight line method as shown in Fig (5·39)

The straight line parameters being

$$\Delta s = 8 \cdot 1 \times 10^{-1} \text{ m}$$

$$t_o = 8 \cdot 0 \times 10^1 \text{ min} = 5 \cdot 56 \times 10^{-2} \text{ days}$$

Using the Eqs (5·11) and (5·12),

$$T = 32 \cdot 6 \text{ m}^2/\text{day}$$

$$S_Y = 2 \cdot 17 \times 10^{-3}$$

say 0·22 %

Example - 5:

[After Rao, A. A. (1980)]

Pumping test conducted at "Kodigehali" site located in Vedavati River Basin, Karnataka State, India, is presented here as a field example where a fractured hard rock aquifer behaved as a leaky confined aquifer with water released from storage in aquitards. At 'Kodigehali,' water bearing weathered and fractured Hornblende Chlorite Schist occurs within a depth of 90 metres. The depth to water level is about 4 metres below land surface.

A pumping test of 10,000 minutes duration was conducted in 1978, at a constant discharge rate of 172·8 m³/day (2 L.P.S.) and drawdown/recovery data was collected from the pumped well and an observation well located at a distance of 5·15 metres from the pumped well. The "time-drawdown" curve of observation well suggests existence of leaky confined conditions with release of water from storage in aquitards. Hence, the drawdown data was analysed by Hantush's modified method (Fig 5·40). The calculation of aquifer parameters are given below,

Hentush's Modified Method :

(Time-drawdown curve)

$$Q = 172 \cdot 8 \text{ m}^3/\text{day}$$

$$r = 5 \cdot 15 \text{ m}$$

'Match-point' co-ordinates being,

$$H(u, \beta) = 10^0$$

$$\frac{1}{u} = 10^3$$

$$s = 7 \cdot 3 \times 10^{-1} \text{ m}$$

$$t = 4 \cdot 0 \times 10^1 \text{ min.} = 2 \cdot 77 \times 10^{-2} \text{ days}$$

$$\beta = 0 \cdot 01$$

Using Eqs. (5·40) and (5·42) we get

$$T = \frac{Q}{4\pi s} H. (u, \beta)$$

$$= 19 \text{ m}^2/\text{day}$$

and
$$S = \frac{4 T tu}{r^2}$$

$$= 7 \cdot 90 \times 10^{-5}$$

Example - 6:

[Taken from report No. 1·11 of SIDA ASSISTED GROUND WATER-PROJECT IN NOYIL, AND AMARAVATI-RIVER BASINS, TAMIL NADU, AND KERALA (1980)]

This example is to illustrate erratic behaviour of an aquifer in hard rock during pumping and how such data could be used for determination of aquifer parameters.

Pumping test conducted at 'Rudrampalaiyam' site located in Noyil river basin, Tamil Nadu state, India, is presented here to illustrate a field example where a fractured hard rock aquifer behaved in an erratic manner— drawdown data beyond 30 minutes of pumping not amenable to interpretation by any standard method. At 'Rudrampalayam', water bearing weathered and fractured biotite gneiss with pegmatite veins occur within a depth of 160 metres. The depth to water level is about 17 metres below land surface.

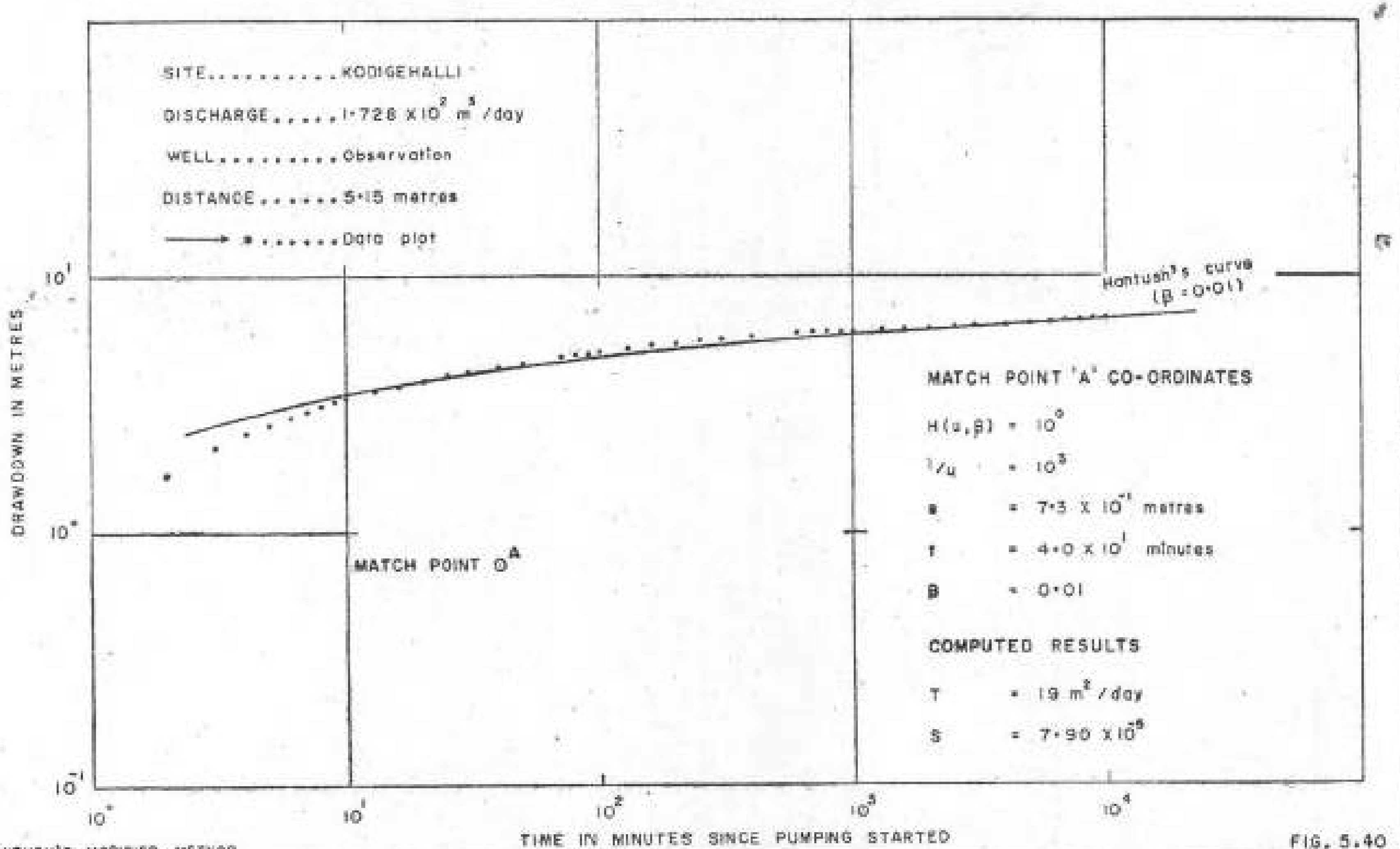
A pumping test of 3000 minutes duration was conducted in Sept. 1978 at a constant discharge rate of 576 m³/day (6.6 L.P.S.) and drawdown /recovery data was collected from the pumped well and an observation well located at a distance of 24·00 metres from the pumped well. The 'time-drawdown" curve of the observation well suggests existence of confined conditions upto about 20 minutes of pumping, after which the curve indicates existence of a barrier boundary followed by a recharge boundary, and finally an erratic rising water level trend. The initial 20 minutes drawdown data has been analysed by Theis's method for determination of local short term T value for the fractures encountered in the borehole. The calculations are given below :

Fig (5·41) depicts the 'time-drawdown' curve of the observation well, analysed by Theis's type curve method.

$$Q = 576 \text{ m}^3/\text{day}$$

$$r = 24 \text{ m}$$

TIME Vs DRAWDOWN CURVE

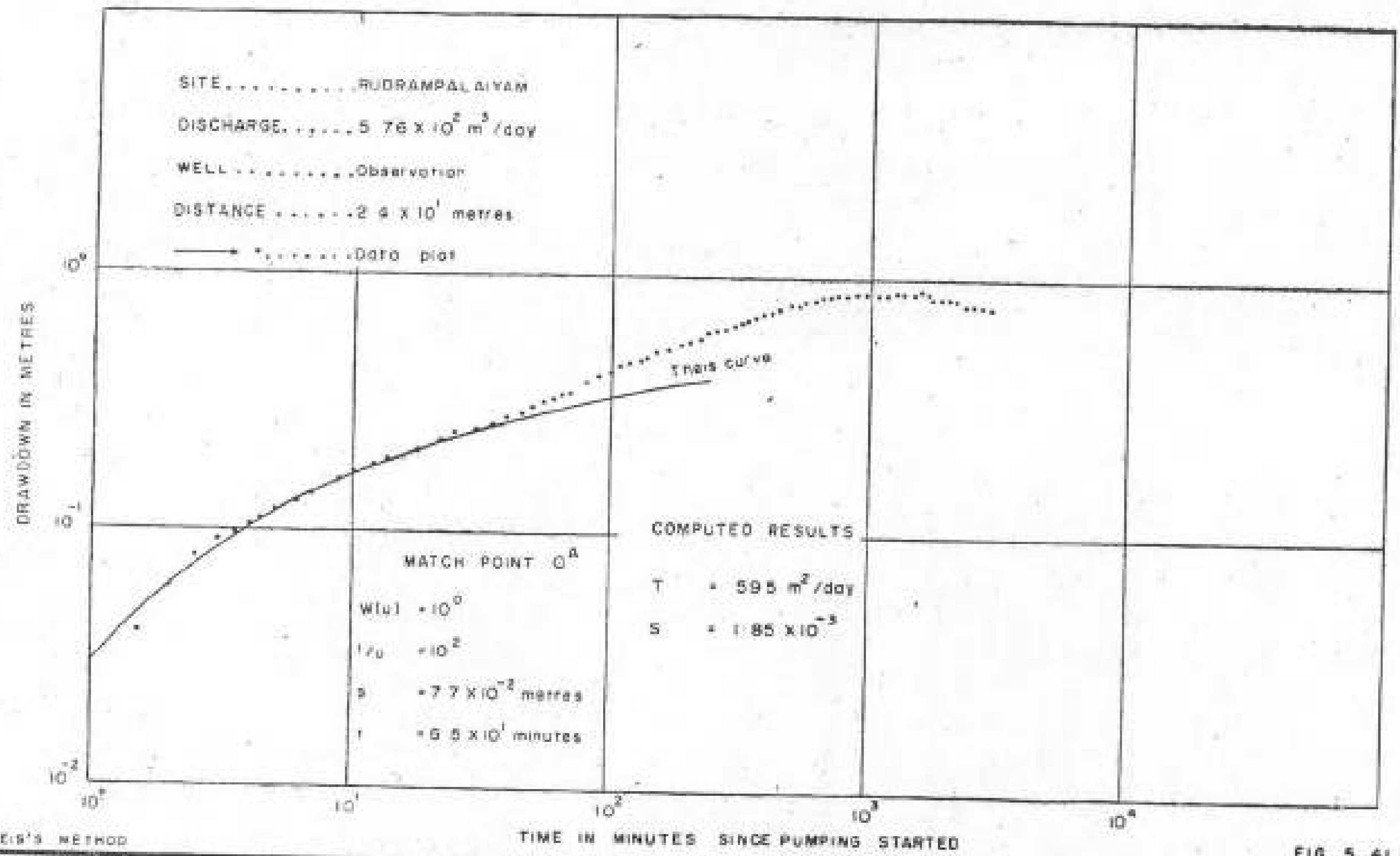


HANTUSH'S MODIFIED METHOD

FIG. 5.40

FROM- Dr. Rao A.A. (1980

TIME Vs DRAWDOWN CURVE



THEIS'S METHOD

FIG. 5.41

Data from report I II, Sida Ground Water Project, INDIA (1980)

'Match-point' co-ordinates being,

$$W(u) = 10^{0}$$

$$\frac{1}{u} = 10^{2}$$

$$s = 7{\cdot}7 \times 10^{-2} m$$

$$t = 6{\cdot}5 \times 10^{1} \text{ min.} = 4{\cdot}51 \times 15^{-2} \text{ days.}$$

Using Theis's Eqs. (5·8) and (5·9)

$$T = 595 \text{ m}^2/\text{day}$$

$$S = 1{\cdot}85 \times 10^{-3}$$

5·3·5 Tests in Non-penetrating Wells (Cavity Wells)

Kanwar and Khepar, Chauhan et, al, and Jaiswal et, al have studied the hydraulics of cavity wells for unsteady-state flow conditions. According to Kanwar and Khepar, assuming a hemispherical cavity and making assumptions similar to those for unsteady-state listed in Section 5·2, the drawdown in an observation well located at a distance, r is given by,

$$s = at^{-1/2} + C \tag{5·89}$$

where,

a and C are constants expressed as,

$$a = \frac{Q\sqrt{Ss}}{4\sqrt{\pi}\, K^{3/2}} \tag{5·90}$$

and

$$C = \frac{Q}{2\pi K r} \tag{5·91}$$

All symbols as defined earlier.

Eq. (5·89) is the equation of a straight line between s and $1/\sqrt{t}$. Therefore, if a plot of observed drawdown, s, and the corresponding square root of time, $1/\sqrt{t}$ are plotted on a simple arithmatic paper, it will give a straight line and the values of constants a and C can be found out from the slope and the intercept of the straight line respectively. Once the values of a and C are known, the values of K and S can be calculated from the Eqs. (5·89) and (5·90).

$$K = \frac{Q}{2\pi C r}$$

and

$$Ss = \frac{16a^2\pi K^3}{Q^2}$$

The values of T and S can thereafter be calculated by multiplying the values of K and S s by the thickness of the aquifer, b respectively.

Example:

The data obtained from a pumping test conducted on a cavity well has been analysed by Kanwar and Khepar., which is reproduced here as an example. Table (5·14) gives the pumping test data.

TABLE 5·14

Time t, since pumping began (hrs)	$1/\sqrt{t}$ (hrs)$^{-1/2}$	Drawdown in observation pipe (cm)	Remarks
(1)	(2))	(3))	(4)
0·0166	7·74	23·75	
0·0333	5·50	28·00	Q = 3·67
0·0493	4·50	31·75	$\times 10^7$ cm/hr
0·0666	3·88	35·00	r = 10m
0·0834	3·46	37·50	
0·166	2·44	40·00	
0·250	2·02	41·25	
0·333	1·77	42·50	
0·416	1·55	43·60	
0·500	1·44	43·90	
0·583	1·31	43·90	
0·666	1·23	44·25	
0·750	1·16	44·25	

Fig (5·42) depicts "time-drawdown" plot of s Vs $1/\sqrt{t}$. The slope and the intercept of the best fit straight line are found to be 1·99 cm hr$^{1/2}$. and 50 cms respectively

Therefore,

$$a = 1{\cdot}99 \text{ cm hr}^{1/2}$$

and $C = 50$ cms.

Using the Eqs (5·90) and (5·91),

$$K = \frac{Q}{2\pi r C}$$

$$= \frac{3{\cdot}67 \times 10^7}{2 \times 3{\cdot}14 \times 10^3 \times 50}$$

$$= 1{\cdot}168 \times 10^2 \text{cm/hr}$$

$$\text{and } Ss = \frac{16a^2\pi K^3}{Q^2}$$

$$= \frac{16 \times (1{\cdot}99)^2 \times 3{\cdot}14 \times (1{\cdot}168 \times 10^2)^3}{(3{\cdot}67 \times 10^7)^2}$$

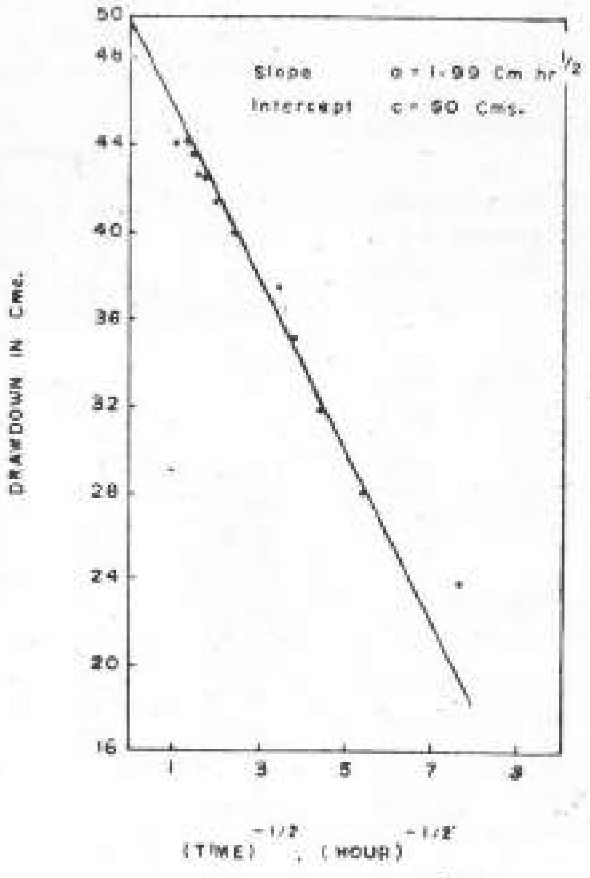
$$= 2{\cdot}29 \times 10^7 \text{ per cm.}$$

The mathematical model developed by Jaiswal et. al. 1977 (a) for flow to a nonpenetrating well and

FIG. 5-42

TIME - DRAWDOWN PLOT

(CAVITY WELL)



FROM- Kanwar R.S. and Khepar S.D. (1976)

techniques developed may be used for determination of aquifer parameters S, T and thickness of aquifer, b. The relationship for flow to a non-penetrating well with hemispherical bottom of vanishing radius and discharging at constant rate may be reproduced below, as

$$s = \frac{Qb}{2\pi T r} \left[C(\sqrt{u}, r/b)\right] \quad (5 \cdot 92)$$

Where,

$$C(\sqrt{u}, r/b) = \mathrm{erfc}(\sqrt{u}) + \sum_{n=1,2,3}^{\infty} \frac{2\,\mathrm{erfc}(\sqrt{1+(2nb/r)^2}\,\sqrt{u})}{\sqrt{1+(2nb/r)^2}}$$

$$u = \frac{r^2 S}{4Tt}$$

erfc = Complementary error function

All other symbols as defined earlier.

Equation (5·92) was developed for non-penetrating well and holds good for observation well that terminates at the points just below the upper confining layer.

If the pump test data i.e. drawdown in an observation well with time with constant discharge rate is available, then aquifer parameters may be determined using equation (5·92) as below :

Procedures :

— Draw a curve of log s vs log $(1/\sqrt{t})$ on a graph paper, this curve may be termed as data curve.

— Draw another curve of log $C(\sqrt{u}, r/b)$ vs log $\sqrt{u}$ in another graph paper having the cycles similar to that of data curve. This curve may be termed as type curve No. 11.

— Superimpose the data on the type curve by keeping the horizontal and vertical axes parallel.

— The match point is obtained from the portion of the curve where maximum match is obtained.

— The coordinate of the 'match point', s, $C(\sqrt{u}, r/b)$, $\sqrt{u}$ and $1/\sqrt{t}$ are read from the data and type curves. The value of r/b is also read for which the match point is obtained.

— The aquifer parameters may be obtained using coordinates of $C(\sqrt{u}, r/b)$, $\sqrt{u}$, $1/\sqrt{t}$, s and r/b as,

$$T = \frac{Q\ b\ C(\sqrt{u}, r/b)}{2\pi s r} \quad (5 \cdot 93)$$

$$S = \frac{4Ttu}{r^2} \quad (5 \cdot 94)$$

Thus S, T and b may be determined from the superimposition technique using the data from constant discharging nonpenetrating wells. The method presented will give the results provided the bottom of the observation well is just below the upper confining layer and the observations are taken in a single observation well at a distance, r from the centre of the well. The illustrated procedure for determination of aquifer parameters from pump test data of a non-penetrating well is given as an example.

Example :

The hypothetical data of pumping test for non-penetrating well given in Jaiswal et al [1977 (b)] may be used to illustrate the procedure for determination of S, T and thickness of the aquifer. The data given below pertains to a non-penetrating well discharging at a constant rate of 102·52 m³/hr. for the values of T/S = 2331·7 m²/hr and K = .3054 m/hr. The drawdown at a distance r = 10m for 10m and 20m thick aquifer are given below.

TABLE 5.15

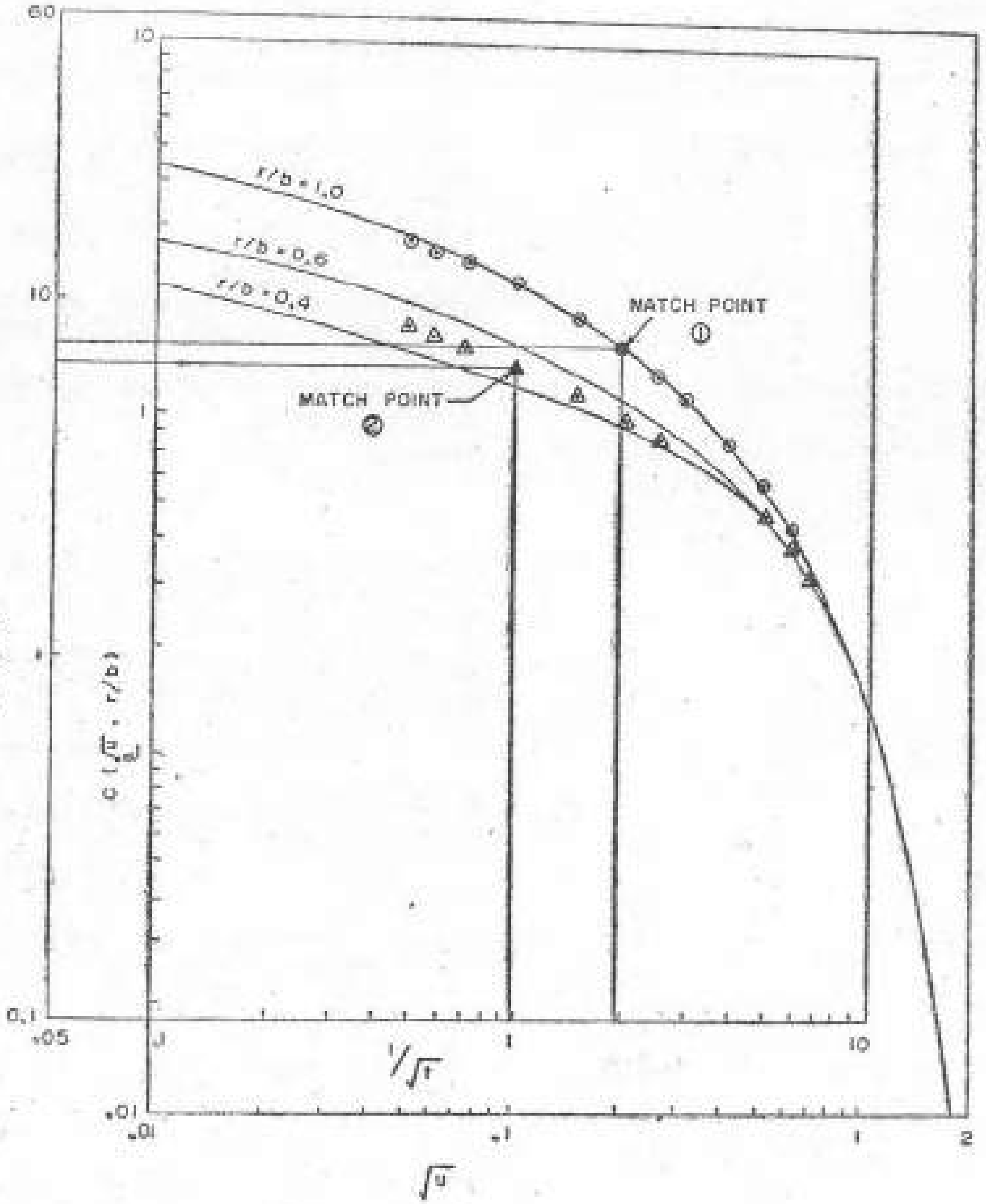
Drawdown for different values of time t for
T/S = 2331·7 m²/hr.
K = 0·3054 m/hr and
Q = 102·52 m³/hr.

Time in hour	$1/\sqrt{t}$ (hrs)$^{-1/2}$	Drawdown (s) (metres) 20 m thick (1) aquifer	10 m thick (2) aquifer
107·21	0·0966	12·68	23·38
104·28	0·4834	8·47	14·79
103·00	0·5744	7·99	13·85
102·00	0·7070	7·45	12·77
101·072	0·9658	6·62	11·11
100·476	1·449	5·56	8·98
100·268	1·931	4·81	7·48
100·171	2·418	4·24	6·35
100·111	2·898	3·79	5·45
100·067	3·863	3·10	4·09
100·042	4·879	2·57	3·11
100·029	5·872	2·11	2·39
100·220	6·742	1·72	1·85
100·017	7·669	0·48	0·84

The data curve and the type curve are matched in Fig. (5·43). The 'match points' are shown for the two sets of the data. It may be observed that set (1) falls in-between the curves for r/b = 0·4 and r/b = 0·6 and set (2) falls on the curve, r/b = 1·0. The coordinates for 'match point' for set (2) are s = 7·45, $C(\sqrt{u}, r/b) = 1 \cdot 38$, $\sqrt{u} = 0 \cdot 205$, $1/\sqrt{t} = 1 \cdot 93$ and r/b = 1·0 and for set (1) are s = 6·62, $C(\sqrt{u}, rb) = 1 \cdot 22$ $\sqrt{u} = 0 \cdot 102$, $1/\sqrt{t} = \cdot 965$ and r/b = 0·5

FIG. 5.43

SUPERIMPOSED TYPE AND DATA CURVES



FROM- Jaiswal et al (1977)

Using equations (5·93) & (5·94), the values for set (1) and set (2) may be calculated as,

set (2)

$$T = \frac{102\cdot52\times1\cdot38}{2\times7\cdot45\times3.14} = 3\cdot0224 \text{m}^2/\text{hr}$$

$$S = \frac{4\times(0\cdot205)^2\times3\cdot0224}{(1\cdot93)^2\times100} \text{—} 0\cdot00136397$$

K $= 0\cdot3052$ m/hr

T/S $= 2216$ m²/hr

%error $= -5\cdot00\%$

Set (1)

$$T = \frac{102\cdot52\times1\cdot22}{2\times3.14\times6\cdot62\times0\cdot5} = 6\cdot0140 \text{ m}^2/\text{hr}$$

$$S = \frac{4\times(0\cdot102)^2\times6\cdot014}{(\cdot965)^2\times100} = \cdot002687$$

K $= \cdot3052$ m/hr

T/S $= 2238$ m²/hr

%error $= -4\cdot02\%$

The method illustrated predict the values within an error of —5%. This error is introduced because of the graphical method and acceptable for practical purposes.

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6. APPROXIMATE METHODS

6.1 Introduction

The aquifer parameters like transmissivity (T), storativity (S) are usually determined by discharging well method; that is, by observing the performance of aquifer over a long period of pumping at a given rate.

In many cases, especially reconnaissance type of groundwater investigations, it may not be practical or feasible to construct, test wells and conduct time consuming pumping tests and apply the conventional type of analysis for the evaluation of aquifer parameters. Moreover, some of the modern quantitative techniques such as those for which electric analog models or mathematical models are contemplated, a sufficiently large number of values of T & S are required. In all such cases, quick and short cut/approximation methods may have to be resorted to, in the determination of aquifer characteristics. The hydraulic properties can be estimated with reasonable degree of accuracy by some of the indirect methods based on water level fluctuations, specific capacity (yield per unit drawdown) data of wells, well log data and areal estimates etc.

Different short cut methods presently available for estimation of hydraulic properties of the aquifer without going into detailed pumping tests are enumerated in this section.

6.2 Estimation of Transmissivity :

6.2.1 Bailer Method :

Skibitzke (1958) proposed a method for determining the transmissivity from the recovery of water level in a well that has been bailed. The following equation is applicable at any given point on the recovery curve.

$$T = \frac{V}{4\pi s't\; e^{(r_w^2 S/4Tt)}} \qquad (6\cdot1)$$

Where;

- s' = Residual drawdown in metre (L)
- V = Volume of water removed in one bailing cycle in m^3 (L^3)
- t = Length of time since bailing stopped in days. (T)
- r_w = Effective radius of the well, in metres. (L)

As r_w is small the term in brackets in equation (6·1) approaches e^0 or unity, as 't' increases. Therefore for large values of 't' the equation (6·1) can be rewritten as :

$$T = \frac{V}{4\pi s't} \qquad (6\cdot2)$$

Remarks:

This method should give satisfactory estimates of T for confined aquifers having sufficiently shallow water level to permit short time intervals between bailing cycles. For wells in unconfined aquifers or wells having relatively deep water levels, this method should be used with considerable precautions.

6.2.2 'Slug' Method :

In cases of non-availability of pumps or wells fitted with pumps, it is desirable to obtain an estimation of transmissivity of aquifer by using this method. In this method a known volume or 'slug' of water is suddenly injected into or removed from a well and the decline or recovery of water level is measured at repeated closely spaced intervals.

Under the above conditions, and with usual assumptions, Cooper, Bredehoeft, and Papadopulous (1967) derived the following equation for the response of a finite-diameter well to such an instantaneous 'slug' of water.

$$H = 8Ho \quad \frac{\alpha}{\pi^2} \int_0^\infty \exp\left(\frac{-\beta_u^2}{\alpha}\right) \frac{du}{\Delta u} \qquad (6\cdot3)$$

Where,

H = Head inside the well at time 't' after injection or removal of the 'slug' above or below the initial head, in metres.

Ho = Head inside the well above or below initial head at instant of injection or removal of 'slug', in metres.

and

$$\alpha = \frac{r_s^2 S}{r_c^2} \qquad (6\cdot4)$$

Where,

- r_s = radius of well screen or open hole in metres. (L)
- r_c = radius of casing in interval over which water level fluctuates, in metres (L)

and in Eq. (6·3)

$$\beta = \frac{Tt}{r_c^2} \qquad (6\cdot5)$$

$$\Delta u = \frac{[uJ_0(u)-2J_1(u)]^2}{[u\,Y_0\,(u)-2\alpha Y_1(u)]^2}$$

where,

J_0 = Bessel function of first kind, zero order.

J_1 = Bessel function of first kind, first order.

Y_0 = Bessel function of second kind, zero order.

Y_1 = Bessel function of second kind, first order.

and

u = Variable of integration.

Values of H/Ho versus Tt/r^2_c for five different values of α obtained by numerical solution of Eq. (6·3) given by Cooper, Bredehoeft, and Papadopulous (1967, table I) are plotted as a family of semi-logarithmic curves. Type Curve No. 10. 'V' the measured volume of water injected or removed from the well, obviously, is equal to $Ho\pi r_c^2$

$$\therefore \quad Ho = \frac{V}{\pi r^2_c} \qquad (6\cdot6)$$

Procedure : "slug injection"

— Inject a known volume (V) of water into the well/borehole.

— Measure the decline of water level at closely spaced intervals, during ensuing minute or two, i.e. H values are recorded at repeated intervals.

— Estimate the value of Ho, i.e. the rise in water level because of injection of water using the Eq. (6·6).

— Compute the values of H/Ho for various time intervals.

— Plot on a semi-logarithmic paper of the same scale as the Type Curve of H/Ho Vs Tt/r_c^2 the values of H/Ho against the corresponding time t, in seconds (t on logarithmic scale.)

— Superpose the field data curve on the type curve and while keeping the axes parallel, select, a 'match line' for the value of t at $Tt/r_c^2 = 1\cdot0$ (Match point values = of H/Ho are not needed).

— Use the following equation for calculation of T value

$$T = \frac{1\cdot0 r^2_c}{t} \qquad (6\cdot7)$$

— Use Eq. (6·4) to compute the value of S.

$$S = \frac{r_c^2}{r_s^2}$$

Remarks:

(i) Determination of S value by this method has questionable reliability since the value of S will change by an order of magnitude when data plot is moved from one type curve to another, whereas, determination of T is not so sensitive to the choice of the curves to be matched.

(ii) This method is applicable only to fully penetrating or fully screened wells in confined aquifers of rather low transmissivity (say less than 100 m^2/day).

(iii) For partially penetrating wells, the value of transmissivity obtained, generally would apply only to that part of the aquifer in which the well is screened or open.

(iv) Application of this method to wells in unconfined aquifers would require considerable judgement and the results are likely to be of doubtful nature.

Example :

[(After Rao, A.A. (1980)]

Data of a 'Slug' test on a well tapping a fracture at 22 metres depth at E. M. Palya site located in Vedavati river basin, India, is presented here to illustrate the applicability of 'Slug' test method. Fig. (6·1) shows a H/Ho Vs t in seconds, plot, matched with the Type Curve of H/Ho Vs Tt/r_c^2. The match line values and calculation of T value are given below:

For $Tt/r_c^2 = 1\cdot0$

t = 2,550 seconds.

$\alpha = 10^{-3}$

Now r_c being 0·019 metres, using the Eq. (6·7).

$$T = \frac{1\cdot0\ r_c^2}{t}$$

$$= \frac{0\cdot019 \times 0.019 \times 60 \times 1440}{2550}$$

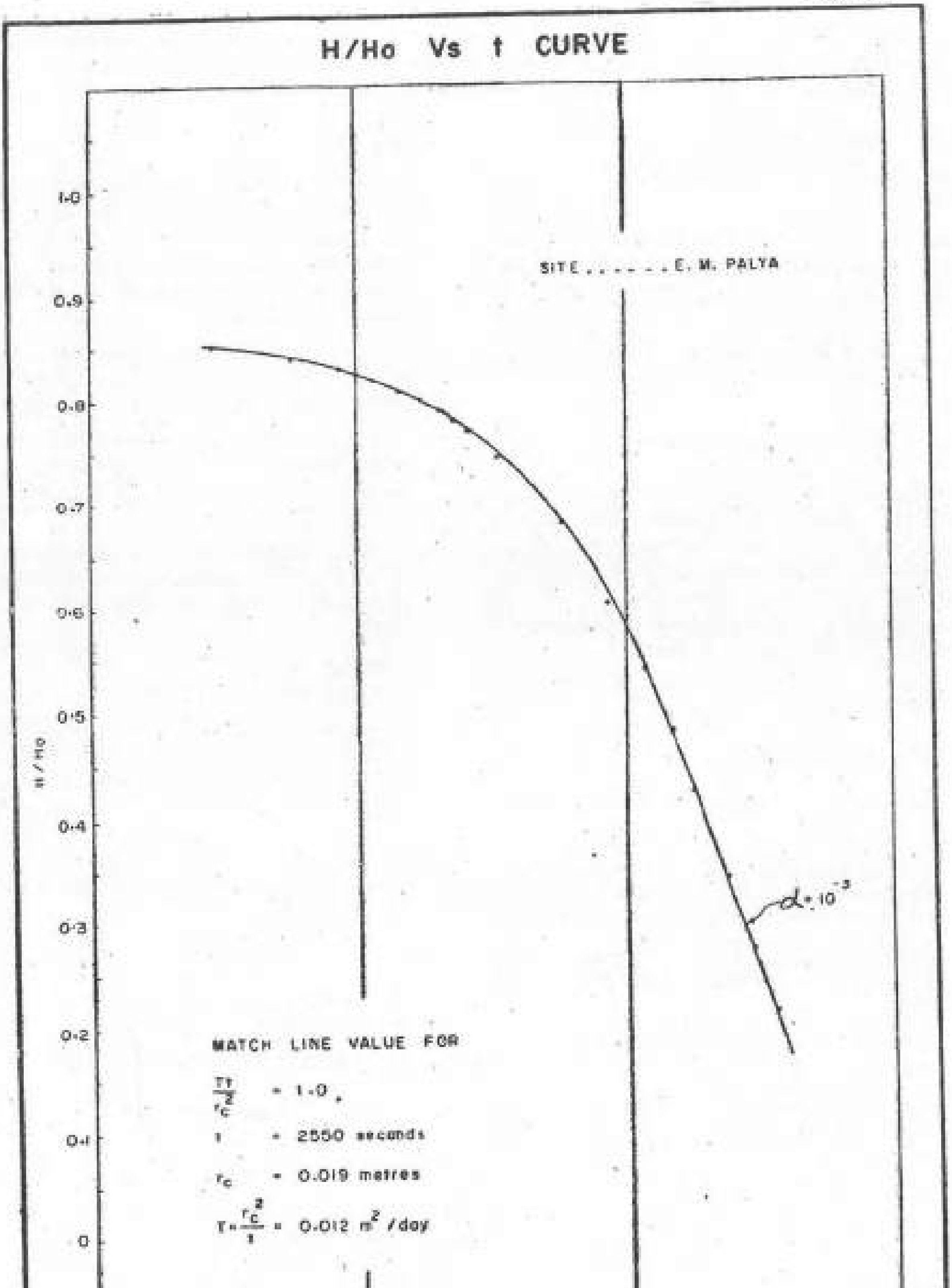
$$= 0\cdot012\ m^2/day.$$

6.2.3 Specific Capacity Methods :

Several methods for estimating transmissivity from specific capacity data have been published, some of which are cited below.

Theis and others (1963) demonstrated that the theoretical specific capacity of a 100 percent efficient

FIG. 6·1



FROM- Dr. Rao A A (1980)

well can be determined from the abbreviated non-equilibrium equation.

$$T = \frac{Q}{4\pi s_w}\left(-0.5772 - l_n \frac{r^2 S}{4\,Tt}\right)$$

$$\text{or} \quad \frac{Q}{s_w} = \frac{4\pi T}{2.30 \log 2.25\, Tt/r^2_w S} \qquad (6.8)$$

Where,

s_w =drawdown in a 100 percent efficient pumped well, in metres (L)

r_w =radius of the pumped well, in metres (L)

$\frac{Q}{s_w}$ = Specific capacity in m³/day per m of drawdown (L^2T^{-1})

All other terms as defined earlier.

From the above equation, it is seen that the theoretical specific capacity of a 100 percent efficient well is directly proportional to T and inversely proportional to log t, log $1/r^2_w$ and log 1/S. Hence, large changes in T cause corresponding large changes in specific capacity; whereas, large changes in t, r_w and S cause comparatively small changes in specific capacity. Also, since r_w is constant for a well, T and S being constants, the Eq. (6.8) may be written as,

$$\frac{Q}{s_w} = \frac{B}{\log t} \qquad (6.9)$$

where,

B =a constant for the well.

However, no well is 100 percent efficient. Jacob (1947) has approximated the well loss to be nearly equal to CQ^2 where C=a constant of proportionality.

Thus, the drawdown in a pumped well which is not 100 percent efficient, would be equal to (s_w+CQ^2) Substituting this instead of s_w in Eq. (6.9)

$$\frac{Q}{s_w+CQ^2} = \frac{B}{\log t} \qquad (6.10)$$

Therefore, specific capacity of a well which is not 100 percent efficient, diminishes with time t, as well as pumping rate, Q.

In an uncased well, r_w may be assumed equal to the radius of the well, but in screened well in unconsolidated material in which the finer particles have been removed near the screen by well development, or in gravel packed well the effective radius, r_w is generally larger than the screen diameter. Jacob (1947) described a method for determining the effective radius r_w and the well loss (CQ^2) from a multiple step-drawdown test.

Theis (1963) gave equation and a chart, based upon the Theis equation, for estimating T from specific capacity for constant S and variable t, with allowance for variable well diameter but not the well efficiency.

Meyer (1963) gave a chart for estimating T from the specific capacity at the end of 1 day of pumping, for different values of S and for well diameters of 0.152, 0.305 and 0.6lm.

Bedinger and Emmett (1963) gave equations and a chart for estimating T from specific capacity, based upon a combination of the Thiem and Theis equations and the average values of T and S for a specific area for well diameters of 0.152, 0.305 and 0.61 m.

None of these methods include corrections for well efficiency. However, a thumb rule relationship between specific capacity and transmissivity in American hydrologic units, is available. The specific capacity is expressed in USGPM/ft of drawdown, and transmissivity in USGPD/ft. In semi-consolidated formations, confined aquifers may have T values in the range of 2000 to 3000 times the specific capacity (t=1 day); whereas, for unconfined aquifers, T values may be 2000 times. In case of hard rocks the relationship is 75 to 100 times the specific capacity (Fig. 6.2). These factors are also valid when the specific capacity is expressed in m³/min per metre of drawdown and T in m²/day. Rao, A.A. (1980) has developed a graphical representation to estimate T in hard rocks if discharge and available drawdown ratio are known (Fig. 6.3).

6.2.4 Other Single Drawdown Observation Methods:

The specific capacity methods discussed in section 6.2.3 permit estimation of transmissivity from a single drawdown observation in the pumped well. In this section, some more methods which can be used for estimation of transmissivity values from a single drawdown observation made in the pumped or an observation well, are discussed.

6.2.4.1 Logan's Method:

Logan's method is applicable under steady-state flow conditions in a confined aquifer satisfying the general assumptions. If a well in a confined aquifer is pumped for quite a long duration, its steady-state drawdown can be used for estimation of an approximate value of transmissivity.

According to Logan (1964) the Theim equation for a confined aquifer (Eq. 5.2) may be written as :

$$T = \frac{2.30\, Q \log r_{max}/r_w}{2\pi s_m} \qquad (6.11)$$

FIG. 6.2

THEORETICAL RELATION BETWEEN SPECIFIC CAPACITY AND TRANSMISSIVITY

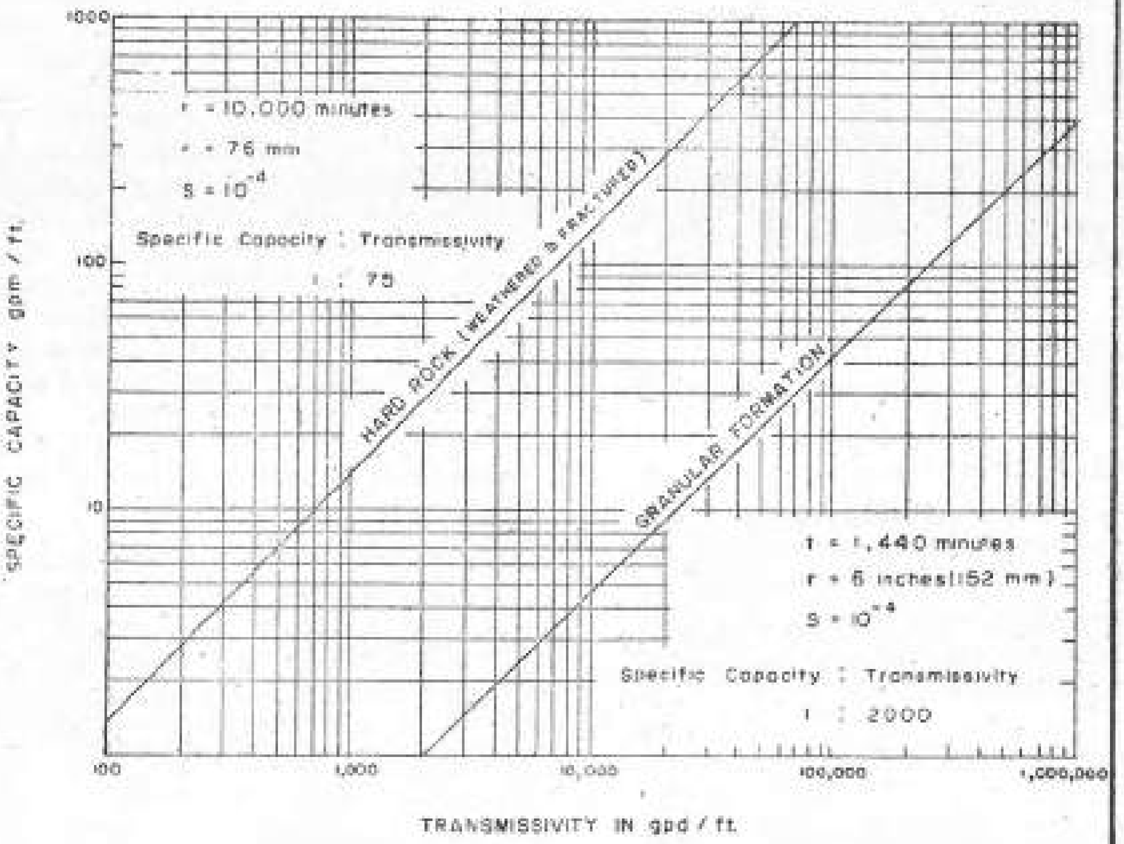
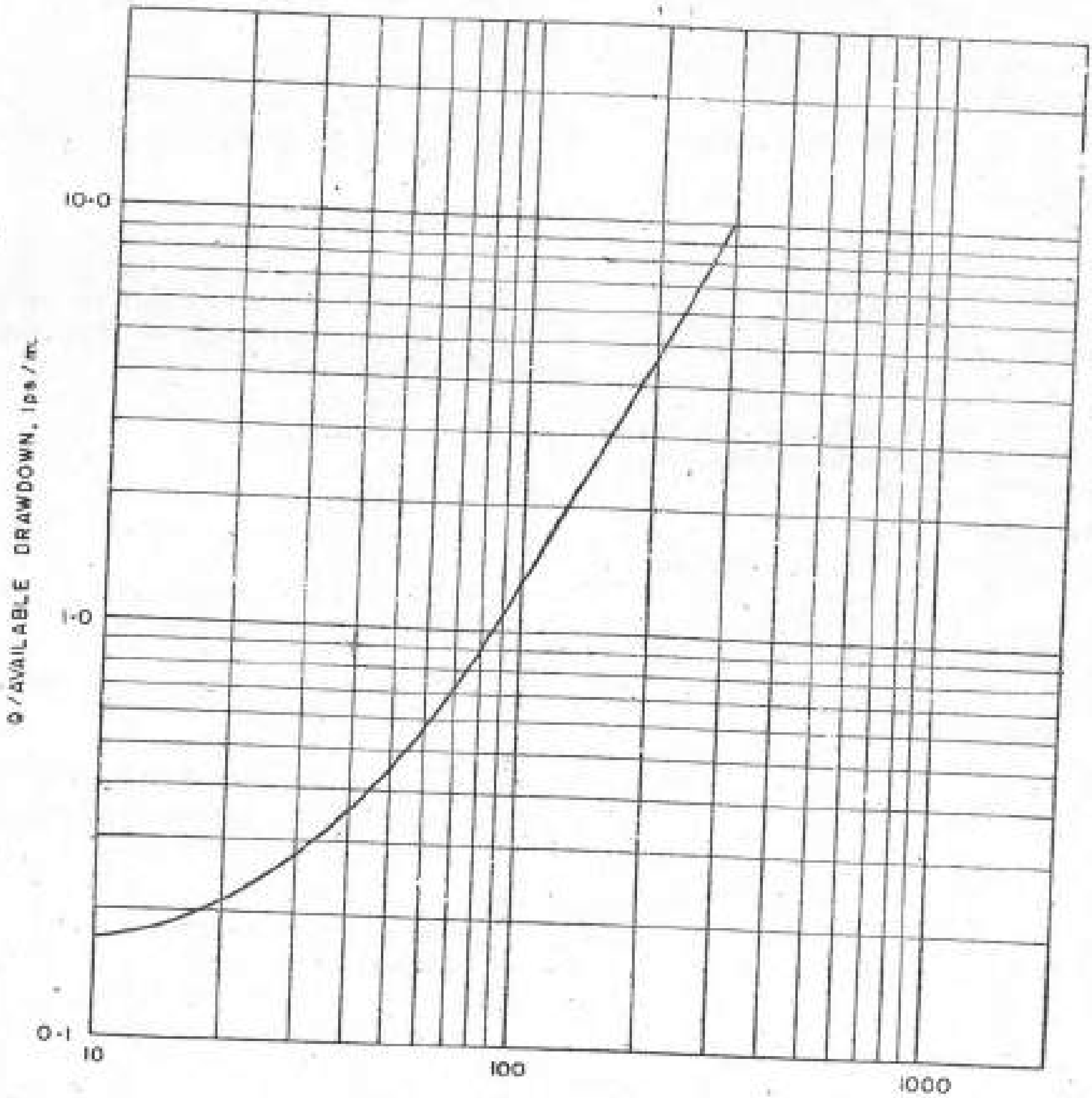


FIG. 8·3

ESTIMATION OF TRANSMISSIVITY BY KNOWN DISCHARGE AND AVAILABLE DRAWDOWN RATIO



FROM - Dr. Rod A. A. (1980)

where,

r_w = radius of the pumped well in metres (L)

r_{max} = radius of influence in metres (L)

s_m = maximum drawdown in the pumped well in metres (L).

The accuracy of calculation depends on the accuracy of measurement of s_m (which includes well losses) and on the accuracy of the ratio r_{max}/r_w. This ratio can't be determined accurately without the use of observation wells. However, although the variations in r_{max} and r_w may be substantial, the variation in the logarithm of their ratio is much smaller. Therefore, assuming average conditions of radii, a value of 3·33 for the log ratio may be taken as a rough approximation.

Substitution of this value into Eq. (6·11) gives,

$$T = \frac{1 \cdot 22\ Q}{s_m} \qquad (6 \cdot 12)$$

This equation may also be applied in unconfined aquifer, wherein the corrected steady-state drawdown s'_m ($s'_m = s_m - s^2_m/2b$) has to be substituted into the Eq. (6·12).

Remarks :

It should be noted that the determined value of T by this method is very approximate — which may be erroneous upto 50% or more.

6.2.4.2 Hurr's Method:

Hurr's method is based on Theis non-equilibrium equation and can be used for calculation of transmissivity value from a single drawdown observation provided the storativity value could be assumed with reasonable accuracy.

The Theis equation being,

$$s = \frac{Q}{4\pi T} W(u) \qquad (6 \cdot 13)$$

where,

$$u = \frac{r^2 S}{4\ T t} \qquad (6 \cdot 14)$$

Eq. (6·13) may be expressed as,

$$W(u) = \frac{4\pi T s}{Q} \qquad (6 \cdot 15)$$

Hurr (1966) demonstrated that the multiplication of both sides of Eq. (6·15) by u results in the disappearance of T from the right hand member,

$$u.W(u) = \frac{4\pi T s}{Q} \times \frac{r^2 S}{4\ T\ t}$$

$$= \frac{\pi r^2 s}{t} \times \frac{S}{Q} \qquad (6 \cdot 16)$$

A table and a graph of corresponding values of u and u.W(u) are given in Annexure-XIII and Fig. (6.4).

Procedure :

— Calculate from Eq. (6·16) the value of u.W(u) for an assumed value of S and measured values of r, t, s and Q.

— Obtain either from Annexure-XIII or Fig. (6·4) the corresponding value of u.

— Substitute the values of u, r, t and S into Eq. (6·14) and calculate T.

Remarks :

The precision required for storage co-efficient declines with declining value of u. For $u/S < 0 \cdot 001$ the influence of S on the calculated value of T becomes negligible.

6.2.5 Closed Contour Method:

A water level contour map containing closed contours around a well or group of wells of known discharge rate may be used to estimate the transmissivity of an aquifer under steady-state flow conditions. From Darcy's law

$$Q = -\frac{K.A.\ \triangle h}{\triangle r}$$

$$= -\frac{T L \triangle h}{\triangle r} \qquad (6 \cdot 17)$$

Eq. (6·17) for any two concentric closed contours of length L_1 and L_2 may be written as:

$$T = -\frac{2Q}{(L_1 + L_2)\ \triangle h / \triangle r} \qquad (6 \cdot 18)$$

where,

$\triangle h$ = Contour interval in **metres** (L)

$\triangle r$ = Average distance between the two closed contours, in metres (L).

Example :

A hypothetical example is given below to illustrate the application of this method.

Assume that two irregularly shapped close contours have the following measured lengths (as by wheel type map measure)

L_1 = 1000 metres.

L_2 = 2000 metres.

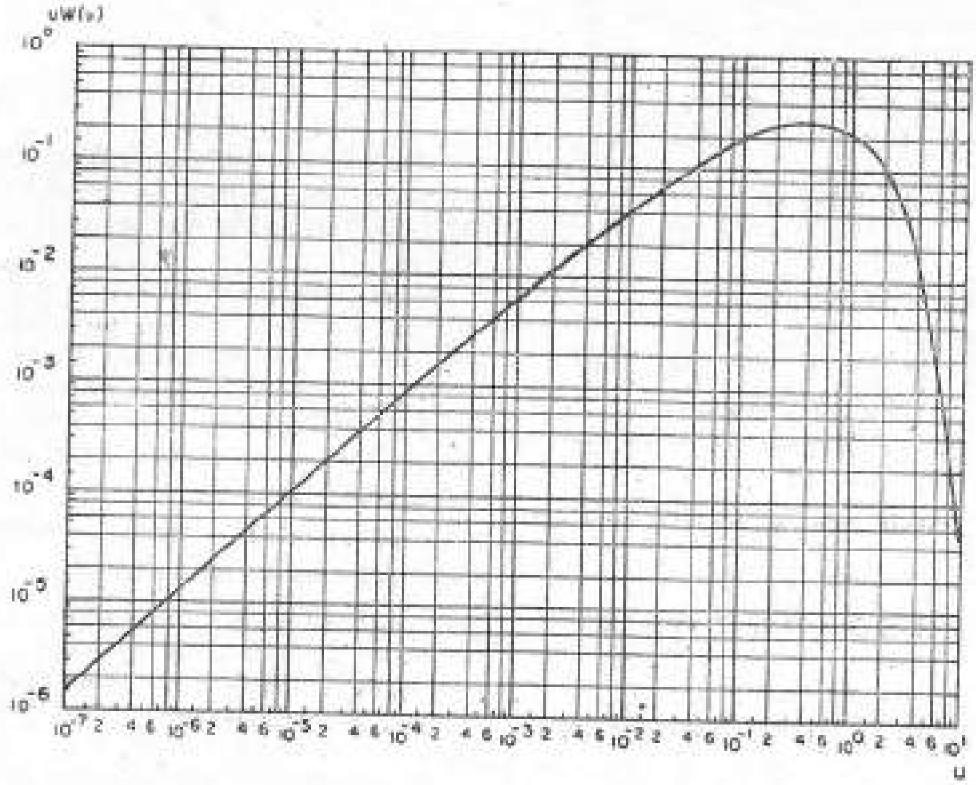
Contour interval ($\triangle h$) = 2 metres

Average distance between two close contours

$$\triangle r = \frac{300 + 500 + 350 + 450}{4} = 400 \text{ metres.}$$

FIG. 6-4

TYPE CURVE FOR CORRESPONDING VALUES OF u AND uW (u)



HURR'S METHOD

Rate of withdrawal from a well field within the lowest closed contour = 2000 m^3/day

Using the Eq. (6·18), we get

$$T = -\frac{[2 \times 2000]}{(3000)[-(2)/400)]}$$

$$L = 266 \quad m^2/day$$

Remarks :

The regularity or irregularity of the shape and spacing of the contours; the density and accuracy of the water level data, and the accuracy to which, Q is known, control the accuracy of transmissivity values.

6.2.6 Dupuit's Formula :

The hydraulic continuity equation could be applied for unconfined flow by using Dupuit's assumptions. Dupuit's assumptions are reasonable for mild curvatures of the free surface.

The continuity equation can be written as

$$1/2 \quad \nabla^2 h^2 \, K = S_Y \frac{\partial h}{\partial t} - W \qquad (6·19)$$

where,

K = Hydraulic Conductivity

$\nabla^2 = \partial^2 + \frac{\partial^2}{\partial}$ (x, y are horioztal orthogonal coordinates)

S_Y = Specific yield (estimated or determined by other methods)

W = Net rate of recharge per unit horizontal area.

$\frac{\partial h}{\partial t}$ = rate of change of head with time.

During the period of observation, if there is an interval with no recharge discharge then W will be 0 and,

$$K = \frac{2 S_Y \frac{\partial h}{\partial}}{\nabla^2 h^2} \qquad (6·20)$$

The transmissivity T of the equifer for a saturated thickness 'b' is given by

$$T = \frac{2 S_Y b \frac{\partial h}{\partial t}}{\nabla^2 h^2} \qquad (6·21)$$

For computation of $\nabla^2 h^2$ in equation (6·21) the numerical methods may be used for computing the curvature of the water table and equate to gradients and find relation between the water table gradients by drawing water table contours of the area.

V.V. Dhruvanarayana (1972) computed the above relation in the vicinity of each observation well. The water table gradients are obtained from the contour maps and the relation developed is as follows.

$$\nabla^2 h^2 = \frac{\nabla^2 h^2}{\partial x^2} + \frac{\nabla^2 h^2}{\partial y^2} = 2\left(\frac{dh}{dl}\right)^2 \qquad (6·22)$$

where,

$\frac{b}{dl}$ = Water table gradient in the direction of groundwater movement

Then the equation (6·21) becomes

$$T = \frac{S_Y b\left(\frac{\partial h}{\partial t}\right)}{\left(\frac{dh}{dl}\right)^2} \qquad (6·23)$$

However, the equation (6·23) is developed for the alluvial areas whereas for the other hydrogeological environments the equation (6·22) needs modification.

The technique of computation requires preparation of water table contour map of the area. The gradients are then computed. The gradient with respect to time can be computed by measuring the water level decline in a network of observation wells. Then solution of equation (6·23) will give the transmissivity value.

6.2.7 Numerical Analysis of Water levels :

6.2.7.1 Stallman & Jenkins Method:

A new method for estimation of transmissivity by numerical analysis of water levels in different observation wells was developed by R.W. Stallman and C.T. Jenkins (1969). This method is applicable under the following conditions.

(i) The aquifers should be homogeneous and isotropic in nature.

(ii) There should not be any recharge from precipitation.

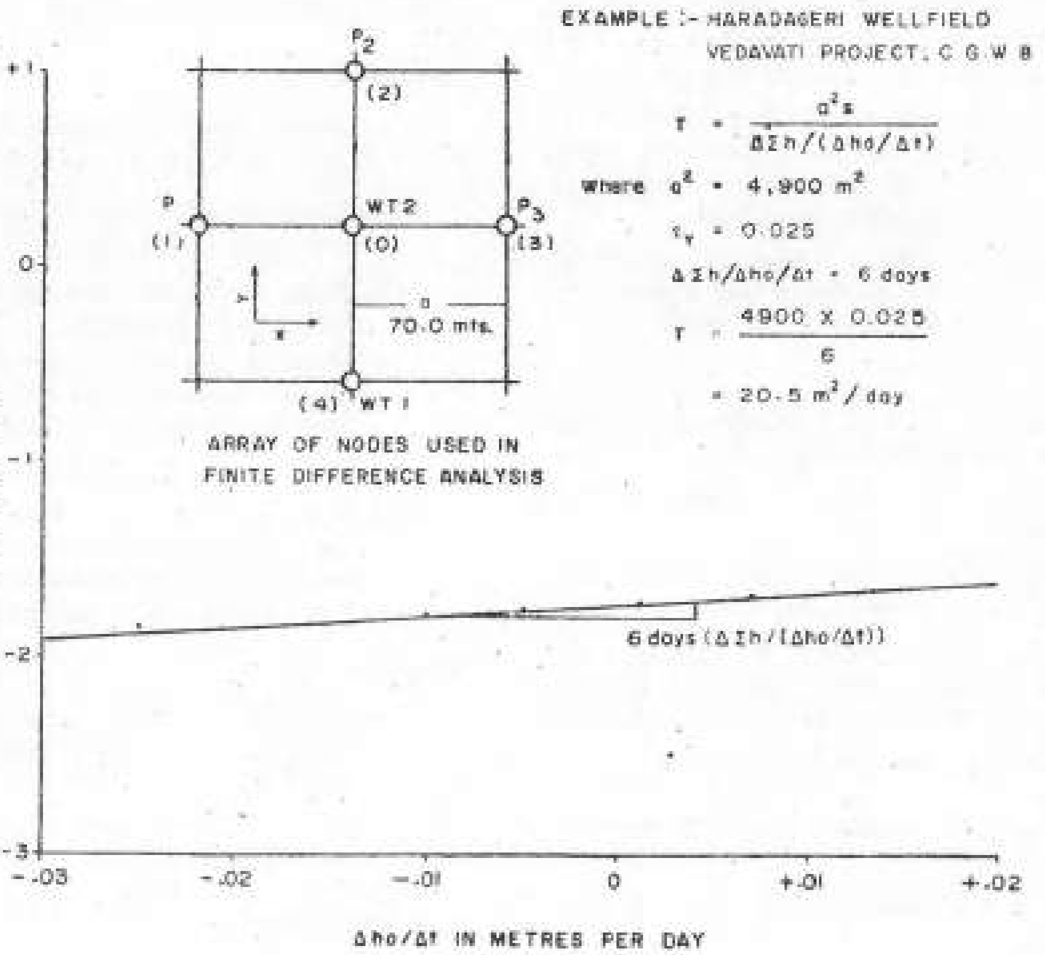
(iii) There should be little or no transpiration from plants.

To satisfy the above conditions (ii) & (iii) the analysis of water levels during winter months is preferable and for (i) it may be assumed, departure due to this may be around 30 to 40 per cent.

(iv) This method requires the wells in a particular pattern as shown in the Fig. (6·5).

FIG- 6-5

AQUIFER TEST BY AREAL METHODS, NUMERICAL ANALYSIS



FROM- Stallman R.W.

The plan of well disposition is shown. The area is to be divided by two systems of equally spaced parallel lines. At right angles to each other, one system is oriented in the 'X' direction and the other in the 'Y' direction.

Wells are constructed at the intersections or the nodes (obs. wells). The spacing of lines equals to (a).

Equation being:

$$T = \frac{a^2 S_Y}{\Delta\Sigma h/(\Delta h_o/\Delta t)} \qquad (6\cdot24)$$

where,

a = Distance between grid lines, in metres (L)

S_Y = Specific yield (estimate based on well logs.)

$\Delta\Sigma h/(\Delta h_o/\Delta t)$ = Slope of the line (T)

where, $\Sigma h = h_1 + h_2 + h_3 + h_4 - 4\ h_o$

h_o, h_1, h_2, h_3, h_4 = Heads of nodes (obs. wells 1 2 3 4 above a convenient arbitrary datum line)

Δh_o = Change in Head at node (Well No. 0) during time interval ..t (Decline at well No. 0).

Procedure :

(i) Fix the convenient equally spaced grid lines as shown in Fig. (6·5.)

(ii) Measure the distance 'a' between the grid lines.

(iii) Determine elevation of observation wells (nodes) by precise survey.

(iv) Fix convenient datum line (elevation) for the area and measure water level in wells above this datum line (h_o, h_1, h_2, h_3, h_4).

(v) Plot on linear graph Σh Vs $\frac{\Delta h_o}{\Delta t}$

Fit a best straight line passing through the points.

(vi) Find the slope of this line in days Fig. (6. 5).

(vii) Determine or estimate the specific yield of formation by any standard procedure.

(viii) Substitute the values of a, S_Y and $\Delta\Sigma h/(\Delta h_o/\Delta t)$ in the equation for solving the transmissivity.

Example : [a well field constructed in Vedavati Project is used [Fig (6.5)]

[After Rao, A.A. (1980)]

Using the Eq. (6·24)

$$T = \frac{a^2 S_Y}{\Delta\Sigma h/(\Delta h_o/\Delta t)}$$

$$= \frac{4900 \times \cdot025}{6}$$

$$= 20\cdot5\ .m^2/day$$

(Pumping test T value = 32 m^2/day).

6.2.8 Air Test

An air test can be conducted subsequent to drilling of exploratory borehole for estimating transmissivity. In view of the nature of the test, measurements of recovery only are possible in this type of test.

Procedure :

(i) After drilling is completed, allow the borehole to recoupe for some time.

(ii) Lower the drill pipe to the top of the producing aquifer zone.

(iii) Pump water by air lift method for a duration of say 100 minutes.

(iv) Note the starting and operating pressure of the compressor. This will help to compute the approximate drawdown in well.

(v) Measure the discharge periodically.

(vi) Stop the pumping and carry out recovery measurements (after pumping stopped) and continue these measurements at least for a period equal to pumping (100 min.)

Precautions :

(i) Discharge during the test should be kept constant or use an average discharge value.

(ii) Recovery measurements should be as accurate as possible.

Analysis of Data :

(i) Plot the values of residual drawdown Vs time ratio $\frac{t}{t'}$ on a semi log paper and draw a best fit straight line through the plotted points.

(ii) Generally 2 or 3 breaks in slope can be observed. Estimate the intermediate slope value for one log cycle of time.

(iii) Substitute values in Theis's recovery formula and calculate the T_A apparent transmissivity,

$$T_A = \frac{2.30 \times Q_A}{4\pi \triangle s} \qquad (6.25)$$

where,

T_A = Apparent transmissivity (m^2/day)

Q_A = Average pumping rate (m^3/day)

$\triangle s$ = Residual drawdown per log cycle of t/t' (m)

If Q_A is expressed in L. P. S., the Eq. (6.25) reduces to,

$$T_A = \frac{15.81 \times Q_A}{\triangle s} \qquad (6.26)$$

where,

Q_A = Average discharge in L. P. S. (1 L.P.S. = 86.4 m^3/day)

This equation can also be used for short duration pumping tests by substituting drawdowns instead of recovery values. Use of this method may yield reliable results upto 70 to 80 per cent.

6.2.9 Study of Well Logs :

Based on the observations of well log (drill cutting) sample descriptions and the available pumping test details, it is possible to assign the values of K to each bed of known thickness. Initially logs of the wells where pumping test results are available, have to be carefully studied and the values of T obtained in the pumping test should be carefully compared with the water bearing bed/beds; as T = Kb, the total has to be distributed by cut and try among several beds according to the equation.

$$T = \Sigma_1^n K_m b_m = K_1 b_1 + K_2 b_2 + K_3 b_3 + \ldots K_n b_n \qquad (6.27)$$

Based on these studies, a standard table for known hydraulic conductivity values has to be prepared for the study area. Then the log of wells where no pumping tests are available has to be carefully examined and K values assigned to each beds. The value of K assigned may be, equal to, or more than or less than the values in the table (depending upon cleanliness, sorting etc.) and thus necessarily involves judgement. With experience, K and T could be estimated with reasonable accuracy.

Laboratory determination for 'K' of cores of consolidated rocks can be used in place of estimates. Reconstitution of disturbed samples is not practically possible. As such, laboratory estimates are not very reliable. However, they may indicate relative values for comparison.

Tables (6.1) and (6.2) give the ranges of Hydraulic Conductivity of unconsolidated/semi-consolidated and consolidated/fractured rocks respectively.

TABLE 6.1

Ranges of Hydraulic Conductivity of Unconsolidated to Semi-Consolidated Formations

Nature of Aquifer Material	Range of Hydraulic Conductivity (K) (m/day)
Gravel :	
Coarse	50—100
Medium	40—50
Fine	30—40
Sand :	
Gravel to very coarse	40—50
Very coarse	30—40
Very corase to coarse	25—30
Coarse	20—25
Coarse to medium	10—20
Medium	5—10
Medium to fine	3—5
Fine	1—3
Loam	0.1—0.5
Clay :	
Clay	$\leqslant$ 0.001

TABLE 6.2

Ranges of Hydraulic Conductivity of Consolidated and Fractured Rocks

(Based on studies carried out in hardrock area projects, Central Ground Water Board. These are average values only)

Nature of Aquifer Material	Range of Hydraulic Conductivity (m/day)
A. Granites/Gneisses etc.	
1. Highly weathered granites with aluvium	25 to 20
2. Weathered and fractured	20 to 10
3. Partly weathered and fractured	10 to 5
4. Relatively fresh and fractured	5
B. Metamorphic Rocks (Schists Phyllites etc.)	
1. Highly weathered and fractured	< 1
2. Weathered and fractured	1 to 5
3. Relatively fresh and fractured	5 to 10
C. Basalt and Other Associated Formations :	
1. Voggey laterite	$\leqslant$ 5
2. Clayey laterite	$\leqslant$ 1
3. Weathered basalt	$\leqslant$ 1
4 Vescular basalt	1 to 5
D. Limestones (Non-Cavernous) }	> 1
E. Sandstone (Shaly) }	> 1

6.3 Estimation of Hydraulic Conductivity

6.3.1 Hooghoudt Formula for Estimation of Hydraulic Conductivity of Phreatic Aquifer:

Assumptions :

(i) Water table near the well does not fall.

(ii) Flow through the well is laminar/horizontal.

(iii) Flow through the bottom of the well is vertical.

Procedure :

(i) The water level in the water table well/borehole is allowed to be stabilised for sometime.

(ii) The entire storage in the well/hole is pumped out quickly (care should be taken to see that no drawdown is created, only storage has to be bailed out).

(iii) The pumping is then stopped and the recovery measurements are taken at close intervals.

Equation being :

$$K = \frac{2 \cdot 3\, r_w C}{(2b + r_w)t} \log \frac{s}{s'} \qquad (6 \cdot 28)$$

$$C = \frac{r_w b}{0 \cdot 19} \qquad (6 \cdot 29)$$

where,

r_w = Radius of the well in metres (L)

b = Saturated thickness of unconfined aquifer, in metres (L)

t = Time of recovery, in days (T)

s = Total drawdown, in metres (L)

s′ = Residual drawdown, in metres (L)

In case where the borehole is drilled down to the impervious layers (i.e. there is no flow from the bottom of the well) the flow of water is horizontal through the walls only. Then K is estimated by following equation.

$$K = \frac{2 \cdot 3 r_w C}{2tb} \log \frac{s}{s'} \qquad (6 \cdot 30)$$

Example:

Fig. (6.6) depicts an example from Vedavati River Basin (India.)

6.4 Estimation of Specific Yield :

The specific yield of the formations generally ranges between 0·01 to 0·30. Unconsolidated to semi-consolidated formations have specific yield values 10 to 20 per cent whereas, for fractured and weathered rocks these are around 1 to 5 per cent. In the absence of any determination, as in rapid reconnaissance surveys, it would not be far off, in assuming that, for supposedly long period of draining, the specific yield of water table aquifers in unconsolidated and semi-consolidated formations, is about 15 per cent

Better estimates of specific yield (might be slightly more or less than the average) could be obtained from:

(i) Careful study of the grain sizes and degree of sorting from observation of well logs.

(ii) Data from few reliable pumping tests.

(iii) Values obtained from the use of neutron moisture probes. The values estimated by this method may not be absolutely correct. They may have to be checked up with values obtained from reliable pumping or flow tests, then extrapolated to similar formations.

(iv) Laboratory determination of values of Sy for disturbed samples, which are likely to be larger than those obtained in the field.

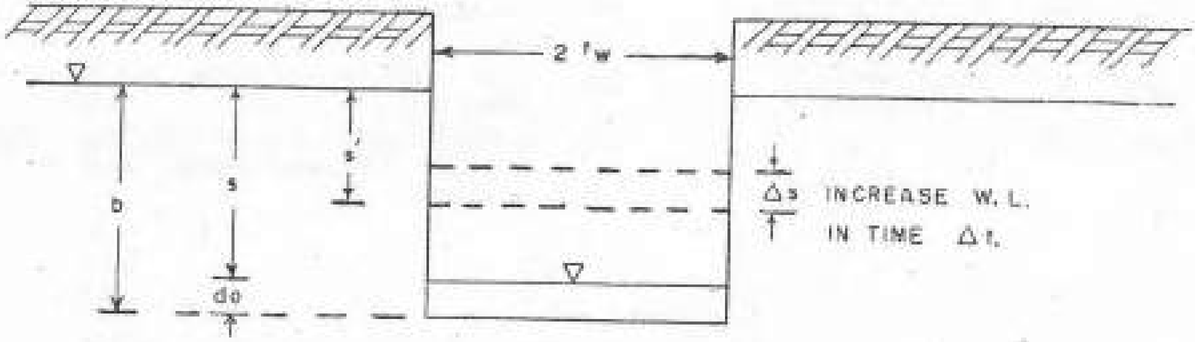
6.5 Estimation of Storativity :

In examining well logs of confined aquifers or measuring sections of exposed rocks that dip down beneath confining bed to become confined aquifers the storage coefficient may be estimated from the following rule of thumb relationship.

b (ft)	S	$\frac{S}{b}$ (ft^{-1})
1	10^{-6}	10^{-}
10	10^{-5}	
100	10^{-4}	
1000	10^{-3}	

One may either multiply the thickness in feet times 10^{-6} ft^{-1} or interpolate between values in the first two columns; thus for b = 300 ft, $S = 3 \times 10^{-4}$, and so on. When thickness of aquifer is expressed in metres, it is to be multiplied by $3 \cdot 3 \times 10^{-6} m^{-1}$; thus, for b = 100 m, $S = 100 \times 3 \cdot 3 \times 10^{-6} = 3 \cdot 3 \times 10^{-4}$. Values thus estimated are not absolutely correct, as no allowances have been made for porosity or for compressibility of the aquifer, but they are fairly reliable for most purposes. Such estimates may be improved upon by comparison with values obtained from reliable pumping or flow tests, then extrapolated to other parts of an aquifer with adjustments for thickness if needed.

FIG. 6.6



DEFINITION SKETCH FOR PERMEABILITY TEST

EXAMPLE :- Haraganadona water table well
Vedavati Project, C.G.W.B.

r_w = 0.0756 metres

b = 9.44 metres

s = 6.477 metres

s' = 5.272 metres

t = 100 minutes.

$$K = \frac{2.3\, r_w C}{2\, tb} \log \frac{s}{s'} \qquad \text{Where } C = \frac{r_w b}{0.19}$$

$$= \frac{2.3 \times 0.0756 \times 0.0756 \times 9.44 \times 1440}{2 \times 100 \times 9.44 \times 0.19} \log \frac{6.447}{5.22}$$

$$= 0.045 \text{ m/day}$$

(BASED ON HOOGHOUDT FORMULA)

FROM - Dr. Rao A. A. (1980)

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7. OTHER METHODS

7·1 Introduction :

In addition to the methods described in earlier chapters, there are some other comparatively less conventional methods available for determination of aquifer parameters. These can be enumerated as:

(i) Laboratory Methods
(ii) Borehole Geophysical Methods
(iii) Inverse Techniques
(iv) Tracer Techniques

7·2 Laboratory Methods :

In the laboratory, the permeability can either be determined from grain size parameters or directly by using permeameters.

7·2·1 Relationship of Permeability of Samples with Grain Size Parameters and related Aspects.

The following two lines of approach have been adopted in this regard :

(i) Fair and Hatch (1933), Kozney (1953) and Marshall (1958) formulated theoretically permeability equations involving factors like porosity, packing factor, sand shape factor etc. However, methods of determining some of these factors are fairly involved and time consuming and thus, severely limit the usage of the equations for determining the permeability.

(ii) Krumbein and Monk (1942), Griffiths (1955), Press and Todd (1963), Cohen (1963) and Masch and Denny (1966) have used an empirical approach for sand sized samples. These workers determined experimentally the permeability of samples and tried to relate the permeability with the various grain size parameters such as mean sorting, D_{60}, D_{10}, D_{50} etc. obtained from grain size distribution curves. These studies indicate that for coarse sand, standard deviation is the controlling factor for permeability, whereas for silt and finer sediments, mean size is the determining factor for permeability value.

7·2·2 Prediction of Field Permeability from Laboratory Permeability

(a) There are number of difficulties in predicting field permeabilities from laboratory permeability such as :

(i) The samples get disturbed, when these are removed from the field to laboratory.

(ii) The field permeabilities are always less than the laboratory permeabilities as even in unconsolidated sediments, some cementation is always there.

(iii) Since permeability may vary considerably both laterally as well as vertically within a formation, large number of samples need to be analyzed.

(b) Most intensive research into relation of laboratory permeability and aquifer tests have upto date been conducted by Johnson and Greenkorn (1960, 1963). They studied 2000 samples from 56 wells from the Pennsylvanian sandstone in Central Oklahoma (USA) in an eight acre plot. Large variations in permeability were noted in well with the average permeability determined by laboratory tests. Local variations in permeability were controlled chiefly by differences in grain size of sandstones.

7·3 Borehole Geophysical Methods :

The different characteristics of aquifer can be determined with the help of geophysical borehole techniques.

7·3·1 Effective Porosity :

This involves determination of formation factor 'F' relating resistivity of saturated sand and the resistivity of the electrolyte filling the pores. This relationship is expressed according to Archie's law

$$Ro = FRw \qquad (7\cdot1)$$

where Ro is the resistivity of saturated rock and Rw that of the electrolyte. 'F' is found to be related to the effective porosity 'f' through the relation,

$$F = \frac{1}{f^m} \qquad (7\cdot2)$$

The factor 'm' is called the cementation factor. This was found to lie between 1·8 to 2·0 for consolidated sandstones and 1·3 for clean unconsolidated sands packed in the laboratory. Another form of the above relationship is

$$F = \frac{C}{f^m} \qquad (7\cdot3)$$

with 'C' varying from rock to rock.

The formation factor can be determined by measuring the resistivity of the mud invaded zone in the drill hole in permeable formations, provided the mud filtrate resistivity is known. It is possible to calibrate an electrical resistivity log in terms of formation factor when it measures the resistivity of mud invaded zone. Effective porosity can then be determined by using any one of the relations (7·2) or (7·3).

If another electric log measures R_o then knowing F it is possible to calculate Rw, the formation water resistivity. If some experimental data regarding the resistivities of water as a function of dissolved salts is available, this enables us to determine the salinity of formation water and thus the chemical quality of water in granular aquifers.

7·3·2 Total Porosity : This can be calculated from readings made on a gamma-gamma log. This log measures formation density in bulk, which is a simple weighted average of the densities of the rock matrix and the pore fluid. The porosity can be determined as

$$\theta = \frac{P_r - P_b}{P_f} \qquad (7\cdot5)$$

where θ is total porosity, P_r is density of the rock matrix, P^f is the density of the fluid and P_b is the measured bulk density.

On a qualitative basis a device measuring natural gamma radiation can distinguish units having a high clay content, which causes a low effective porosity relative to the total porosity.

7·3·3 Hydraulic conductivity :

It cannot be measured directly by logging. However, empirical relations involving hydraulic conductivity, porosity and water saturation have been developed and could be used in certain cases. Hydraulic conductivity is related to grain size as well as porosity it has been demonstrated by Alger (1966) that the formation factor in fresh water sands increases as the grain size increases. If premeability could be estimated, its multiplication with the thickness of the aquifer would yield the transmissivity.

7·3·4 Specific Yield of Unconfined Aquifers :

Neutron logs, especially when properly calibrated in terms of moisture content, helps to determine Specific Yield of unconfined aquifers.

7·4 Inverse Techniques :

7·4·1 Introduction :

The use of simulative flow models of ground water basins has become an established management tool. The aquifer models which may represent flow of water, chemicals or heat through an aquifer are widely used both to design real world water management systems as well as to gain better understanding of the prototype itself. A large class of ground water models, however, involve differential equations embedded with unknown parameters which must be known; a priori before these can be used for system management. This part of model building known as calibration has not yet been reduced to a routine task. Considerable amount of highly qualified hydrologic and geologic attention is currently required to make a mathematical model plausible.

One of the most important source of uncertainty in model preparation is the unavoidable lack of definitive measurements of the two principal geohydrologic parameters; storage capacity and permeability to flow. The aggregated and *in situ* character of these parameters makes laboratory measurement of little use. The current and classical methods of field testing such as pumping, dye-tracer and other tests are of limited use in providing values that can be directly used or reliably extrapolated to the large scale phenomena, i. e. basin models, as these give point informations; are costly if conducted in large numbers and the assumptions implicit in these tests may not hold in the regional models. In real life, rather large deviations from the theoretical conditions and assumptions inherent in the classical methods may occur and any deviation from the theoretical conditions will lead to an error in the computations. It is, therefore, imperative to evolve better practical methods for estimating geohydrologic parameters in order to use the aquifer models as effective management and decision tools. The Inverse Method is a step forward in this direction.

7·4·2 The Inverse Problem :

As has been mentioned earlier, application of ground water system models to real-life management issues demands precise information regarding distribution of system parameters. The set of methodologies used to determine these unknown parameters of the model from appropriate experimental and a field data is usually known as system identification or Inverse Problem. Abundant literature is presently available on system identification, particularly when the model is expressed in terms of ordinary differential equations. When the model is given in terms of partial differential equations, which is the usual case in sub-surface hydrology, the problem becomes one of identification of distributed systems. Unfortunately, this problem has been explored less than the previous one.

The general equation of three dimentional, non-steady ground water flow under stressed condition is given by

$$\frac{\partial}{\partial x}\left[T(x,y,z)\frac{\partial \varnothing}{\partial x}\right]+\frac{\partial}{\partial y}\left[T(x,y,z)\frac{\partial \varnothing}{\partial y}\right]$$
$$+\frac{\partial}{\partial z}\left[T(x,y,z)\frac{\partial \varnothing}{\partial z}\right]-S(x,y,z)\frac{\partial \varnothing}{\partial t}+W(x,y,z,t) \quad (7\cdot5)$$

where $\varnothing$ is the field variable, T and S are the aquifer parameters (Transmissivity and Storativity). W is the source/sink term and x, y, z, t are the space and time co-ordinates.

The problem of solving for the dependent variable $\varnothing = \varnothing(x, y, z, t)$ from equation (7·5) is referred to as the direct problem. The above equation can be solved analytically for very simple flow and boundary conditions. Such geometries do not normally occur in large scale natural systems and hence analytical solutions are extremely difficult, if not impossible, to obtain. For such situation, the equation is usually, solved by numerical techniques on a digital computer. However, an unique solution to the direct problem even by numerical methods can be obtained if and only if the parameters, source/sink term and initial and boundary conditions are known. As has been mentionede arlier, frequently these quantities are not easily and completely known, which causes indeterminacy in the direct problem. The quantity that is most easily observable in the field is the dependent variable $\varnothing$ itself. Assuming that the observed values of $\varnothing$ are a solution of equation (7·5), the problem of estimating the parameters, the source/sink term and the initial and boundary conditions is known as the Inverse Problem. Thus in Inverse Problem $\varnothing$ the dependent variable in the direct problem now becomes the independent variable and the problem is solved in reverse.

Budhisagar et-al (1973) categorise the Inverse Problem into the following types :—

Inverse Problem Type I	Determination of Parameters.
Inverse Problem Type II	Determination of Initial conditions.
Inverse Problem Type III	Determination of Boundary conditions.
Inverse Problem Type IV	Determination of Source/ Sink or Input/Output.
Inverse Problem Type V.	A mixture of the above.

In the hydrologic literature, much stress has been laid on the estimation of parameters by assuming that the source/sink term and the initial and boundary conditions are known. The objective of the present manual is to determine the aquifer parameters, and hence only the Inverse Problem Type I is considered here. Readers interested in other types also may refer to Lattes and Lions (1969) for Type II, Phillipson (1971) for Type III, Moench and Kisiel (1970) for Type IV and Budhisagar et-al (1973) for type V.

7·4·3 Solution of Inverse Problem :

The Inverse Problem, as defined, is shown schematically in Figure (7·1) and the usual method for solving it is shown in Figure (7·2). The method consists of the following steps :—

1. An approximate value of unknown parameters may be determined by some handy method. However, better the initial approximation, quicker will be the convergence and thus less computing time.
2. Solve the direct problem with this approximate estimate knowing initial and boundary conditions and flux terms.
3. Compare the results obtained in Step (2) with the actual observation.
4. If the two do not correspond within a certain limit as per the criterion function formulated then with the help of a suitable algorithm (adjustment algorithm), change the values of parameters in Step (1).
5. Repeat steps (2) through (4) until satisfactory value of criterion function, expressing the difference between the observed and the computed values, is obtained.

Different adjustment algorithms in step (4) and criterion functions in step (5) have been used by different investigators.

7·4·3·1 Criterion Functions :

If b is a true parameter vector and β the estimated one, the different criteria for optimality can be :

(a) the minimization of some function of (β-b.) As b is unknown, the expectation of this difference can only be minimized if sufficient a prior knowledge is available.

(b) The minimization of some function or functional of $e = \hat{\varnothing}\;\; \hat{\hat{\varnothing}}$ (Fig. 7·2) i.e. the difference between the measured out put (including noise) and the model output. This error can be used because e can be made measurable. $\hat{\varnothing}$ is the observed value of $\varnothing$ and $\hat{\hat{\varnothing}}$ the computed one.

(c) The minimization of some functional containing the measurable process outputs and the estimates of the state vector and the parameter vector.

SCHEMATIC REPRESENTATION OF THE INVERSE PROBLEM

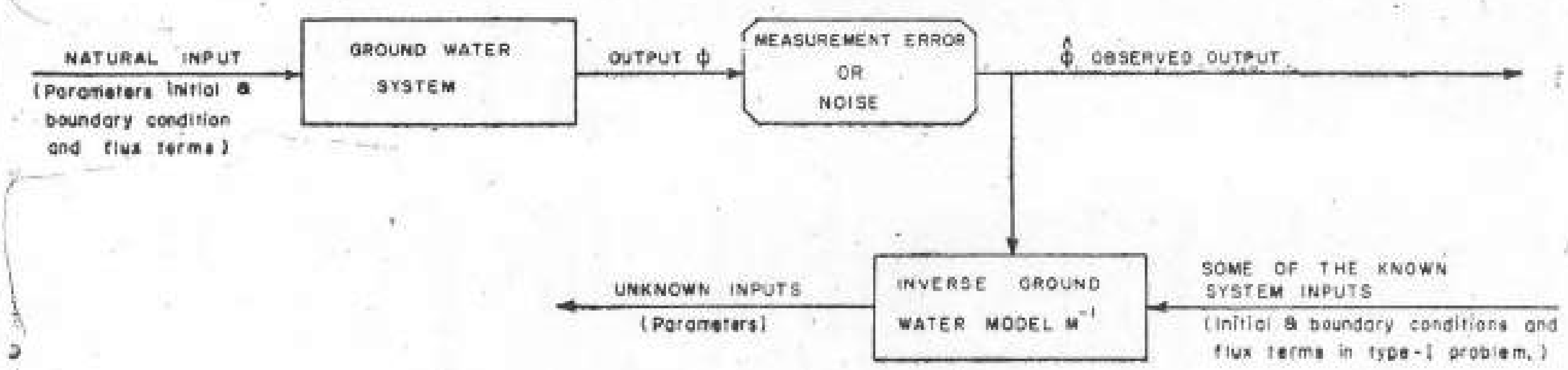

FIG. 7.1

SOLUTION OF INVERSE PROBLEM (SCHEMATIC REPRESENTATION)

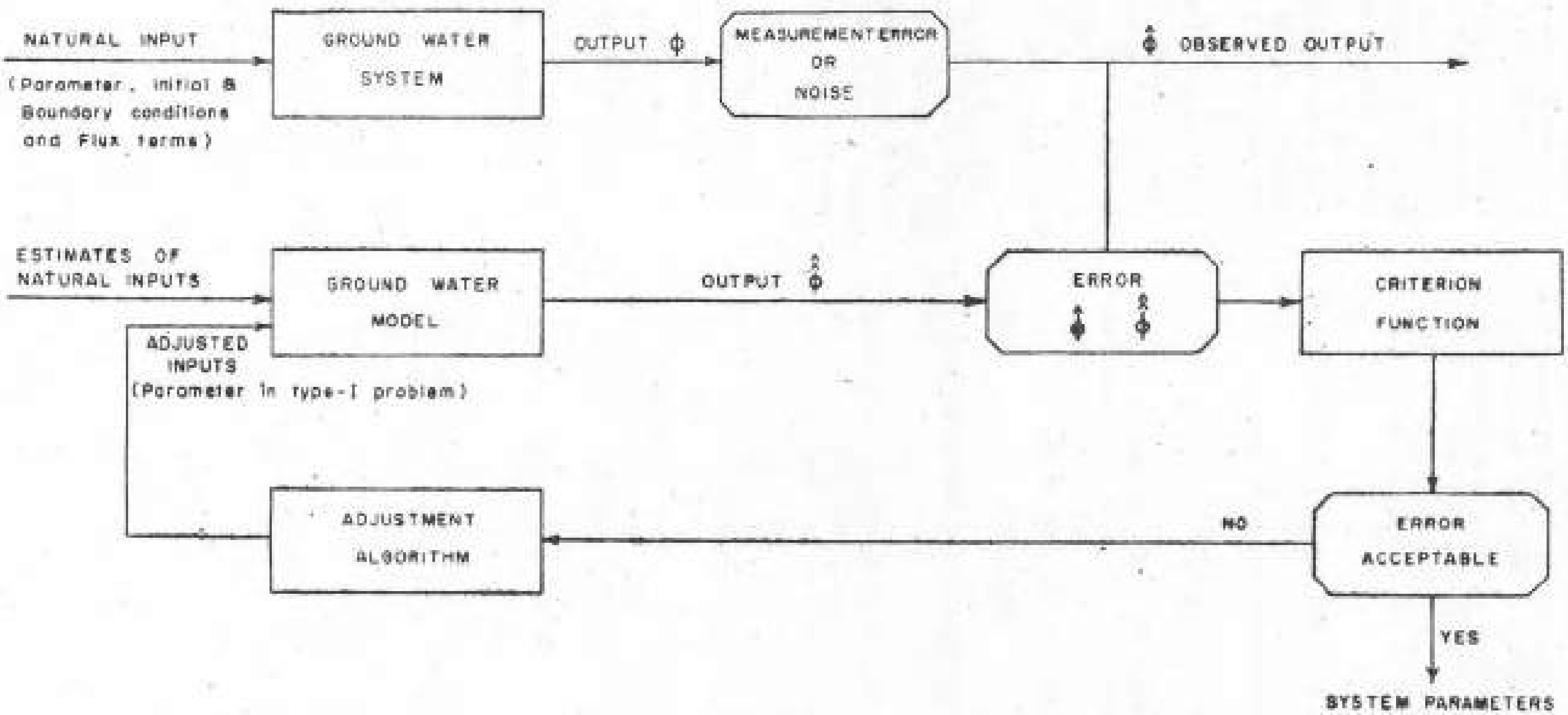

FIG. 7.2

Often, the second criterion is used because the correspondence of input-output relations is considered more important than parameter correspondence. This is specifically true if the main aim of modelling is prediction in the future and not basic understanding of the process itself. Using a least square criterion, the problem then reduces to the variational problem of finding β so that the functional $J(\beta)$ is extremized.

$$J(\beta) = \| \phi(\beta) - \phi(\beta) \| \qquad (7 \cdot 6)$$

Where $\| \quad \|$ denotes the norm of the vector

7·4·3·2 Adjustment Algorithm

In ground water hydrology, the method of gradients has been largely used. It has equation (7·6) as the optimality condition and is based on the algorithm that lets $(i + 1)$ estimate be :—

$$b^{(i+1)} = b^{(i)} - K \nabla b\, J(b^{(i)}) \qquad (7 \cdot 7)$$

Where $\nabla b\, J$ is the gradient vector and K is a suitable constant.

Usually, it is desired that the adjustment algorithm converge fast. If possible, it should also guarantee that the parameter values to be obtained in any iteration be definitely better than those obtained in the previous iteration, i.e. $J(\beta)$ in equation (7·6) should be a continually decreasing function.

The important point in this methodology is that it is required to solve the partial differential equation of flow repeatedly in step (2).

7·4·3·3 Different Algorithms for Inverse Problem Solutions.

Various algorithms have been proposed by different investigators to solve the Inverse Problem. A summary of these techniques with their relative features and assumptions is given in Table 7·1. These methods can be classified as direct and indirect. The basic premise of the direct method is that there is no need to solve the partial differential equation of flow shown in equation (7·5). With a knowledge of the first and second order derivatives of the dependent variable at the points of interest from observed records, the partial differential equation can be reduced to a set of algebraic equations, the solution of which gives the parameters. The indirect methods use optimization techniques for getting the optimal set of parameters. These again are either automatic or are based on trial and error procedure. In automatic techniques, the well known methods of optimization have been used with varying degree of success Table (7·1). The trial and error method is based on subjective consideration A suitable numerical scheme is set up to solve equation (7·5). The unknown parameters, the recharge and discharge terms are assigned values based on best available information and equation (7·5) is solved to obtain ϕ at the grid points. The calculated ϕ is compared to the observed ϕ at the same points. Any of the parameters are now changed subjectively (without any authomated algorithm) to obtain a better fit between ϕ and ϕ at as many grid points as possible. This method is perhaps the simplest and therefore is preferred by field hydrologists who have a considerable knowledge of the aquifer thay are working in. The subjective method is made efficient by sensitivity analysis.

7·4·4 Conclusions.

Although many models have been proposed and used in the last two decades, a general theory for their calibration and validation is yet to be developed. The various optimization techniques put forth require the partial differential equation to be solved repeatedly. This causes a number of difficulties such as :—

(1) Complicated cases anisotropy etc. may be difficult to solve and may require large computers.

(2) When more than one parameter is to be determined, no unique answer is guaranteed.

(3) When solving for parameters, initial and boundary conditions and inputs will have to be exactly known and vice-versa.

In the direct method, spline functions have been suggested to approximate the piezometric surface. The method, however, has the limitations that it can not evaluate time dependent parameters and that one of the parameters (storativity in Budhisagar's work) has to be assigned a value.

Thus, notwithstanding the potentiality of various inverse techniques, the method is still at research stage and its wider and general application in real life situation may take some more time. Till then, the aquifer models would have to be calibrated either by the subjective considerations or in limited cases by the methods given in Table 7·1 where similar situation exists.

TABLE 7·1

Tabulation of known Methods for Solving Inverse Problem

Author	Method	General Assumptions made	Results and Comments
1	2	3	4
Klenecke (1971)	Linear Programming	1. Linear Objective Function. 2. Isotropic Medium with two dimensional flow. 3. T & S independent of time.	1. No guidelines on sub-division of the basin. 2. No way to force all the un-known into the optimum solution.
Yen and Tauxe (1971)	Quasi-linearization	1. Infinite homogenous and isotropic aquifer. 2. Drawdown data on a constant discharge well is required. 3. A good initial estimate of the parameters for the method to converge.	1. Proposed as an alternative to the type curve method but not applicable to leaky aquifers. 2. Not applicable to a regional identification of the parameters. 3. If initial estimates are bad the method may not converge.
Vemuri and Karplus (1969)	Hybrid Computations.	1. Unconfined aquifer. 2. Initial Estimate of the values of the parameters required. 3. Sufficient subjective knowledge of of S and boundaries needed.	1. No guarantee of a global minimum of the objective function. 2. Two of the three parameters are arbitrarily adjusted to obtain a best fit in the T parameter thus unicity not guaranteed. 3. Hybrid computor required.
Vemuri et-al (1969)	Sensitivity analysis.	1. Scalar problem i.e. only lumped estimates are to be obtained.	1. Not applied to the partial differential equation of ground water flow. 2. Lots of computational difficulties.
Ensellem and Marsily (1971)	Automatic Solution of Inverse Problem.	1. Two dimensional isotropic medium. 2. T&S are continuous functions of space. 3. T & S are independent of time.	1. No initial estimate required.
Haimes et-al (1968)	Decomposition & Multi-level Optimization.	1. The total region can be divided into Wedge-shaped homogeneous isotropic portions each enclosing a single production well.	1. Initial estimates of S&T are required.
Gates (1972)	Subjective trial	1. Enough subjective knowledge is available. 2. Valid for confined and un-confined aquifers.	1. Solely based on the investigator's experience and subjective knowledge ; there is no specific algorithm to adjust the parameter values.
Lovell Duckstein and Kiesel (1972)	Subjective optimization.	1. Subjective knowledge can be quantified by defining certain notions.	1. Not posed as an optimization problems. 2. Certain parameters quantiyfying subjective knowledge have to be defined.
Nelson (1968)	Method of energy dissipation.	1. Two dimensional isotropic medium. 2. Storage co-efficient is zero. 3. Permeability values are available at a point on every stream line.	1. It will be hard to treat anisotropic aquifers. 2. Only applicable to steady state flows.

1	2	3	4
Budhisagar, Kiesel and Duckstein (1973)	Direct Algebraic Method using Spline functions.	1. No change in inputs and parameters with time. 2. Linear flow. 3. Data at all points are observed simultaneously. 4. Enough data available for estimation by splines.	1. Applicable to non-homogenous anisotropic medium. 2. Explicit information of initial and boundary conditions not required. 3. It requires at least one of the parameters.

7·5 Tracer Techniques :

7·5·1 Introduction :

The tracer technique is direct and independent method for the determination of aquifer parameters. The method is based on the assumption that the aquifer is homogeneous and isotropic, flow is horizontal and concentration of tracer is kept homogeneous during measure events. Artificial radio isotopes can be measured in extremely low concentrations and often *in situ*, making possible the design of convenient and efficient field experiments. These radioactive tracers are replacing the conventional non-radio active tracers such as salts or dyes etc. However, radioisotopes may present the problem of health hazard. Therefore, before considering the use of an artificial radioactive tracer, it must be ensured that health hazard is non-existent and other non- radioactive tracers would not meet the needs of the problem.

The choice of the artificial radioactive tracer to be used depends on the features of the problem. Some of the most frequently used radioactive tracers are : $^{3}H(T)$, ^{24}Na, ^{51}Cr, ^{58}Co, ^{82}Br, ^{110}Ag, ^{131}I, ^{198}Au

The introduction of tracer into a borehole may be done by pouring through a thin pipe, by crushing an ampoule at the depth of interest, or by using a special injection device.

Some of the important local characteristics of the aquifers which may be determined by using tracer techniques are briefly discussed in the following sections.

7·5·2 Effective Porosity of an Aquifer :

The principle of the method of porosity determination in a saturated zone is based on the approximate equality of porosity θ (Void Volume/Total Volume) and partial volume of water, f = volume of water/total volume.

Tracer is introduced into a well and a second well, at a distance r, is pumped. Disregarding the dispersion of tracer on its path between wells, its arrival at the second well signifies that the volume of water pumped, V, is

$$V = \pi r^2 b/f \qquad (7·8)$$

where,

V = Volume of water pumped (L^3)
r = Distance between wells (L)
b = Aquifer thickness (L)
f = Effective porosity (dimensionless)

Prime requirements are that the distance between wells be large compared to the aquifer thickness ($r > b$), radial pumping velocities be larger than the natural velocities, and the cone of depression at the pumping well be small compared with the volume of water pumped.

7·5·3 Transmissivity :

Mercado and Halevy (1966) have found that the volume of water pumped referred in section 7·5·2, measured until the tracer peak arrives at the next well, is inversely proportional to the value of transmissivity. They have determined transmissivity of two layers of the same aquifer by means of two injection and one observation well. Transmissivity then was found from the relationships :

$$V_1 = \pi r_1^2 b_1 f_1 T/T_1 \qquad (7·9)$$
$$V_2 = \pi r_2^2 b_2 f_2 T/T_2 \qquad (7·10)$$
$$T_1 + T_2 = T \qquad (7·11)$$

where,

T = total transmissivity ($L^2 T^{-1}$)
$T_{1,2}$ = partial transmissivity of the layers $_1$ and $_2$ ($L^2 T^{-1}$)
$r_{1,2}$ = distances between injection and observation wells (L)
$b_{1,2}$ = thickness of layers $_1$ and $_2$ (L)
$f_{1,2}$ = effective porosity of layers $_1$ and $_2$ (dimensionless)

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8. EVALUATION OF THE EXISTING METHODS

8·1 General

Techniques of aquifer parameter evaluation broadly fall into the following distinct groups :

(i) Laboratory methods

(ii) *Insitu* determinations through field pumping tests

(iii) Analysis of Tracer transport

(iv) Analysis of water level changes

(v) Borehole geophysical techniques

Each of these broad groups has been treated and discussed earlier in this monograph. Present chapter is intended to evaluate the applicability and limitations of these techniques from practical view point. Areas of future research needs have also been identified.

8·2 Laboratory methods

Application of laboratory techniques in aquifer parameter evaluation is severelly limited in view of the fact that laboratory can hardly simulate the actual field conditions of natural geostatic pressures and other factors which are not directly determinable. However, in areas where pumping tests data are not available, laboratory analysis of aquifer samples of porous formations may suggest some values of a preliminary nature for a reconnaissance level work. To enhance the utility of laboratory methods further studies on following lines need to be carried out for aquifer samples from India.

(i) documentation of field determined and laboratory estimated values of aquifer parameters for Indian aquifers ;

(ii) correlation of size and shape analysis data for different aquifer samples in India with field determined aquifer parameters and development of regression equations.

8·3 Pumping tests

In the present state of knowledge field pumping tests are most reliable for aquifer parameter evaluation. Most of the analytical techniques developed for aquifer parameter evaluation have been developed for porous media flow. Application of these techniques of pumping tests analysis for fractured media flow has restricted applicability since fractured rocks are, characterised by high degree of heterogenity and anisotropy. Fractures, jointings and solution characteristics may result in development of secondary porosity—permeability features such as infinite permeability in one direction and absence of permeability in other directions which would largely distort the cone of influence around the pumped well. Additionally, the fracture features may have infillings of porous materials and the ground water flow will be some combination of porous and fractured media flow properties. Thus various combinations of fracture location, fracture width, fracture content and volume of ground water in storage in the vicinity of the well can result in a time drawdown curve not amenable to analysis by the currently available analytical techniques. Moreover, the observation well associated with the pumping well in fractured media may tap an entirely different fracture system than the one tapped in the pumped well, rendering observation well data of little consequence. Often pumping tests in fractured formations where fracturing and weathering has resulted in uniform development of secondary porosity—permeability system, pumping test data analysis by available analytical techniques may yield meaningful results.

Several hundreds of pumping tests carried out in fractured rocks in India have brought out the prevalence of aquifer conditions such as, confined aquifer, leaky confined aquifer with release of water from storage in aquitards, unconfined aquifer showing delayed yield, and aquifer of limited areal extent.

In fractured formations which constitute large part of the country dug wells and dug-cum-bored wells constituted a large and most important source of supply. Evaluation of hydrological characteristics of fractured formations assumes great significance in evaluation of ground water resources potential of such formations. Most methods of analysis of aquifer tests of large diameter wells have serious theoretical and practical short comings. Recent work (Karanjac, 1975 Ground Water 13 (4) has brought to light a semi-empirical technique what has been called "Optimum Yield" estimation of large diameter dug wells. In an area where numerous dug wells exists, a comparative study of "Optimum Yields" of wells may suggest areas of favourable hydro-geological conditions (higher T and S values) for accumulation and development of shallow ground water. The method involves pumping a well at uniform rate Q_p, for a brief period of time (t_p) and measuring the time for complete recuperation (t_r) after shutting down.

The optimum yield (Q_R) which is an average inflow rate from the aquifer during the time of well recovery is given by :

$$Q = Q \frac{t_p}{t_p + tr}$$

Higher optimum yield is suggestive of higher permeability. A plot of variations of Q_R in the area will delineate high permeable area suitable for well field development.

8·4 Analysis of Tracer Transport

Tracer techniques (both dye tracers and Radioactive) are helpful tools for aquifer parameter evaluations. Tracers techniques must be used in conjunctions with hydrochemical and hydrogeological data. Independently, the results may not always be dependable because of various interfering causes.

Radio active tracers require specialised equipments and require severe precautions to be taken. However, they give a deep insight into the hydraulic properties of the aquifer materials.

8·5 Analysis of water Level Changes

It is possible to estimate the hydraulic properties of the aquifers from the analysis of changes in water levels. Aquifer diffusivity, aquifer storativity, transmissivity, barometric and tidal efficiencies may be computed from the relationships between changes of surface water stages and the resultant ground water stages. The results obtained are quite dependable and costs involved are ordinarily less. However, this requires careful monitoring of water levels.

8·6 Borehole Geophysical Technique

The different aquifer characteristic viz., effective porosity, total porosity, hydraulic conductivity and specific yield can approximately be estimated with the help of geophysical techniques.

It will be advantageous to develop integrated well logging equipment to log shallow boreholes. Such a device could incorporate the SP log, the short normal resistivity log, gamma log, gamma-gamma log, neutron log, temperature and sonic log. There is need to develop such multipurpose equipment from the point of view of economy and different conditions existing in water wells. The equipment so developed will have to be smaller and lighter than that used in logging oil wells.

Development of interpretive techniques of the various kinds of logs in accordance with the prevailing geologic and hydrologic conditions will have to be taken up. There is some information for wells drilled in sedimentary regions but in hard rock areas, new techniques have to be developed. In some cases it may become necessary to develop and fabricate different equipments for hard rock areas.

9. PRESENTATION OF REPORT

The report for presenting results of pumping tests should have the following paras :

9·1 Introduction :

This should deal with the objectives of the test, location, and other introductory remarks.

9·2 Topography and Hydrogeology of the area :

Under this should be described the general topographic features of the test area, drainage system, climatic factors and other physio-graphical features, etc. It should also give in detail the geology of the area and the general hydrogeological setting.

9·3 Lay-out for the Tests :

Here the detailed lay-out of test wells and observation wells should be given. The construction of these wells and nature of assembly lowered, etc. should also be described. It should also include details about the design of the well including grading and thickness of the gravel packs, pack aquifer ratio, size of the slot, open area of screen, etc.

9·4 Observations made during the Tests :

Under this para a detailed account should be given regarding the manner in which tests were carried out and significant observations made and recorded. The observation recorded for time and draw-down, etc. should be given in the tabular form.

9·5 Analysis of Data :

Test data should be analysed for the following conditions :

9·5·1 Theis type curve method, Jacob straight line method and Recovery head method assuming "Non-leaky isotropic artesian aquifer with fully penetrating wells and constant discharge conditions."

9·5·2 Hantush method assuming "leaky artesian aquifer with fully penetrating wells without water released from storage in aquitard and constant discharge conditions".

9·5·3 Boulton method assuming "Water-table aquifer with fully/partially penetrating wells and constant discharge conditions"

9·5·4 Any other method.

9·6 Discussions of Results :

Results obtained by various methods should be tabulated and discussed in detailed for deciding the most representatives value for S & T for the aquifer in the test area.

9·7 Conclusions :

Under this para important conclusions derived from the tests and recommendations for future works should be given.

9·8 References

9·9 Maps and Charts

Under this the following should be included :

9·9·1 Index map showing the location of the pump test sites and a map showing lay-out of test wells and observation wells

9·9·2 Strata chart giving graphical representations of the formation

9·9·3 A chart showing the assembly of the pump well and observation wells

9·9·4 A table and a graph giving results of mechanical analysis of aquifers samples

9·9·5 Chart showing calculations of drawdown from the recovery data plot

9·9·6 Time vs drawdown curves

9·9·7 Chart indicating the results of step drawdown test

9·9·8 Specific capacity vs. drawdown curve

9·9·9 Specific drawdown vs. discharge

RECOMMENDED SYMBOLS, UNITS AND OTHER USEFUL TABLES IN DATA REPORTING OR REPORT WRITING

A. Linear

Units Recommended	*in place of*
1. Millimetre (mm) 2. Centimetre (cm)	Inch
3. Metre (m)	Foot and Yard
4. Kilometre (km)	Mile and Furlong

B. Area

1. Sq. Centimetre (sq. cm. or cm^2)	Sq. Inch
2. Sq. Metre (sq. m or m^2)	Sq. Foot
3. Sq. Kilometre (sq. km or km^2)	Sq. Mile
4. Hectare (ha.)	Acre

C. Volume

1. Litre (l)	Gallon (Ukgal or USgal)
2. Cubic centimetre (cm^3)	Cubic Inch
3. Cubic metre (m^3)	Cubic Foot or Cubic Yard
4. Million cubic metre (MCM)	Million cubic foot
5. Thousand Million cubic metre (TMCM)	Thousand Million cubic feet
6. Hectare-metre (ha-m)	Acre Foot

D. Rate of Flow

1. Litre per second (lps)	Gallon per minute (GPM) or
Litre per minute (lpm)	Gallon per hour (GPH)
Cubic metre per hour (m^3/hr)	
2. Cubic metre per second (cumec)	Cubic feet per second (cusec)
3. Metre per second (m/s)	Foot per second
4. Hectare metre per day (ham/d)	Acre foot per day
5. Cubic kilometre per day (km^3/day)	Cubic mile/day

E. Aquifer Parameter Units

1. Transmissivity, metre square per day (m^2/day)	USGPD per foot (USGPD/ft) or IGPD per foot (IGPD/ft.)
2. Hydraulic conductivity, metre per day (m/day)	USGPD/ft^2 or IGPD/ft^2
3. Specific Capacity, Litre per metre of drawdown (l/m/dd) (mention time 1 day; 1 h)	Gallon per foot of drawdown

F. Other Parameter Units

1. Millimetre (mm)	Inch
Millimetre/hour (mm/h)	Inch/hour
2. Temperature Celsius (°C)	Fahrenheit (°F)
3. Atmospheric Pressure Millibar (mb) or Millimetre of Mercury (mmHg)	Inch of Hg

(1 cm of mercury column at 0°C is equal to a pressure of 1333·3 dyne/cm² or 1·33 mb; 1 mb is equal to 1000 dyne/cm²)

4. Wind Velocity	
Kilometre per hour (kmph)	Mile per hour (mph)
5. Evaporation/Evapotranspiration Millimetre per day (mm/day)	Inch/Day
6. Litre per second (lps)	Gallon per minute or Gallon per hour
7. Duration of Pumping	
Number of Minute or Hour or Day (Log cycle expression recommended) (First, Second, Third or Fourth log cycle)	Hour/Day
8. Drilling Rate	
Minute per metre of drilling (min/m)	Minutes per foot of drilling
9. *Air Pressure*	
Kilogram per sq. centimetre (kg/cm^2)	Pound per sq. Inch (psi)
10. Weight	
Milligram (mg) Gram (gm) Kilogram (kg)	Ounce/pound or grain

Table 10.1

UNITS AND CONVERSION FACTORS

1 inch = 2·54 centimetres

1 foot = 12 inches
= 30·48 centimetres

1 mile = 5,280 feet
= 1·609 kilometres

1 acre = 4,840 sq. yards

1 sq. mile = 640 acres
=259 hectares

1 sq. kilometre = 100 hectares
= 247·1 acres
= 0·386 square mile

1 Cubic foot =0·028 cubic metre
= 6·229 gallons
=28·316 litres

1 million cubic feet = 11·574 cusec day
= 22.957 acre feet
= 6·229 UK million gallons
= 28,316·8 cubic metres

1 thousand million cubic feet = 22,956·84 acre-feet
= 385·8 cusec for one month
= 31·71 cusec for one year
= 6,228·8 million gallons
= 28·317 million cubic metres
= 2,832 thousand hectare metres
= 14 cm depth over 20,000 ha

1 million acre-feet = 43·56 thousand cubic feet

1 cusec-day
=0·0864 million cubic feet
=1·9834 acre-feet
= 2,446·57 cubic metres
= 0·538 million gallons

1 cusec-month (30 days)
= 59·50 acre-feet
= 2·592 million cubic feet
= 73,397·19 cubic metres

1 cusec-year (365 days)
= 723·97 acre-feet
= 31·54 million cubic feet
= 0·89 million cubic metres

1. LINEAR

10 millimetres = 1 centimetre
= 0·394 inch

1 metre =100 centimetres
= 3·281 feet

1 kilometre = 1,000 metres
= 0·621 mile

2. AREA

1 hectare = 100 metres × 100 metres
= 10,000 sq. metres
= 2·471 acres

3. VOLUME

1 cubic metre = 1,000 litres
= 35·315 cubic feet
= 219·969 UK gallons

1 million cubic metre = 10^6 cubic metre
= 100 hectare metre
= 35·31 million cubic feet
= 810·71 acre feet
= 219·969 million gallons

1 cubic kilometre = 10^6 cubic metre
= 1 milliard cubic metres
= 1 million cubic metres

1 hectare metre = 10,000 cubic metres
= 0·353 million cubic feet
= 8·10 acre-feet

1 cubic metre per second for one day
= 0·0864 million cubic metres
=8·64 hectare metres
= 70·044 acre feet
= 3·051 million cubic feet

1 cusec-month (30 days)
= 2,101·36 acre feet
= 91·54 million cubic feet

1 cumec year/cubic metre per second for one year (365 days)
= 31·54 million cubic metres
= 25,569·79 acre-feet
= 1·11 thousand million cubic feet

1 gallon=4·55 litres
1 litre= 0·22 gallon

4. RATES OF FLOW

1 cubic foot per second
= 0·028 cubic metre per second
28·317 litres per second
= 6·229 gallons per second
= 22,423·68 gallons per hour
= 0·54 million gallons per day

1 acre-foot per day
= 0·504 cusec
= 0·014 cubic metre per second
= 14·16 litres per second

10,000 gallons per hour
= 0·45 cusec
= 0·88 acre-foot per day

1 million gallons per day
= 1·86 cusecs
= 3·69 acre-feet per day

1 cubic metre per second
= 35·315 cusec
= 219·969 gallons epr second
= 70·05 acre-feet per day

1 cubic klometre per day
= 0·409 million cusec
= 0·811million acre-feet per day

1 Hectare metre/day
= 0·116 cumec
= 4·087 cusecs

1 litre per second
= 0·0353cusec
= 791·90 gallons per hour

1 litre per second per day
= 86·4 cubic metres

Note :—Gallon is Imperial Gallon
1 Imperial Gallon = 1·20 US Gallon
Milliard = 1,000 million = Billion

Source : ISI Publication No. IS:786-1967, "Conversion Factors and Conversion Tables"—1968—New Delhi.

Table 10.2
DISCHARGE RATE

	l/sec	m^3/day	m^3/sec	Imp. gal/day	U.S. gal/day	ft^3/day
1 l/sec	1·000	86·40	$1·000 \times 10^{-3}$	$1·901 \times 10^{4}$	$2·282 \times 10^{4}$	$3·051 \times 10^{3}$
1 m^3/h	0·2777	24·00	$2·777 \times 10^{-4}$	$5·279 \times 10^{3}$	$6·340 \times 10^{3}$	$8·476 \times 10^{2}$
1 m^3/day	$1·157 \times 10^{2}$	1·000	$1·157 \times 10^{-5}$	$2·200 \times 10^{2}$	$2·642 \times 10^{2}$	35·32
1 m^3/sec	$1·000 \times 10^{3}$	$8·640 \times 10^{4}$	1·000	$1·901 \times 10^{7}$	$2·282 \times 10^{7}$	$3·051 \times 10^{6}$
1 Imp. gal./day	$5·262 \times 10^{-5}$	$4·546 \times 10^{-3}$	$5·262 \times 10^{-8}$	1·000	1·201	0·1605
1 U.S. gal./day	$4·381 \times 10^{-5}$	$3·785 \times 10^{-3}$	$4·381 \times 10^{-8}$	0·8327	1·000	0·1337
1 Ft^3/day	0·3277	$2·832 \times 10^{-2}$	$3·277 \times 10^{-7}$	6·229	7·481	1·000

HYDRAULIC CONDUCTIVITY

	m/day	m/sec	Cm/h	Imp. gal/day-ft^2	U.S. gal/day-ft^2	Imp. gal/min.ft^2	U.S. gal/min.ft^2
1 m/day	1·000	$1·157 \times 10^{-5}$	4·167	20·44	24·54	$1·419 \times 10^{-2}$	$1·704 \times 10^{-2}$
1 m/sec	8.640×10^{4}	1,000	$3·600 \times 10^{-5}$	$1·766 \times 10^{-6}$	$2·121 \times 10^{6}$	$1·226 \times 10^{3}$	$1·472 \times 10^{3}$
1 cm/h	0·2400	$2·777 \times 10^{-6}$	1·000	4·905	5·890	$3·406 \times 10^{-3}$	$4·089 \times 10^{3}$
1 Imp. gal/ 1 day-ft^2	$4·893 \times 10^{-2}$	$5·663 \times 10^{-7}$	0·2039	1·000	1·201	$6·944 \times 10^{-4}$	$8·339 \times 10^{-4}$
1 U.S. gal/ 1 day-ft^2	$4·075 \times 10^{-2}$	$4·716 \times 10^{-7}$	0·1698	0·8327	1·000	$5·783 \times 10^{-4}$	$6·944 \times 10^{-4}$
1 Imp. gal/ 1 1min-ft^2	70·46	$8·155 \times 10^{-2}$	$2·936 \times 10^{2}$	$1·440 \times 10^{3}$	$1·729 \times 10^{3}$	1·000	1·201
1 U.S. gal/ 1 min-ft^2	58·67	$6·791 \times 10^{-3}$	$2·445 \times 10^{2}$	$1·195 \times 10^{3}$	$1·440 \times 10^{3}$	0·8326	1·000

TRANSMISSIVITY

	m^2/day	m^2/sec	Imp. gal/day-ft.	U.S. gal/day-ft	Imp. gal/min-ft	U.S. gal/min-ft.
1 m^2/day	1·000	$1{\cdot}157 \times 10^{-5}$	67·05	80·52	$4{\cdot}656 \times 10^{-2}$	$5{\cdot}592 \times 10^{-2}$
1 m^2/sec	$8{\cdot}64 \times 10^{4}$	1·000	$5{\cdot}793 \times 10^{6}$	$6{\cdot}957 \times 10^{6}$	$4{\cdot}023 \times 10^{3}$	$4{\cdot}831 \times 10^{3}$
1 Imp. gal/ day-ft	$1{\cdot}491 \times 10^{-2}$	$1{\cdot}726 \times 10^{-7}$	1·000	1·201	$6{\cdot}944 \times 10^{-4}$	$8{\cdot}339 \times 10^{-4}$
1 U.S. gal/ day-ft	$1{\cdot}242 \times 10^{-2}$	$1{\cdot}437 \times 10^{-7}$	0·8326	1·000	$5{\cdot}783 \times 10^{-4}$	$6{\cdot}944 \times 10^{-4}$
1 Imp. gal/ min-ft	21·48	$2{\cdot}486 \times 10^{-4}$	$1{\cdot}440 \times 10^{3}$	$1{\cdot}729 \times 10^{3}$	1·000	1·201
1 U.S. gal/ min-ft	17·88	$2{\cdot}070 \times 10^{-4}$	$1{\cdot}199 \times 10^{3}$	$1{\cdot}440 \times 10^{3}$	0·8326	1·000

Abbreviations : ft=foot; in=inch; l=liter; Imp. gal=Imperial gallon; U.S. gal=U.S. gallon.
Source : Krusenan, G.P. and de Ridder, N.A. (1970), "Analysis and evaluation of pumping test data"—ILRI, Wageningen, The Netherlands—P : 184.

Table 10.3

COMMON MAP SCALES

Scale of Map	Reduced Factor (R.F.)	Approximate metric ratio	Area on the Map for one square inch (in acres)
1" = 16 miles	1 : 10,13,760	1 : 1000,000 (1 cm = 10 km)	163,840
1" = 4 miles	1 : 2,53,440	1 : 250,000 (1 cm = 2·5 km)	10,240
1" = 2 miles	1 : 1,26,720	1 : 125,000 (1 cm = 1·25 km)	2,560
1" = 1 mile 1" = 8 furlongs 1" = 5280'	1 : 63,360	1 : 50,000 (1 cm = 500 m)	640
1" = 4 furlongs or 2640'	1 : 31,680	1 : 25,000 (1 cm = 250 m)	160
1" = 2 furlongs or 1320'	1 : 15,840	1 : 15,000 (1 cm = 150 m)	40
1" = 1 furlong or 660'	1 : 7,920	1 : 10,000 (1 cm = 100 m)	10
1" = 330' or 1" = 1/16 mile 1" = 1/2 furlong	1 : 3,960	1 : 5,000 (1 cm = 50 m)	2·5

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ORIFICE TABLES

For measurement of water through pipe orifice with free discharge.

Head/orifice in incues.

Discharge in U.S.G.P.M.

1 inch=25·4 mm

1 USGPM=3·785 L.P.M.

=5·45 m³ /day.

Head in Inches	3" Orifice		4" Orifice		5" Orifice		6" Orifice		7" Orifice	8" Orifice	Head in inches
	4 in. pipe	6 in. pipe	6 in. pipe	8 in. pipe	6 in. pipe	8 in. pipe	8 in. pipe	10 in. pipe	10 in. pipe	10 in. pipe	
1	2	3	4	5	6	7	8	9	10	11	12
5	100	76	145	140	280	220	380	320	—	—	5
5·5	104	79	153	145	293	230	394	333	—	—	5·5
6	108	82	160	150	305	240	408	345	—	—	6
6·5	111	85	167	155	316	250	421	358	—	—	6·5
7	115	88	172	160	328	260	433	370	—	—	7
7·5	119	91	179	165	339	270	446	383	—	—	7·5
8	122	94	185	170	350	280	458	395	600	935	8
8·5	125	96	190	175	361	289	471	408	617	963	8·5
9	128	99	195	180	372	298	483	420	633	992	9
9·5	130	102	200	185	383	307	495	433	650	1016	9·5
10	133	104	205	190	393	316	508	445	666	1040	10
10·5	137	107	210	195	402	324	521	458	682	1060	10·5
11	140	109	215	200	412	330	533	470	698	1080	11
11·5	143	111	220	204	421	338	545	480	713	1100	11·5
12	146	114	225	208	430	346	556	490	728	1120	12
12·5	149	116	230	212	439	354	567	500	743	1139	12·5
13	151	118	234	216	448	362	578	510	757	1158	13
13·5	154	121	239	219	457	369	589	520	771	1176	13·5
14	157	123	243	224	465	376	599	530	785	1194	14
14·5	159	126	247	227	473	383	609	540	799	1212	14·5
15	162	128	250	231	480	390	618	550	812	1230	15
15·5	164	130	254	234	488	396	627	459	825	1248	15·5
16	167	132	257	238	495	402	636	568	838	1266	16
16·5	170	134	261	241	503	408	645	577	851	1284	16·5
17	172	136	264	245	510	414	654	586	863	1302	17
17·5	175	138	268	249	517	420	663	595	875	1319	17·5
18	178	140	271	252	524	426	672	604	887	1336	18
18·5	180	142	275	256	530	432	681	612	899	1353	18·5
19	183	144	278	259	536	438	690	620	910	1370	19
19·5	185	146	282	263	542	444	699	628	922	1387	19·5
20	187	148	285	266	548	449	708	636	933	1404	20
20·5	190	150	289	270	554	455	717	643	945	1421	20·5
21	192	152	292	273	560	460	726	650	956	1438	21
21·5	195	154	295	275	566	465	735	657	968	1455	21·5
22	197	156	299	279	572	470	744	664	979	1471	22
22·5	199	158	302	282	578	475	752	671	990	1486	22·5
23	201	160	305	285	584	479	760	678	1001	1500	23
23·5	203	162	307	288	590	484	768	685	1012	1515	23. 5'
24	205	164	310	291	596	488	776	692	1022	1529	24
24·5	207	165	314	294	602	492	784	699	1033	1543	24·5
25	210	167	317	297	608	496	791	706	1043	1557	25
25·5	212	169	320	300	614	500	798	713	1059	1571	25·5
26	214	171	323	303	620	504	805	720	1064	1585	26
26·5	216	173	326	305	626	508	812	727	1074	1599	26·5
27	219	174	329	308	632	512	818	734	1084	1613	27
27·5	221	176	332	311	638	516	825	741	1094	1627	27·5
28	222	177	335	314	644	520	831	747	1104	1641	28

1	2	3	4	5	6	7	8	9	10	11	12
28·5	224	179	337	317	650	524	838	754	1114	1655	28·5
29	226	180	340	320	656	528	844	760	1124	1669	29
29·5	228	182	343	323	662	532	851	767	1134	1683	29·5
30	230	183	346	325	668	[illegible]	857	773	1143	1697	30
30·5	232	185	348	328	674	540	763	780	1153	1711	30·5
31	235	186	351	330	680	544	869	786	1162	1735	31
31·5	236	188	354	333	686	548	876	793	1172	1739	31·5
32	239	189	357	335	692	552	882	799	1181	1753	32
32·5	240	191	360	338	697	556	889	806	1191	1767	32·5
33	242	192	363	340	703	560	895	812	1200	1791	33
33·5	244	194	366	342	709	564	901	818	1209	1795	33·5
34	246	195	369	345	715	568	907	824	1218	1809	34
34·5	248	196	372	347	720	572	913	830	1227	1823	34·5
35	250	197	375	349	726	576	919	836	1235	1837	35
35·5	252	198	377	351	732	580	925	842	1243	1851	35·5
36	254	200	380	354	737	584	931	847	1251	1865	36
36·5	256	201	383	356	743	588	937	852	1259	1879	36·5
37	257	203	385	358	748	592	943	857	1266	1893	37
37·5	259	204	388	360	754	596	949	862	1274		37·5
38	260	205	390	363	759	600	955	867	1281		38
38·5	262	206	393	365	765	604	961	872	1289		38·5
39	263	208	396	367	770	608	967	877	1295		39
39·5	265	209	398	369	776	612	974	882	1304		39·5
40	266	210	401	371	781	616	979	887	1311		40
40·5	267	211	403	373	786	620	985	891	1319		40·5
41	269	212	406	375	790	624	990	896	1326		41
41·5	271	213	408	378	795	628	996	901	1334		41·5
42	272	214	411	380	800	631	1001	906	1341		42
42·5	274	216	413	382	805	635	1007	910	1349		42·5
43	275	217	415	384	810	638	1012	915	1356		43
43·5	277	218	418	386	815	642	1018	920	1364		43·5
44	278	219	420	388	820	645	1023	925	1371		44
44·5	280	220	422	390	824	649	649	929	1379		44·5
45	281	222	425	392	828	652	1034	934	1387		45
45·5	283	223	427	394	832	656	1040	939	1394		45·5
46	284	224	429	396	837	659	1045	944	1401		46
46·5	285	225	432	399	842	663	1051	948	1409		46·5
47	287	227	434	401	847	666	1056	953	1416		47
47·5	289	228	437	403	851	669	1062	958	1424		47·5
48	290	229	440	405	855	672	1067	936	1431		48
48·5	292	230	442	407	859	676	1073	967	1439		48·5
49	293	231	444	409	863	679	1078	972	1446		49
49·5	294	232	446	411	868	683	1084	977	1454		49·5
50	296	234	448	413	872	686	1089	982	1461		50
50·5	298	235	430	415	876	690	1095	986	1469		50·5
51	300	236	453	417	880	693	1100	991	1476		51
51·5	301	237	455	419	884	697	1105	996	1484		51·5
52	302	238	457	421	888	700	1110	1000	1491		52
52·5	303	239	459	423	892	704	1115	1005	1499		52·5
53	304	240	461	425	896	707	1120	1009	1506		53
53·5	305	241	463	427	900	711	1125	1014	1513		53·5
54	307	243	465	429	904	714	1130	1018	1520		54
54·5	309	244	467	431	908	718	1135	1023	1527		54·5
55	310	246	469	433	912	721	1140	1027	1534		55
55·5	311	247	471	435	915	725	1145	1032	1541		55·5
56	313	248	472	437	919	727	1150	1036	1548		56
56·5	314	249	474	439	923	730	1155	1040	1554		56·5
57	315	250	476	441	927	733	1160	1044	1560		57
57·5	316	251	478	443	930	736	1165	1046	1567		57·5
58	317	252	480	445	934	739	1170	1052	1574		58
58·5	319	253	482	447	938	742	1175	1056	1580		58·5
59	320	254	485	449	942	745	1180	1060	1586		59

1	2	3	4	5	6	7	8	9	10	11	12
59·5	321	256	487	451	945	748	1185	1064	1592		59·5
60	323	257	489	453	948	751	1190	1068	1598		60
60·5	324	258	491	455	951	754	1195	1072			60·5
61	325	259	492	457	955	757	1200	1076			61
61·5	326	261	494	459	958	760	1205	1080			61·5
62	328	262	496	461	961	763	1209	1084			62
62·5	329	263	498	463	964	766	1214	1088			62·5
63	330	264	500	465	968	769	1218	1092			63
63·5	331	265	502	467	971	772	1223	1096			63·5
64	333	266	504	469	974	775	1227	1099			64
64·5	334	267	507	471	977	778	1232	1103			64·5
65	335	268	509	472	981	781	1236	1106			65
65·5	336	269	511	474	984	784	1241	1110			65·5
66	338	271	513	475	988	787	1245	1113			66
66·5	339	272	515	477	991	790	1250	1117			66·5
67	340	273	517	479	995	793	1254	1120			67
67·5	341	274	518	481	998	796	1259	1124			67·5
68	343	275	520	483	1002	799	1263	1127			68
68·5	344	276	521	485	1005	802	1268	1131			68·5
69	346	277	523	487	1009	805	1272	1134			69
69·5	347	278	524	489	1012	808	1276	1137			69·5
70	349	280	535	491	1016	811	1280	1140			70

Table of values of W(u) corresponding to values u and 1/u, after WALTON (1962)

		1/u=	n	n(1)	n(2)	n(3)	n(4)	n(5)	n(6)	n(7)	n(8)	n(9)	n(10)
n	N	u=	N	N(—1)	N(—2)	N(—3)	N(—4)	N(—5)	N(—6)	N(—7)	N(—8)	N(—9)	N(—10)
1·000	1·0	W(u)=	2·194(—1)	1·823	4·038	6·332	8·633	1·094(1)	1·324(1)	1·554(1)	1·784(1)	2·015(1)	2·245(1)
0·833	1·2		1·584(—1)	1·660	3·858	6·149	8·451	1·075(1)	1·306(1)	1·536(1)	1·766(1)	1·996(1)	2·227(1)
0·666	1·5		1·000(—1)	1·465	3·637	5·927	8·228	1·053(1)	1·283(1)	1·514(1)	1·744(1)	1·974(1)	2·204(1)
0·500	2·0		4·890(—2)	1·223	3·355	5·639	7·940	1·024(1)	1·255(1)	1·485(1)	1·715(1)	1·945(1)	2·176(1)
0·400	2·5		2·491(—2)	1·044	3·137	5·417	7·717	1·002(1)	1·232(1)	1·462(1)	1·693(1)	1·923(1)	2·153(1)
0·333	3·0		1·305(—2)	9·057(—1)	2·959	5·235	7·535	9·837	1·214(1)	1·444(1)	1·674(1)	1·905(1)	2·135(1)
0·286	3·5		6·970(—3)	7·942(—1)	2·810	5·081	7·381	9·683	1·199(1)	1·429(1)	1·659(1)	1·889(1)	2·120(1)
0·250	4·0		3·779(—3)	7·024(—1)	2·681	4·948	7·247	9·550	1·185(1)	1·415(1)	1·646(1)	1·876(1)	2·106(1)
0·222	4·5		2·073(—3)	6·253(—1)	2·568	4·831	7·130	9·432	1·173(1)	1·404(1)	1·634(1)	1·864(1)	2·094(1)
0·200	5·0		1·148(—3)	5·598(—1)	2·468	4·726	7·024	9·326	1·163(1)	1·393(1)	1·623(1)	1·854(1)	2·084(1)
0·166	6·0		3·601(—4)	4·544(—1)	2·295	4·545	6·842	9·144	1·145(1)	1·375(1)	1·605(1)	1·835(1)	2·066(1)
0·142	7·0		1·155(—4)	3·738(—1)	2·151	4·392	6·688	8·990	1·129(1)	1·360(1)	1·590(1)	1·820(1)	2·050(1)
0·125	8·0		3·767(—5)	3·106(—1)	2·027	4·259	6·555	8·856	1·116(1)	1·346(1)	1·576(1)	1·807(1)	2·037(1)
0·111	9·0		1·245(—5)	2·602(—1)	1·919	4·142	6·437	8·739	1·104(1)	1·334(1)	1·565(1)	1·795(1)	2·025(1)

Table of corresponding values of u, W(u) and F (u).

u	W(u)	F(u)	u	W(u)	F(u)	u	W(u)	F(u)
5	1·14(—3)	7·34(—2)	9(—2)	1·92	9·13(—1)	9(—4)	6·44	
4	3·78(—3)	8·98(—2)	8(—2)	2·03	9·56(—1)	8(—4)	6·55	
3	1·30(—2)	1·17(—1)	7(—2)	2·15	1·00	7(—4)	6·69	
2	4·89(—2)	1·57(—1)	6(—2)	2·30	1·06	6(—4)	6·84	
1	2·19(—1)	2·50(—1)	5(—2)	2·47	1·13	5(—4)	7·02	
			4(—2)	2·68	1·21	4(—4)	7·25	
9(—)	2·60(—1)	2·76(—1)	3(—2)	2·96	1·33	3(—4)	7·53	
8(—1)	3·11(—1)	3·01(—1)	2(—2)	3·35	1·49	2(—4)	7·94	
7(—1)	3·74(—1)	3·27(—1)	1(—2)	4·04	1·77	1(—4)	8·63	$F(u) = \frac{W(u)}{2{\cdot}30}$
6(—1)	4·54(—1)	3·60(—1)						
5(—1)	5·60(—1)	4·01(—1)	9(—3)	4·14	1·82	9(—5)	8·74	
4(—1)	7·02(—1)	4·55(—1)	8(—3)	4·26	1·87	8(—5)	8·86	
3(—1)	9·06(—1)	5·32(—1)	7(—3)	4·39	1·92	7(—5)	8·99	
2(—1)	1·22	6·47(—1)	6(—3)	4·54	1·99	6(—5)	9·14	
1(—1)	1·82	8·74(—1)	5(—3)	4·73	2·07	5(—5)	9·33	
			4(—3)	4·95	2·16			
			3(—3)	5·23	2·28			
			2(—3)	5·64	2·46			
			1(—3)	6·33	2·75			

Table of functions e^x e^{-x}, K_0 (x) and e^x K_0 (x), after Hantush (1956)

x	e^x	e^{-x}	K_0 (x)	e^x K_0 (x)	x	e^x	e^{-x}	K_0 (x)	e^x K_0 (x)	x	e^x	e^{-x}	K_0 (x)	e^x K_0 (x)
0·010	1·010	0·990	4·721	4·769	0·10	1·105	0·905	2·427	2·682	1·0	2·718	0·368	0·421	1·144
11	1·011	0·989	4·626	4·677	11	1·116	0·896	2·333	2·605	1·1	3·004	0·333	0·366	1·098
12	1·012	0·988	4·539	4·594	12	1·127	0·887	2·248	2·534	1·2	3·320	0·301	0·318	1·057
13	1·013	0·987	4·459	4·517	13	1·139	0·878	2·169	2·471	1·3	3·669	0·272	0·278	1·021
14	1·014	0·986	4·385	4·447	14	1·150	0·869	2·097	2·412	1·4	4·055	0·247	0·244	0·988
15	1·015	0·985	4·316	4·381	15	1·162	0·861	2·030	2·358	1·5	4·482	0·223	0·214	0·958
16	1·016	0·984	4·251	4·320	16	1·173	0·852	1·967	2·309	1·6	4·953	0·202	0·188	0·931
17	1·017	0·983	4·191	4·263	17	1·185	0·844	1·909	2·262	1·7	5·474	0·183	0·165	0·906
18	1·018	0·982	4·134	4·209	18	1·197	0·835	1·854	2·219	1·8	6·050	0·165	0·146	0·883
19	1·019	0·981	4·080	4·158	19	1·209	0·827	1·802	2·179	1·9	6·686	0·150	0·129	0·861
0·020	1·020	0·980	4·028	4·110	0·20	1·221	0·819	1·753	2·141	2·0	7·389	0·135	0·114	0·842
21	1·021	0·979	3·980	4·064	21	1·234	0·811	1·706	2·105	2·1	8·166	0·122	0·101	0·823
22	1·022	0·978	3·933	4·021	22	1·246	0·802	1·662	2·071	2·2	9·025	0·111	8·93(—2)	0·806
23	1·023	0·977	3·889	3·979	23	1·259	0·794	1·620	2·039	2·3	9·974	0·100	7·91(—2)	0·789
24	1·024	0·976	3·846	3·940	24	1·271	0·787	1·580	2·008	2·4	1·102(1)	9·07(—2)	7·02(—2)	0·774
25	1·025	0·975	3·806	3·902	25	1·284	0·779	1·541	1·979	2·5	1·218(1)	8·21(—2)	6·23(—2)	0·760
26	1·026	0·974	3·766	3·866	26	1·297	0·771	1·505	1·952	2·6	1·346(1)	7·43(—2)	5·54(—2)	0·746
27	1·027	0·973	3·729	3·831	27	1·310	0·763	1·470	1·925	2·7	(1·488(1)	6·72(—2)	4·93(—2)	0·733
28	1·028	0·972	3·692	3·797	28	1·323	0·756	1·436	1·900	2·8	1·644(1)	6·08(—2)	4·38(—2)	0·721
29	1·029	0·971	3·657	3·765	29	1·336	0·748	1·404	1·876	2·9	1·817(1)	5·50(—2)	3·90(—2)	0·709
0·030	1·030	0·970	3·623	3·734	0·30	1·350	0·741	1·372	1·853	3·0	2·009(1)	4·98(—2)	3·47(—2)	0·698
31	1·031	0·969	3·591	3·704	31	1·363	0·733	1·342	1·830	3·1	2·220(1)	4·50(—2)	3·10(—2)	0·687
32	1·032	0·968	3·559	3·675	32	1·377	0·726	1·314	1·809	3·2	2·453(1)	4·08(—2)	2·76(—2)	0·677
33	1·034	0·967	3·528	3·647	33	1·391	0·719	1·286	1·788	3·3	2·711(1)	3·69(—2)	2·46(—2)	0·667
34	1·035	0·967	3·499	3·620	34	1·405	0·712	1·259	1·768	3·4	2·996(1)	3·34(—2)	2·20(—2)	0·658
35	1·036	0·966	3·470	3·593	35	1·419	0·705	1·233	1·749	3·5	3·312(1)	3·02(—2)	1·96(—2)	0·649
36	1·037	0·965	3·442	3·568	36	1·433	0·698	1·207	1·731	3·6	3·660(1)	2·73(—2)	1·75(—2)	0·640
37	1·038	0·964	3·414	3·543	37	1·448	0·691	1·183	1·713	3·7	4·045(1)	2·47(—2)	1·56(—2)	0·632
38	1·039	0·963	3·388	3·519	38	1·462	0·684	1·160	1·696	3·8	4·470(1)	2·24(—2)	1·40(—2)	0·624
39	1·040	0·962	3·362	3·495	39	1·477	0·677	1·137	1·679	3·9	4·940(1)	2·02(—2)	1·25(—2)	0·617
0·040	1·041	0·961	3·336	3·473	0·40	1·492	0·670	1·114	1·663	4·0	5·460(1)	1·83(—2)	1·12(—2)	0·609
41	1·042	0·960	3·312	3·450	41	1·507	0·664	1·093	1·647	4·1	6·034(1)	1·00(—2)	1·00(—2)	0·602
42	1·043	0·959	3·288	3·429	42	1·522	0·657	1·072	1·632	4·2	6·669(1)		8·9(—3)	0·595
43	1·044	0·958	3·264	3·408	43	1·537	0·650	1·052	1·617	4·3	7·370(1)		8·0(—3)	0·589
44	1·045	0·957	3·241	3·387	44	1·553	0·644	1·032	1·602	4·4	8·145(1)		7·1(—3)	0·582
45	1·046	0·956	3·219	3·367	45	1·568	0·638	1·013	1·589	4·5	9·002(1)		6·4(—8)	0·576
46	1·047	0·955	3·197	3·348	46	1·584	0·631	0·994	1·575	4·6	9·948(1)		5·7(—3)	0·570
47	1·048	0·954	3·176	3·329	47	1·600	0·625	0·976	1·562	4·7	1·099(2)		5·1(—3)	0·564
48	1·049	0·953	3·155	3·310	48	1·616	0·619	0·958	1·549	4·8	2·215(2)		4·6(—3)	0·559
49	1·050	0·952	3·134	3·292	49	1·632	0·613	0·941	1·536	4·9	1·343(2)		4·1(—3)	0·553
0·050	1·051	0·951	3·114	3·274	0·50	1·649	0·606	0·924	1·524	5·0	1·484(2)		3·7(—3)	0·548
51	1·052	0·950	3·094	3·256	51	1·665	0·600	0·908	1·512					
52	1·053	0·949	3·075	3·239	52	1·682	0·594	0·892	1·501					
53	1·054	0·948	3·056	3·223	53	1·699	0·589	0·877	1·489					
54	1·055	0·947	3·038	3·206	54	1·716	0·583	0·861	1·478					
55	1·056	0·946	3·019	3·190	55	1·733	0·577	0·847	1·467					
56	1·058	0·945	3·001	3·174	56	1·751	0·571	0·832	1·457					
57	1·059	0·945	2·984	3·159	57	1·768	0·565	0·818	1·446					
58	1·060	0·944	2·967	3·144	58	1·786	0·560	0·804	1·436					
59	1·061	0·943	2·950	3·129	59	1·804	0·554	0·791	1·426					

0·060	1·062	0·942	2·933	3·114	0·60	1·822	0·549	0·777	1·417
61	1·063	0·941	2·916	3·100	61	1·840	0·543	0·765	1·407
62	1·064	0·940	2·900	3·086	62	1·859	0·538	0·752	1·398
63	1·065	0·939	2·884	3·072	63	1·878	0·533	0·740	1·389
64	1·066	0·938	2·869	3·058	64	1·896	0·527	0·728	1·380
65	1·067	0·937	2·853	3·045	65	1·915	0·522	0·716	1·371
66	1·068	0·936	2·838	3·032	66	1·935	0·517	0·704	1·363
67	1·069	0·935	2·823	3·019	67	1·954	0·512	0·693	1·354
68	1·070	0·934	2·809	3·006	68	1·974	0·507	0·682	1·346
69	1·071	0·933	2·794	2·994	69	1·994	0·502	0·671	1·338
0·070	1·072	0·932	2·780	2·981	0·70	2·014	0·497	0·660	1·330
71	1·074	0·931	2·766	2·969	71	2·034	0·492	0·650	1·322
72	1·075	0·930	2·752	2·957	72	2·054	0·487	0·640	1·315
73	1·076	0·930	2·738	2·945	73	2·075	0·482	0·630	1·307
74	1·077	0·929	2·725	2·934	74	2·096	0·477	0·620	1·300
75	1·078	0·928	2·711	2·923	75	2·117	0·472	0·611	1·293
76	1·079	0·927	2·698	2·911	76	2·138	0·468	0·601	1·285
77	1·080	0·926	2·685	2·900	77	2·160	0·463	0·592	1·278
78	1·081	0·925	2·673	2·889	78	2·181	0·458	0·583	1·272
79	1·082	0·924	2·660	2·879	79	2·203	0·454	0·574	1·265
0·080	1·083	0·923	2·647	2·868	0·80	2·225	0·449	0·565	1·258
81	1·084	0·922	2·635	2·857	81	2·248	0·445	0·557	1·252
82	1·085	0·921	2·623	2·847	82	2·270	0·440	0·548	1·245
83	1·086	0·920	2·611	2·837	83	2·293	0·436	0·540	1·239
84	1·088	0·919	2·599	2·827	84	2·316	0·432	0·532	1·233
85	1·089	0·918	2·587	2·817	85	2·340	0·427	0·524	1·226
86	1·090	0·918	2·576	2·807	86	2·363	0·423	0·516	1·220
87	1·091	0·917	2·564	2·798	87	2·387	0·419	0·509	1·214
88	1·092	0·916	2·553	2·788	88	2·411	0·415	0·501	1·209
89	1·093	0·915	2·542	2·779	89	2·435	4·111	0·494	1·203
0·090	1·094	0·914	2·531	2·769	0·90	2·460	0·407	0·487	1·197
91	1·095	0·913	2·520	2·760	91	2·484	0·402	0·480	1·192
92	1·096	0·912	2·509	2·751	92	2·509	0·398	0·473	1·186
93	1·097	0·911	2·499	2·742	93	2·534	0·395	0·466	1·181
94	1·099	0·910	2·488	2·733	94	2·560	0·391	0·459	1·175
95	1·100	0·909	2·478	2·725	95	2·586	0·387	0·452	1·170
96	1·101	0·908	2·467	2·716	96	2·612	0·383	0·446	1·165
97	1·102	0·908	2·457	2·707	97	2·638	0·379	0·440	1·159
98	1·103	0·907	2·447	2·699	98	2·664	0·375	0·433	1·154
99	1·104	0·906	2·437	2·691	99	2·691	0·372	0·427	1·149

Annexure-V

Table of values of the function W (u, r/L) after HANTUSH (1956)

W(u, r/L)=W(o, r/L)

W(u, r/L)= W (u,0)

u	1/u	r/L 0	0·005	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0,09
0		∞	1·08(1)	9·44	8·03	7·25	6·67	6·23	5·87	5·56	5·29	5·06
1(—6)	1·00(6)	1·32(1)										
2(—6)	5·00(5)	1·25(1)										
4(—6)	2·50(5)	1·18(1)	1·07(1)									
6(—6)	1·66(5)	1·14(1)	1·06(1)									
8(—6)	1·25(5)	1·12(1)	1·05(1)	9·43								
1(—5)	1·00(5)	1·09(1)	1,04(1)	9·42								
2(—5)	5·00(4)	1·02(1)	9·95	9·30								
4(—5)	2·50(4)	9·55	9·40	9·01	8·03							
6(—5)	1·66(4)	9·14	9·04	8·77	7·98	7·24						
8(—5)	1·25(4)	8·86	8·78	8·57	7·91	7·23						
1(—4)	1·00(4)	8·63	8·57	8·40	7·84	7·21						
2(—4)	5·00(3)	7·94	7·91	7·82	7·50	7·07	6·62	6·22	5·86			
4(—4)	2·50(3)	7·25	7·23	7·19	7·01	6·76	6·45	6·14	5·83	5·55		
6(—4)	1·66(3)	6·84	6·83	6·80	6·68	6·50	6·27	6·02	5·77	5·51	5·27	5·05
8(—4)	1·25(3)	6·55		6·52	6·43	6·29	6·11	5·91	5·69	5·46	5·25	5·04
1(—3)	1·00(3)	6·33		6·31	6·23	6·12	5·97	5·80	5·61	5·41	5·21	5·01
2(—3)	5·00(2)	5·64		5·63	5·59	5·53	5·45	5·35	5·24	5·12	4·89	4·85
4(—3)	2·50(2)	4·95		4·94	4·92	4·89	4·85	4·80	4·74	4·67	4·59	4·51
6(—3)	1·66(2)	4·54			4·53	4·51	4·48	4·45	4·40	4·36	4·30	4·24
8(—3)	1·25(2)	4·26			4·25	4·23	4·21	4·19	4·15	4·12	4·08	4·03
1(—2)	1·00(2)	4·04			4·03	4·02	4·00	3·98	3·95	3·92	3·89	3·85
2(—2)	5·00(1)	3·35				3·34	3·34	3·33	3·31	3·30	3·28	3·26
4(—2)	2·50(1)	2·68					2·67	2·67	2·66	2·65	2·65	2·64
6(—2)	1·66(1)	2·29							2·28	2·28	2·27	2·27
8(—2)	1·25(1)	2·03							2·02	2·01	2·01	2·01
1(—1)	1·00(1)	1·82								1·81	1·81	1·81
2(—1)	5·00(1)	1·22										1·22
4(—1)	2·50(1)	7·02(—1)										7,00(—1)
6(—1)	1·66(1)	4·54(—1)										
8(—1)	1·25(1)	3·11(—1)										

u	1/u	r/L	0,1	0,2	0,3	0,4	0,6	0,8
0			4·85	3·50	2·74	2·23	1·55	1·13
1(—4)	1·00(4)	8·63						
2(—4)	5·00(3)	7·94						
4(—4)	2·55(3)	7·25						
6(—4)	1·66(3)	6·84						

W(u, r/L)=W (o, r/L)

8(—4)	1·25(3)	6·55	4·84					
1(—3)	1·00(3)	6·33	4·83					
2(—3)	5·00(2)	5·64	4·71					
4(—3)	2·50(3)	4·95	4·42	3·48				
6(—3)	1·66(2)	4·54	4·18	4·43				
8(—3)	1·25(2)	4·26	3·98	3·36	2·73			
1(—2)	1·00(—2)	4·04	3·81	3·29	2·71	2·22		
2(—2)	5·00(—1)	3·35	3·24	2·95	2·57	2·18		
4(—2)	2·50(1)	2·68	2·63	2·48	2·27	2·02	1·52	
6(—2)	1·66(1)	2·29	2·26	2·17	2·02	1·84	1·46	1·11
8(—2)	1·25(1)	2·03	2·00	1·93	1·83	1·69	1·39	1·08
1(—1)	1·00(1)	1·82	1·80	1·75	1·67	1·56	1·31	1·05
2(—1)	5·00	1·22	1·21	1.19	1·16	1·11	9·96(—1)	8·58(—1)
4(—1)	2·50	7·02(—1)	7·00(—1)	6·93(—1)	6·81(—1)	6·65(—1)	6·21(—1)	5·65(—1)
6(—1)	1·66	4·54(—1)	4·53(—1)	4·50(—1)	4·44(—1)	4·36(—1)	4·15(—1)	3·87(—1)
8(—1)	1·25	3·11(—1)	3·10(—1)	3·08(—1)	3·05(—1)	3·01(—1)	2·89(—1)	2·73(—1)
1	1·00		2·19(—1)	2·18(—1)	2·16(—1)	2·14(—1)	2·07(—1)	1·97(—1)
	5·00(—1)		4·88(—2)	4·87(—2)	4·85(—2)	4·82(—2)	4·73(—2)	4·60(—2)
u	l/u r/L	O	1.0	2.0	3.0	4.0	5·0	6.0
0			8·42(—1)	2.28(—1)	6.95(—2)	2·23(—2)	6·4(—3)	2.5(—3)
1(—2)	1·00(2)	4.04						
2(—2)	5·00(1)	3.35						
4(—2)	2.50(1)	2.68						
6(—2)	1·66(1)	2·29	8.39(—1)					
8(—2)	1·25(1)	2·03	8·32(—1)					
1(—1)	1,00(1)	1·82	8·19(—1)					
2(—1)	5·00	1.22	7.15(—1)	2,27(—1)				
4—1)	2.50	7·02[—1)	5·02(—1)	2,10(—1)	6·91(—2)			
6(—1)	1,66	4.54(—1)	3,54(—1)	1·77(—1)	6.64(—2)	2·22(—2)		
8(—1)	1·25	3·11(—1)	2·54(—1)	1·44(—1)	6·07(—2)	2·18(—2)		
1	1·00	2·19(—1)	1·85(—1)	1·14(—1)	5.34(—2)	2·07(—2)	7.3(—3)	
2	5·00(—1)	4.89(—2)	4·44(—2)	3.35(—2)	2·10(—2)	1.12(—2)	5.1(—3)	2·1(—3)
4	2,50(—1)	3·78(—3)	3·6(—3)	3.1(—3)	2·4(—3)	1·60(—3)	1·0(—3)	6·0(—4)

W(u,r/L)=W(or/L]

Values of H(u, β). (From Hantush, 1964)*

u \ β	(—3)			(—2)			(—1)		
	1	2	5	1	2	5	1	2	5
1(—6)	11·9842	11·4237	10.5908	9·9259	9·2469	8·3395	7·6497	6·9590	6·0463
5(—6)	10·8958	10·4566	9,7174	9,0866	8,4251	7,5284	6,8427	6,1548	5,2459
1(—5)	10·3739	9·9987	9,3203	8·7142	8·0657	7,1771	4,4944	5·8085	4,9024
5(—5)	9·0422	8·8128	8·3171	8,8031	7,2072	6·3523	5·6821	5·0045	4·1090
1(—4)	8·4258	8·2487	7·8386	7·3803	6·8208	5·9906	5·3297	4·6581	3·7700
5(—4)	6·9273	6·8375	6·6024	6·2934	5·8561	5·1223	4·4996	3·8527	2·9933
1(3)	6·2624	6·1969	6·0193	5·7727	5·4001	4·7290	4·1337	3·5043	2·6650
5(—3)	4·6951	4·6649	5·5786	4·4474	4·2231	3·7415	3·2483	2·6891	1·9250
5(—2)	4·0163	3·9950	3·9334	3·8374	3·6669	3·2752	2·8443	2·3325	1·6193
5(—2)	2·4590	2·4502	2·4243	2·3826	2·3040	2·1007	1·8401	1·4872	0·9540
1(—1)	1·8172	1·8116	1·7949	1,7677	1,7157	1·5768	1,3893	1,1202	1·6947
5(—1)	0·5584	0·5570	0·5530	0·5463	0·5333	0·4969	0·4436	0·3591	0,2083
1(0)	0·2189	0·2184	0·2169	0·2144	0·2097	0·1961	0·1758	0·1427	812(—4)
5(0)	115(—5)	114(—5)	114(—5)	112(—5)	110(—5)	104(—5)	934(—6)	763(—6)	423(—6)
10(0)	415(—8)	414(—8)	411(—8)	407(—8)	399(—8)	375(—8)	339(—8)	277(—8)	153(—8)

* The numbers in parenthesis are powers of 10 by which the other numbers are multiplied, e. g.,

u \ β	(0)			(1)			(2)		
	1	2	5	1	2	5	1	2	5
1(—6)	5·3575	4·6721	3·7756	3·1110	2·4671	1·6710	1·1361	0·6879	0·2698
5(—6)	4·5617	3·8836	3·0055	2·3661	1·7633	1·0574	0·6256	0·3091	787(—4)
1(—5)	4·2212	3·5481	2·6822	2·0590	1·4816	0·8285	0·4519	0·1978	388(—4)
5(—5)	3·4394	2·7848	1·9622	1·3943	0·8994	0·4024	0·1685	494(—4)	405(—5)
1(—4)	3·1082	2·4658	8.6704	1.1359	0·6878	0·2678	963(—4)	222(—4)	107(—5)
5(—4)	2·3601	1·7604	1.0564	0·6252	0·3089	787(—4)	166(—4)	169(—5)	129(—7)
1(—3)	2·0506	1·4776	0·8271	0·4513	0·1976	388(—4)	590(—5)	361(—6)	
5(—3)	1·3767	0·8915	0·4001	0·1667	393(—4)	403(—5)	205(—6)	228(—8)	
1(—2)	1·1122	0·6775	0·2670	955(—4)	221(—4)	106(—5)	274(—7)		
5(—2)	0·5812	0·2923	755(—4)	160(—4)	164(—5)	126(—7)			
1(—1)	0·3970	0·1789	359(—4)	552(—5)	340(—6)				
5(—1)	0·1006	325(—4)	288(—5)	151(—6)	171(—8)				
1(0)	365(—4)	993(—5)	547(—6)	151(—7)					
5(0)	167(—7)	309(—7							
10(0)									

488(—4)=0,0488

Table of values of the functions W (u_A, r/B) and W (u_Y, r/B) after BOULTON (1963) **Annexure-VII**

r/B = 0·01		r/B = 0·1		r/B = 0·2		r/B = 0·316	
$1/u_A$	W(u_A, r/B)	$1/u_A$	W(u_A, r/B)	$1/u_A$	W(u_A, r/B)	$1/u_A$	W(u_A, r/B)
1(1)	1·82	1(1)	1·80	5(0)	1·19	1(0)	2·16(−1)
1(2)	4·04	5(1)	3·24	1(1)	1·75	2(0)	5·44(−1)
1(3)	6·31	1(2)	3·81	5(1)	2·95	5(0)	1·15
5(3)	7·82	2(2)	4·30	1(2)	3·29	1(1)	1·65
1(4)	8·40	5(2)	4·71	5(2)	3·50	5(1)	2·50
1(5)	9·42	1(3)	4·83	1(3)	3·51	1(2)	2·62
1(6)	9·44	1(4)	4·85			1(3)	2·65

r/B = 0·4		r/B = 0·6		r/B = 0·8		r/B = 1·0	
$1/u_A$	W(u_A, r/B)	$1/u_A$	W(u_A, r/B)	$1/u_A$	W(u_A, r/B)	$1/u_A$	W(u_A, r/B)
1(0)	2·13(−1)	1(0)	2·06(−1)	5(−1)	4·60(−2)	5(−1)	4·44(−2)
2(0)	5·34(−1)	2(0)	5·04(−1)	1	1·97(−1)	1(0)	1·85(−1)
5(0)	1·11	5(0)	9·96(−1)	2	4·66(−1)	2(0)	4·21(−1)
1(1)	1·56	1(1)	1·31	5	8·57(−1)	5(0)	7·15(−1)
5(1)	2·18	2(1)	1·49	1(1)	1·05	1(1)	8·19(−1)
1(2)	2·22	5(1)	1·55	2(1)	1·12	2(1)	8·41(−1)

r/B = 1·5		r/B = 2·0		r/B = 2·5		r/B = 3·0	
$1/u_A$	W(u_A, r/B)	$1/u_A$	W(u_A, r/B)	$1/u_A$	W(u_A, r/B)	$1/u_A$	W(u_A, r/B)
5(−1)	3·94(−2)	3·33(−1)	1·00(−2)	5(−1)	2·71(−2)	5(−1)	2·10(−2)
1(0)	1·51(−1)	5(−1)	3·35(−2)	1(0)	8·03(−2)	1(0)	5·34(−2)
1·25(0)	1·99(−1)	1(0)	1·14(−1)	1·25(0)	9·61(−2)	1·25(0)	6·07(−2)
2(0)	3·01(−1)	1·25(0)	1·44(−1)	2(0)	1·17(−1)	2(0)	6·81(−2)
5(0)	4·13(−1)	2(0)	1·94(−1)	5(0)	1·25(−1)	5(0)	6·95(−2)
1(1)	4·27(−1)	5(0)	2·27(−1)	1(1)	1·25(−1)	1(1)	6·95(−2)
2(1)	4·19(−1)	1(1)	2·28(−1)				

r/B = 0·01		r/B = 0·1		r/B = 0·2		r/B = 0·316	
1/u	W(u_Y, r/B)	1/u	W(u r/B)	1/u	W(u r/B)	1/u	W(u r/B)
4(2)	9·45	4(0)	4·86	4(−1)	3·51	4(−1)	2·66
4(3)	9·54	4(1)	4·95	4(0)	3·54	4(0)	2·74
4(4)	1·02(1)	4(2)	5·64	2(1)	3·69	4(1)	3·38
4(5)	1·23(1)	4(3)	7·72	4(1)	3·85	4(2)	5·42
4(6)	1·46(1)	4(4)	1·00(1)	1·5(2)	4·55	4(3)	7·72
				4(2)	5·42		

r/B = 0·4		r/B = 0·6		r/B = 0·8		r/B = 1·0	
$1/u_Y$	W(u r/B)	$1/u_Y$	W(u r/B)	1/u	W(u_Y, r/B)	$1/u_Y$	W(u_Y r/B)
1(−1)	2·23	4·44(−1)	1·59	2·5(−2)	1·13	4(−2)	8·44(−1)
1(0)	2·26	2·22(0)	1·71	2·5(−1)	1·16	4(−1)	9·01(−1)
5(0)	2·40	4·44(0)	1·84	1·25(0)	1·26	4(0)	1·36
1(1)	2·55	1·67(1)	2·45	2·5(0)	1·39	4(1)	3·14
3·75(1)	3·20	4·44(1)	3·26	9·37(0)	1·94		
1(2)	4·05			2·5(1)	2·70		

r/B = 1·5		r/B = 2·0		r/B = 2·5		r/B = 3·0	
1/u	W(u r/B)	1/u	W(u_Y, r/B)	1/u	W(u r/B)	$1/u_Y$	W(u_Y, r/B)
7·11(−2)	4·44(−1)	4(−2)	2·39(−1)	2·56(−2)	1·32(−1)	1·78(−2)	7·43(−2)
3·55(−1)	5·09(−1)	2(−1)	2·83(−1)	1·28(−1)	1·62(−1)	8·89(−2)	9·39(−2)
7·11(−1)	5·87(−1)	4(−1)	3·37(−1)	2·56(−1)	1·99(−1)	1·78(−1)	1·19(−1)
2·67(0)	9·63(−1)	1·5(0)	6·14(−1)	9·6(−1)	3·99(−1)	6·67(−1)	2·62(−1)
7·11(0)	1·57	4(0)	1·11	2·56(0)	7·98(−1)	1·78(0)	5·77(−1)

Values of W(—). (From Jacob and Lohman, 1952)

N \ λ	$N \times 10^{-4}$	$N \times 10^{-3}$	$N \times 10^{-2}$	$N \times 10^{-1}$	$N \times 1$	$N \times 10$	$N \times 10^{2}$	$N \times 10^{3}$	$N \times 10^{4}$	$N \times 10^{5}$	$N \times 10^{6}$	$N \times 10^{7}$	$N \times 10^{8}$	$N \times 10^{9}$	$N \times 10^{10}$	$N \times 10^{11}$
1	56·9	18·34	6·13	2·249	0·985	0·534	0·346	0·251	0·1964	0·1608	0·1360	0·1177	0·1037	0·0927	0·0838	0·0764
2	40·4	13·11	4·47	1·716	0·803	0·461	0·311	0·232	0·1841	0·1524	0·1299	0·1131	0·1002	0·0899	0·0814	0·0744
3	33·1	10·79	3·74	1·477	0·719	0·427	0·294	0·222	0·1777	0·1479	0·1266	0·1106	0·0982	0·0883	0·0801	0·0733
4	28·7	9·41	3·30	1·333	0·667	0·405	0·283	0·215	0·1733	0·1449	0·1244	0·1089	0·0968	0·0872	0·0792	0·0726
5	25·7	8·47	3·00	1·234	0·630	0·389	0·274	0·210	0·1701	0·1426	0·1227	0·1076	0·0958	0·0864	0·0785	0·0720
6	23·5	7·77	2·78	1·160	0·602	0·377	0·268	0·206	0·1675	0·1408	0·1213	0·1066	0·0950	0·0857	0·0779	0·0716
7	21·8	7·23	2·60	1·103	0·580	0·367	0·263	0·203	0·1654	0·1393	0·1202	0·1057	0·0943	0·0851	0·0774	0·0712
8	20·4	6·79	2·46	1·057	0·562	0·359	0·258	0·200	0·1636	0·1380	0·1192	0·1049	0·0937	0·0846	0·0770	0·0709
9	19·3	6·43	2·35	1·018	0·547	0·352	0·254	0·198	0·1621	0·1369	0·1184	0·1043	0·0932	0·0842	0·0767	0·0706
10	18·3	6·13	2·25	0·985	0·534	0·346	0·251	0·196	0·1608	0·1360	0·1177	0·1037	0·0927	0·0838	0·0764	0·0704

(Values of Wx, r_w/L) (From Hanuisk, 1989)

r_w/L / λ	0	1×10^{-4}	2×10^{-5}	4×10^{-5}	6×10^{-5}	8×10^{-5}	10^{-4}	2×10^{-4}	4×10^{-4}	6×10^{-4}	8×10^{-4}	10^{-3}	2×10^{-3}	4×10^{-3}
1×10^{2}	0·346													
1×10^{2}	0·274													
1×10^{3}	0·251												0·274	0·274
1×10^{3}	0·210												0·251	0·252
1×10^{4}	0·196												0·210	0·212
5×10^{4}	0·170											0·196	0·197	0·200
1×10^{5}	0·161											0·170	0·173	0·181
5×10^{5}	0·143								0·161	0·162	0·162	0·162	0·167	0·178
1×10^{6}	0·136							0·143	0·143	0·144	0·145	0·148	0·161	
5×10^{6}	0·123						0·136	0·137	0·138	0·139	0·141	0·144	0·159	
1×10^{7}	1·138						0·133	0·124	0·128	0·133				
5×10^{7}	1·108						0·118	0·120	0·127					
1×10^{8}	1·104		0·104	0·104	0·105	0·106	0·108							
5×10^{8}	0·0958	0·0958	0·0966	0·0989										
1×10^{9}	0·0927	0·0930	0·0943	0·0980										
5×10^{9}	0·0864	0·0880	0·0916											
1×10^{10}	0·0838	0·0867	0·0914											
5×10^{10}	0·0785													
10×10^{10}	0·0764	0·0860	0·0924	0·0976	0·102	0·105	0·107	0·116	0·126	0·133	0·138	0·142	0·158	0·177

rw/L / λ	6×10^{-3}	8×10^{-3}	10^{-2}
1×10^{2}			0·346
5×10^{2}	0·275	0·275	0·276
1×10^{3}	0·252	0·254	0·255
5×10^{3}	0·215	0·218	0·222
1×10^{4}	0·204	0·209	0·216
5×10^{4}	0·192		
1×10^{5}			
5×10^{5}			
1×10^{6}			
5×10^{6}			
1×10^{7}			
5×10^{7}			
1×10^{8}			
5×10^{8}			
1×10^{9}			
5×10^{9}			
1×10^{10}			
5×10^{10}	0·191	0·202	0·212

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Table of values of Σ= f (P, 1), after ANONYMOUS (1964)

P	Σ=O	0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45
0·1	0·54	0·54	0·55	0·55	0·56	0·57	0·59	0·61	0·67	1·09
0·2	0·44	0·45	0·46	0·47	0·49	0·52	0·59	0·59	0·89	
0·3	0·37	0·37	0·38	0·39	0·41	0·43	0·50	0·74		
0·4	0·31	0·31	0·32	0·34	0·36	0·42	0·62			
0·5	0·25	0·26	0·27	0·29	0·34	0·51				
0·6	0·21	0·21	0·23	0·27	0·41					
0·7	0·16	0·17	0·20	0·32						
0·8	0·11	0·13	0·22							
0·9	0·06	0·12								

Table of values of M (u, β) , after HANTUSH (1962)

u	1/u \ β	0·1	0·2	0·3	0·4	0·5	0·6	0·7
0		0·1997	0·3974	0·5913	0·7801	0·9624	1·1376	1·3053
1(—6)	1·00(6)	0·1994	0·3969	0·5907	0·7792	0·9613	1·1363	1·3037
2(—6)	5·00(5)	0·1993	0·3967	0·5904	0·7788	0·9608	1·1357	1·3031
4(—6)	2·50(5)	0·1992	0·3965	0·5900	0·7783	0·9602	1·1349	1·3022
6(—6)	1·66(5)	0·1991	0·3963	0·5897	0·7779	0·9596	1·1343	1·3014
8(—6)	1·25(5)	0·1990	0·3961	0·5894	0·7775	0·9592	1·1338	1·3009
1(—5)	1·00(5)	0·1989	0·3959	0·5892	0·7772	0·9588	1·1334	1·3003
2(—5)	5·00(4)	0·1987	0·3954	0·5883	0·7760	0·9574	1·1316	1·2983
4(—5)	2·50(4)	0·1982	0·3945	0·5871	0·7744	0·9553	1·1291	1·2953
6(—5)	1·66(4)	0·1979	0·3939	0·5861	0·7731	0·9537	1·1271	1·2931
8(—5)	1·25(4)	0·1976	0·3933	0·5853	0·7720	0·9523	1·1255	1·2912
1(—4)	1·00(4)	0·1974	0·3929	0·5846	0·7710	0·9511	1·1241	1·2895
2(—4)	5·00(3)	0·1965	0·3910	0·5818	0·7673	0·9465	1·1185	1.2830
4(—4)	2·50(3)	0·1952	0·3883	0·5778	0·7620	0·9398	1·1106	1·2737
6(—4)	1·66(3)	0·1941	0·3863	0·5748	0·7580	0·9348	1·1045	1·2666
8(—4)	1·25(3)	0·1933	0·3846	0·5722	0·7545	0·9305	1·0994	1·2607
1(—3)	1·00(3)	0·1925	0·3831	0·5699	0·7515	0·9267	1·0984	1·2554
2(—3)	5·00(2)	0·1896	0·3772	0·5611	0·7397	0·9120	1·0771	1·2347
4(—3)	2·50(2)	0·1854	0·3689	0·5486	0·7231	0·8912	1·0521	1·2056
6(—3)	1·66(2)	0·1822	0·3625	0·5390	0·7103	0·8752	1·0330	1·1832
8(—3)	1·25(2)	0·1795	0·3571	0·5310	0·6995	0·8618	1·0169	1·1645
1(—2)	1·00(2)	0·1772	0·3524	0·5239	0·6901	0·8500	1·0027	1·1480
2(—2)	5·00(1)	0·1680	0·3340	0·4962	0·6533	0·8040	0·9476	1·0836
4(—2)	2·50(1)	0·1551	0·3083	0·4578	0·6020	0·7400	0·8708	0·9942
6(—2)	1·66(1)	0·1455	0·2890	0·4289	0·5635	0·6919	0·8132	0·9272
8(—2)	1·25(1)	0·1375	0·2731	0·4050	0·5317	0·6522	0·7658	0·8720
1(—1)	1·00(1)	0·1306	0·2993	0·3844	0·5043	0·6181	0·7249	0·8245
2(—1)	5·00	0·1051	0·2084	0·3081	0·4030	0·4920	0·5744	0·6500
4(—1)	2·50	7·39(—2)	0·1462	0·2153	0·2801	0·3397	0·3925	0·4415
6(—1)	1·66	5·44(—2)	0·1074	0·1575	0·2039	0·2458	0·2828	0·3149
8(—1)	1·25	4·10(—2)	8·06(—2)	0·1179	0·1519	0·1821	0·2082	0·2302
1	1·00	3·13(—2)	6·14(—2)	8·95(—2)	0·1148	0·1369	0·1555	0·1709
2	5·00(—1)	9·01(—3)	1·75(—2)	2·51(—2)	3·16(—2)	3·67(—2)	4·07(—2)	4·35(—2)
4	2·50(—1)	9·20(—4)	1·76(—3)	2·44(—3)	2·96(—3)	3·31(—3)	3·53(—3)	3·66(—3)
6	1·66(—1)	1·04(—4)	1·95(—4)	2·64(—4)	3·10(—4)	3·36(—4)	3·50(—4)	3·56(—4)
8	1·25(—1)	1·23(—5)	2·26(—5)	2·99(—5)	3·42(—5)	3·63(—5)	3·72(—5)	3·75(—5)

Annexure—XI

0·8	0·9	1·0	1·2	1·4	1·6	1·8	2·0
1·4653	1·6177	1·7627	2·0319	2,2739	2·4979	2·7009	2·8872
1·4635	1·6157	1·7605	2·0292	2·2728	2·4943	2·6968	2·8827
1·4628	1·6148	1·7595	2·0281	2·2715	2·4929	2·6951	2·8809
1·4617	1·6137	1·7582	2·0265	2·2696	2·4907	2·6927	2·8782
1·4609	1·6127	1·7572	2·0253	2·2682	2·4891	2·6909	2·8762
1·4602	1·6120	1·7563	2·0243	2·2670	2·4877	2·6894	2·8745
1·4596	1·6113	1·7556	2·0234	2·2660	2·4865	2·6880	2·8730
1·4572	1·6086	1·7526	2·0198	2·2618	2·4818	2·6827	2·8671
1·4539	1·6049	1·7485	2·0148	2·2560	2·4751	2·6752	2·8587
1·4513	1·6020	1·7452	2·0110	2·2515	2·4700	2·6694	2·8523
1·4492	1·5996	1·7425	2,0077	2,2477	2·4657	2·6645	2·8469
1·4473	1·5974	1·7402	2·0049	2·2444	2·4619	2·6603	2·8421
1·4398	1·5890	1·7308	1·9936	2·2313	2·4469	2·6434	2·8234
1·4292	1·5771	1·7176	1·9778	2·2128	2·4258	2·6197	2·7970
1·4211	1·5680	1·7075	1·9656	2·1986	2·4095	2·6014	2·7768
1·4143	1·5603	1·6989	1·9554	2·1866	2·3959	2·5860	2·7597
1·4083	1·5535	1·6914	1·9463	2·1761	2·3838	2·5725	2·7446
1·3846	1·5270	1·6619	1·9109	2·1348	2·3367	2·5195	2·6857
1·3513	1·4895	1·6203	1·8610	2·0766	2·2702	2·4447	2·6027
1·3258	1·4608	1·5884	1·8228	2·0320	2·2193	2·3875	2·5393
1·3044	1·4367	1·5616	1·7907	1·9946	2·1766	2·3395	2·4861
1·2855	1·4155	1·5381	1·7625	1·9617	2·1391	2·2975	2·4394
1·2121	1·3329	1·4464	1·6527	1·8340	1·9935	2·1342	2·2587
1·1100	1·2183	1·3193	1·5008	1·6577	1·7932	1·9103	2·0117
1·0336	1·1326	1·2243	1·3877	1·5268	1,6450	1,7454	1·8307
0·9707	1·0621	1·1464	1·2951	1·4200	1·5246	1·6120	1·6848
0·9167	1·0016	1·0795	1·2159	1·3290	1·4223	1·4991	1·5619
0·7186	0·7806	0·8362	0·9297	1·0029	1·0595	1·1026	1·1352
0·4837	0·5203	0·5519	0·6015	0·6363	0·6602	0·6760	0·6863
0·3423	0·3652	0·3842	0·4122	0·4300	0·4408	0·4471	0·4506
0·2484	0·2632	0·2750	0·2913	0·3007	0·3058	0·3084	0·3096
0·1833	0·1929	0·2004	0·2101	0,2151	0·2175	0·2186	0·2191
4·55(—2)	4·69(—2)	4·77(—2)	4·85(—2)	4·88(—2)			
3·72(—3)	3·76 —3)	3·77(—3)					
3·59(—4)	3·60(—4)						
3·76(—5)	3·77(—5)						

M (u, β) = W(u) : see Annex II

Annexure—XI

u	1/u\β	2·2	2·4	2·6	2·8	3·0	3·2	3,4	3·6	3·8
0		3·0593	3·2188	3·3675	3·5064	3·6393	3·7597	3·8757	3·9856	4·0900
1(—6)	1·00(6)	3·0543	3·2134	3·3616	3·5001	3·6301	3·7525	3·8681	3·9775	4·0815
2(—6)	5·00(5)	3·0524	3·2112	3·3592	3·4975	3·6271	3·7495	3·8649	3·9742	4·0779
4(—6)	2·50(5)	3·0494	3·2080	3·3557	3·4938	3·6233	3·7453	3·8604	3,9694	4·0729
6(—6)	1·66(5)	3·0471	3·2056	3·3531	3,4910	3·6203	3·7420	3·8569	3·9658	4·0690
8(—6)	1·25(5)	3·0453	3·2035	3·3509	3·4886	3·6177	3·7393	3·8540	3·9627	4·0658
1(—5)	1·00(5)	3·0436	3·2017	3·3489	3·4865	3·6155	3·7369	3·8515	3·9600	4·0629
2(—5)	5·00(4)	3·0371	3·1946	3·3412	3·4782	3·6066	3·7274	3·8414	3·9493	4·0517
4(—5)	2·50(4)	3·0279	3·1846	3·3304	3·4665	3·5941	3·7140	3·8272	3·9343	4·0358
6(—5)	1·66(4)	3·0209	3·1769	3·3220	3·4575	3·5844	3,7038	3·8163	3·9227	4·0236
8(—5)	1·25(4)	3·0149	3·1704	3·3150	3·4499	3·5763	3·6951	3·8071	3·9130	4·0133
1(—4)	1·00(4)	3·0097	3·1647	3·3088	3·4433	3·5692	3·6875	3·7790	3·9044	4·0043
2(—4)	5·00(3)	2·9891	3·1423	3·2845	3·4171	3·5412	3·6576	3·7673	3·8708	3·9688
4(—4)	2·50(3)	2·9600	3·1106	3·2502	3·3801	3·5015	3·6154	3·7224	3·8233	3·9187
6(—4)	1·66(3)	2·9378	3·0863	3·2238	3·3518	3·4712	3·5830	3·6880	3·7869	3·8802
8(—4)	1·25(3)	2·9190	3·0658	3·2017	3·3279	3·4456	3·5557	3·6590	3·7562	3·8479
1(—3)	1·00(3)	2·9024	3·0478	3·1821	3·3069	3·4231	3·5317	3·6335	3·7292	3·8194
2(—3)	5·00(2)	2·8377	2·9771	3·1056	3·2245	3·3349	3·4377	3·5337	3·6236	3·7080
4(—3)	2·50(2)	2·7464	2·8776	2·9980	3·1087	3·2110	3·3056	3·3936	3·4734	4·5518
6(—3)	1·66(2)	2·6767	2·8018	2·9159	3·0205	3·1166	3·2052	3·2871	3·3629	3·4334
8(—3)	1·25(2)	2·6183	2·7382	2·8472	2·9466	3·0377	3·1213	3·1982	3·2691	3·3346
1(—2)	1·00(2)	2·5671	2·6825	2·7870	2·8820	2·9687	3·0480	3·1206	3·1873	3·2487
2(—2)	5·00(1)	2·3692	2·4675	2,5552	2·6337	2·7041	2·7673	2·8243	2·8756	2·9218
4(—2)	2·50(1)	2·0996	2·1759	2·2423	2·3000	2·3503	2·3942	2·4324	2·4658	2·4949
6(—2)	1·66(1)	1·9031	1·9645	2·0167	2·0610	2·0986	2·1305	2·1574	2·1802	2·1995
8(—2	1·25(1)	1·7455	1·7959	1·8378	1·8725	1·9012	1·9249	1·9444	1·9604	1·9734
1(—1)	1·00(1)	1·6133	1·6552	1·6892	1·7167	1·7389	1·7568	1.7711	1·7825	1·7915
2(—1)	5·00	1·1596	1·1777	1·1909	1·2004	1·2073	0·2122	1·2156	1·2179	1·2195
4(—1)	2·50	0·6928	0·6968	0·6992	0·7006	0·7014	0·7019	0·7021	0·7023	0·7023
6(—1)	1·66	0·4525	0·4535	0·4540	0·4542	0·4543	0·4543			
8(—1)	1·25	0·3102	0·3104	0·3105						

4·0	4·2	4·4	4·6	4·8	5·0	5·2	5·4	5·6	5·8	6·0
4·2841	4·2842	4·3748	4·4616	4·5448	4·6248	4·7018	4·7760	4·8475	4·9167	4·9835
4·1804	4·2747	4·3649	4·4512	4·5340	4·6136	4·6901	4·7638	4·8349	4·9036	4·9700
4·1766	4·2708	4·3608	4·4469	4·5295	4·6089	4·6852	4·7588	4·8297	4·8982	4·9644
4·1714	4·2653	4·3550	4·4408	4·5232	4·6023	4·6784	4·7516	4·8223	4·8905	4·9565
4·1673	4·2610	4·3505	4·4362	4·5183	4·5972	4·6731	4·7462	4·8166	4·8846	4·9504
4·1639	4·2574	4·3467	4·4323	4·5142	4·5929	4·6686	4·7415	4·8118	4·8797	4·9452
4·1609	4·2542	4·3434	4·4288	4·5106	4·5892	4·6647	4·7375	4·8076	4·8753	4·9407
4·1490	4·2418	4·3304	4·4152	4·4964	4·5744	4·6494	4·7215	4·7911	4·8582	4·9230
4·1323	4·2243	4·3120	4·3960	4·4764	4.5533	4·6276	4·6989	4·7677	4·8339	4·9879
4·1195	4·2108	4·2979	4·3812	4·4610	4·5375	4·6110	4·6816	4·7497	4·8153	4·8707
4·1087	4·1994	4·2860	4·3688	4·4480	4·5240	4·5969	4·6670	4·7346	4·7997	4·8625
4·0992	4·1894	4·2756	4·3578	4·4366	4·5121	4·5845	4·6542	4·7212	4·7859	4·8482
4·0618	4·1502	4·2345	4·3149	4·3918	4·4654	4·5360	4·6038	4·6690	4·7317	4·7922
4·0090	4.0948	4·1764	4·2542	4·3283	4·3995	4·4674	4·5326	4·5952	4·6553	4·7132
3·9686	4·0524	4·1320	4·2077	4·2800	4·3940	4·4150	4·4781	4·5387	4·5969	4·6527
3·9345	4·0166	4·0945	4·1686	4·2392	4·3065	4·3708	4·4323	4·4912	4·5477	4·6019
3·9046	3·9852	4·0616	4·1342	4·2033	4·2691	4·3320	4·3920	4·4494	4·5045	4·5572
3·7874	3·8623	3·9329	3·9908	4·0632	4·1233	4·1805	4·2349	4·2867	4·3360	4·3832
3·6233	3·6902	3·7530	3·8120	3·8676	3·9199	3·9694	4·0161	4·0602	4·1020	4·1416
3·4989	3·5599	3·6169	3·6702	3·7200	3·7667	3·8105	3·8517	3·8903	3·9257	3·9609
3·3953	3·4516	3·5038	3·5524	3·5977	3·6398	3·6792	3·7159	3·7502	3·7823	3·8132
3·3052	3·3574	3·4057	3·4503	3·4917	3·5300	3·5656	3·5987	3·6294	3·6580	3·6845
2·9637	3·0015	3·0357	3·0666	3·0946	3·1200	3·1430	3·1638	3·1827	3·1998	3·2153
2·5202	2·5423	2·5615	2·5782	2·5927	2·6052	2·6161	2·6256	2·6337	2·6408	2·6468
2·2157	2·2294	2·2408	2·2504	2·2384	2·2651	2·2706	2·2752	2·2790	2·2821	2·2846
1·9841	1·9928	1·9998	2·0055	2·0101	2·0137	2·0166	2·0189	2·0207	2·0221	2·0733
1·7989	1·8043	1·8087	1·8121	1·8147	1·8168	1·8183	1·8195	1·8204	1·8211	1·8216
1·2206	1·2213	1·2218	1·2221	1·2223	1·2224	1·2225	1·2226			
0·7004										

M(u, β) = W(u) : *see* Annex. II

u	1/u \ β	6·2	6·4	6·6	6·8	7·0	7·2	7·4	7·6	7·8
0		5·0482	5·1109	5·1718	5·2308	5·2882	5·3440	5·3983	5·4511	5·5026
1(—6)	1·00(6)	5·0343	5·0963	5·1569	5·2155	5·2724	5·3278	5·3816	5·4340	5·4851
2(—6)	5·00(5)	5·0285	5·0905	5·1507	5·2091	5·2659	5·3210	5·3747	5·4269	5·4778
4(—6)	2·50(5)	5·0203	5·0821	5·1420	5·2002	5·2566	5·3115	5·3649	5·4169	5·4[illegible]75
6(—6)	1·66(5)	5·0140	5·0756	5·1353	5·1933	5·2495	5·3042	5·3574	5·4092	5·4596
8(—6)	1·25(5)	5·0087	5·0701	5·1297	5·1874	5·2435	5·2981	5·3511	5·4027	5·4529
1(—5)	1·00(5)	5·0040	5·0653	5·1247	5·1823	5·2383	5·2926	5·3455	5·3969	5·4470
2(—5)	5·00(4)	4·9857	5·0464	5·1052	5·1622	5·2176	5·2714	5·3236	5·3745	5·4240
4(—5)	2·50(4)	4·9598	5·0196	5·0776	5·1338	5·1883	5·2413	5·2927	5·3427	5·3914
6(—5)	1·66(4)	4·9399	4·9991	5·0565	5·1120	5·1659	5·2182	5·2690	5·3184	5·3664
8(—5)	1·25(4)	4·9232	4·9618	5·0386	5·0937	5·1470	5·1988	5·2490	5·2979	5·3453
1(—4)	1·00(4)	4·9084	4·9666	5·0229	5·0775	5·1303	5·1816	5·2314	5·2798	5·3268
2(—4)	5·00(3)	4·8506	4·9069	4·9614	5·0141	5·0651	5·1145	5·1624	5·2089	5·2541
4(—4)	2·50(3)	4·7689	4·8227	4·8745	4·9246	4·9730	5·0198	5·0652	5·1091	5·1516
6(—4)	1·66(3)	4·7065	4·7582	4·8081	4·8562	4·9026	4·9475	4·9908	5·0327	5·0733
8(—4)	1·25(3)	4·6540	4·7040	4·7522	4·7987	4·8435	4·8867	4·9284	4·9687	5·0076
1(—3)	1·00(3)	4·6078	4·6565	4·7032	4·7482	4·7915	4·8333	4·8736	4·9124	4·9500
2(—3)	5·00(2)	4·4282	4·4713	4·5125	4·5519	4·5898	4·6260	4·6609	4·6943	4·7264
4(—3)	2·50(2)	4·1792	4·2148	4·2487	4·2808	4·3114	4·3405	4·3682	4·3945	4·4197
6(—3)	1·66(2)	3·9932	4·0236	4·0523	4·0793	4·1048	4·1290	4·1517	4·1733	4·1936
8(—3)	1·25(2)	3·8404	3·8668	3·8914	3·9146	3·9362	3·9566	3·9756	3·9935	4·0103
1(—2)	1·00(2)	3·7093	3·7823	3·7537	3·7737	3·7923	3·8096	3·8258	3·8408	3·8548
2(—2)	5·00(1)	3·2293	3·2419	3·2534	3·2638	3·2731	3·2816	3·2892	3·2961	3·3023
4(—2)	2·50(1)	2·6520	2·6565	2·6603	2·6636	2·6664	2·6688	2·6708	2·6725	2·6740
6(—2)	1·66(1)	2·2867	2·2884	2·2898	2·2909	2·2918	2·2926	2·2931	2·2936	2·2940
8(—2)	1·25(1)	2·0241	2·0248	2·0253	2·0257	2·0260	2·0263	2·0264	2·0266	2·0267
1(—1)	1·00(1)	1·8219	1·8222	1·8224	1·8226	1·8227	1·8227	1·8228	1·8228	1·8229

Annexure—XI

8·0	8·2	8·4	8·6	8·8	9·0	9·2	9·4	9·6	9·8	10·0
5·5529	5·6019	5·6497	5·6965	5·7421	5·7868	5·8305	5·8733	5·9151	5·9562	5·9964
5·5349	5·5834	5·6308	5·6771	5·7223	5·7666	5·8098	5·8521	5·8935	5·9341	5·9739
5·6274	5·5758	5.6230	5·6691	5·7141	5.7581	5·8012	5.8433	5·8846	5·9250	5.9645
5.5168	5.5649	5·6119	5·6577	5·7025	5·7463	5·7890	5·8309	5·8719	5·9120	5·9513
5·5087	5·5566	5·6034	5·6490	5·6936	5·7371	5·7797	5·8114	5·8621	5·9021	5·9412
5·5019	5·5496	5·5962	5·6416	5·6860	5·7294	5·7718	5·8133	5·8539	5·8937	5·9326
5·4958	5·5434	5·5898	5·6352	5·6794	5·7226	5·7649	5·8063	5·8467	5·8863	5·9251
5·4722	5·5192	5·5650	5·6097	5·6534	5·6961	5·7377	5·7785	5·8183	5·8573	5·8955
5·4388	5·4849	5·5299	5·5738	5·6167	5·6585	5·6993	5·7392	5·7782	5·8164	5·8538
5·4131	5·4587	5·5030	5·5463	5·5885	5·6296	5·6698	5·7091	5·7475	5·7850	5·8217
5·3915	5·4365	5·4803	5·5231	5·5647	5·6053	5·6450	5·6837	5·7216	5·7586	5·7948
5·3725	5·4170	5·4604	5·5026	5·5438	5·5840	5·6231	5·6614	5·6988	5·7353	5·7710
5·2980	5·3406	5·3821	5·4225	5·4619	5·5002	5·5375	5·5739	5·6095	5·6441	5·6780
5·1929	5·2330	5·2719	5·3097	5·3464	5·3822	5·4169	5·4508	5·4837	5·5158	5·5471
5·1127	5·1508	5·1877	5·2236	5·2583	5·2921	5·3249	5·3568	5·3879	5·4180	5·4474
5·0453	5·0818	5·1171	5·1513	5·1845	5·2166	5·2478	5·2781	5·3075	5·3361	5·3639
4·9862	5·0213	5·0552	5·0880	5·1197	5·1505	5·1803	5·2092	5·2372	5·2644	5·2908
4·7573	4·7870	4·8155	4·8430	4·8695	4·8950	4·9196	4·9433	4·9662	4·9882	5·0095
4·4436	4·4664	4·4881	4·5089	4·5287	4·5476	4·5656	4·5829	4·5993	4·6150	4·6301
4·2129	4·2311	4·2483	4·2646	4·2800	4·2946	4·3084	4·3214	4·3338	4·3455	4·3566
4·0261	4·0409	4·0548	4·0678	4·0801	4·0916	4·1024	4·1125	4·1220	4·1309	4·1393
3·8679	3·8801	3·8914	3·9020	3·9119	3·9210	3·9296	3·9375	3·9449	3·9518	3·9582
3·3079	3·8130	3·3175	3·3215	3·3252	3·3285	3·3314	3·3340	3·3364	3·3385	3·3403
2·6752	2·6762	2·6771	2·6778	2·6784	2·6789	2·6793	3·6797	2·6800	2·6802	2·6804
2·2943	2·2945	2·2947	2·2948	2·2949	2·2950	2·2951	2·2951	2·2952		
2·0267	2·0268	2·0268	2·0269							

M (u, β,)—W(u) : see Annex II

Annexure—XI

u	1/u|β	12	14	16	18	20	22	24	26	28
0		6·3595	6·6668	6·9333	7·1684	7·3789	7·5692	7·7431	7·9030	8·0511
1(—6)	1·00(6)	6·3325	6·6353	6·8973	7·1279	7·3339	7·5197	7·6891	7·8445	7·9881
2(—6)	5·00(5)	6·3213	6·6223	6·8823	7·1111	7·3152	7·4992	7·6667	7·8202	7·9620
4(—6)	2·50(5)	6·3054	6·6038	6·8612	7·0873	7·2887	7·4701	7·6350	7·7859	7·9250
6(—6)	1·66(5)	6·2932	6·5896	6·8450	7·0691	7·2685	7·4478	7·6106	7·7596	7·8966
8(—6)	1·25(5)	6·2830	6·5775	6·8313	7·0537	7·2514	7·4290	7·5901	7·7374	7·8727
1(—5)	1·00(5)	6·2739	6·5671	6·8193	7·0402	7·2362	7·4125	7·5721	7·7178	7·8517
2(—5)	5·00(4)	6·2385	6·5257	6·7720	6·9870	7·1773	7·3476	7·5013	7·6412	7·7692
4(—5)	2·50(4)	6·1884	6·4673	6·7053	6·9120	7·0940	7·2551	7·4016	7·5332	7·6531
6(—5)	1·66(4)	6·1500	6·4225	6·6542	6·8546	7·0303	7·1861	7·3253	7·4508	7·5644
8(—5)	1·25(4)	6·1177	6·3848	6·6112	6·8063	6·9767	7·1212	7·2613	7·3815	7·4901
1(—4)	1·00(4)	6·0892	6·3617	6·5734	6·7638	6·9296	7·0756	7·2051	7·3208	7·4249
2(—4)	5·00(3)	5·9778	6·2221	6·4257	6·5982	6·7463	6·8747	6·9869	7·0856	7·1729
4(—4)	2·50(3)	5·8214	6·0406	6·2194	6·3677	6·4920	6·5972	6·6868	6·7635	6·8294
6(—4)	1·66(3)	5·7026	5·9031	6·0638	6·1945	6·3019	6·3908	6·4648	6·5266	6·5784
8(—4)	1·25(3)	5·6034	5·7887	5·9348	6·0155	6·1456	6·2219	6·2841	6·3349	6·3763
1(—3)	1·00(3)	5·5168	5·6892	5·8230	5·9281	6·0113	6·0775	6·1303	6·1724	6·2061
2(—3)	5·00(2)	5·1861	5·3123	5·4037	5·4701	5·5184	5·5534	4·5788	5·5970	5·6101
4(—3)	2·50(2)	4·7481	4·8235	4·8714	4·9017	4·9205	4·9320	4·9390	4·9430	4·9454
6(—3)	1·66(2)	4·4396	4·4872	4·5140	4·5288	4·5367	4·5409	4·5429	4·5439	4·5444
8(—3)	1·25(2)	4·1991	4·2300	4·2455	4·2530	4·2565	4·2580	4·2587	4·2589	4·2590
1(—2)	1·00(2)	4·0020	4·0224	4·0316	4·0355	4·0370	4·0376	4·0378		
2(—2)	5·00(1)	3·3507	3·3537	3·3545	3·3547					
4(—2)	2·50(1)	2·6812	2·6812							

Annexure—XI

30	32	34	36	38	40	42	44	46	48	50
8·1890	8·3180	8·4392	8·5535	8·6615	8·7641	8·8616	8·9546	9·0425	9·1286	9·2102
8·1215	8·2460	8·3627	8·4725	8·5761	8·6741	8·7671	8·8556	8·9400	9·0206	9·0977
8·0935	8·2161	8·3309	8·4388	8·5406	8·6367	8·7279	8·8145	8·8971	8·9758	9·0510
8·0539	8·1739	8·2861	8·3913	8·4904	8·5839	8·6725	8·7565	8·8364	8·9125	8·9851
8·0235	8·1415	8·2516	8·3549	8·4519	8·5435	8·6300	8·7120	8·7899	8·8640	8·9346
7·9979	8·1142	8·2226	8·3242	8·4196	8·5094	8·5942	8·6745	8·7507	8·8231	8·8921
7·9753	8·0901	8·1971	8·2972	8·3910	8·4794	8·5628	8·6416	8·7163	8·7872	8·8547
7·8871	7·9960	8·0972	8·1914	8·2795	8·3621	8·4397	8·5127	8·5817	8·6469	8·7087
7·7627	7·8636	7·9566	8·0428	8·1229	8·1975	8·2671	8·3322	8·3933	8·4507	8·5047
7·6679	7·7626	7·8496	7·9298	8·0038	8·0725	8·1362	8·1955	8·2508	8·3024	8·3507
7·5885	7·6781	7·7601	7·8353	7·9044	7·9682	8·0271	8·0817	8·1323	8·1792	8·2229
7·5189	7·6042	7·6818	7·7527	7·8177	7·8773	7·9321	7·9826	8·0292	8·0723	8·1122
7·2594	7·3194	7·3811	7·4364	7·4861	7·5307	7·5709	7·6072	7·6399	7·6695	7·6962
6·8862	6·9353	6·9778	7·0146	7·0465	7·0742	7·0982	7·1191	7·1371	7·1528	7·1663
6·6219	6·6583	6·6890	6·7147	6·7363	6·7545	6·7696	6·7823	6·7929	6·8017	6·8090
6·4103	6·4380	6·4607	6·4791	6·4942	6·5063	6·5162	6·5241	6·5305	6·5357	6·5397
6·2330	6·2543	6·2713	6·2848	6·2954	6·3037	6·3102	6·3153	6·3192	6·3222	6·3246
5·6193	5·6257	5·6302	5·6333	5·6354	5·6368	5·6377	5·6383	5·6387	5·6390	5·6391
4·9467	4·9474	4·9487	4·9480	4·9481						
4·5446	4·5447									
4·2591										

M (u, β) — W(u) see Annex II

u	1/u \ β	52	54	56	58	60	62	64	66	68	70
0		9·2886	9·3641	9·4368	9·5069	9·5747	9·6403	9·7037	9·7653	9·8249	9·8829
1(—6)	1·00(6)	9·1716	9·2426	9·3108	9·3765	9·4398	9·5008	9·5598	9·6168	9·6720	9·7255
2(—6)	5·00(5)	9·1231	9·1922	9·2585	9·3223	9·3838	9·4430	9·5001	9·5553	9·6086	9·6602
4(—6)	2·50(5)	9·0545	9·1210	9·1847	9·2459	9·3047	9·3613	9·4158	9·4684	9·5191	9·5681
6(—6)	1·66(5)	9·0020	9·0665	9·1282	9·1874	9·2442	9·2988	9·3513	9·4019	9·4507	9·4977
8(—6)	1·25(5)	8·9578	9·0206	9·0807	9·1382	9·1933	9·2413	9·2971	9·3460	9·3931	9·4385
1(—5)	1·00(5)	8·9190	8·9803	9·0389	9·0949	9·1486	9·2001	9·2495	9·2970	9·3426	9·3865
2(—5)	5·00(4)	8·7673	8·8229	8·8759	8·9263	8·9743	9·0202	9·0640	9·1059	9·1461	9·1845
4(—5)	2·50(4)	8·5555	8·6035	8·6488	8·6916	8·7321	7·7705	8·8069	8·8414	8·8742	8·9053
6(—5)	1·66(4)	8·3959	8·4383	8·4780	8·5154	8·5505	8·5836	8·6147	8·6440	8·6716	8·6977
8(—5)	1·25(4)	8·2636	8·3016	8·3370	8·3700	8·4009	8·4297	8·4568	8·4821	8·5057	8·5279
1(—4)	1·00(4)	8·1491	8·1833	8·2151	8·2446	8·2720	8·2974	8·3211	8·3431	8·3636	8·3827
2(—4)	5·00(3)	7·7203	7·7421	7·7618	7·7797	7·7958	7·8104	7·8236	7·8355	7·8463	7·8560
4(—4)	2·50(3)	7·1780	7·1881	7·1968	7·2043	7·2108	7·2163	7·2211	7·2251	7·2286	7·2315
6(—4)	1·66(3)	6·8151	6·8201	6·8242	6·8276	6·8304	6·8327	6·8345	6·8360	6·8372	6·8382
8(—4)	1·25(3)	6·5430	6·5456	6·5476	6·5492	6·5504	6·5514	6·5521	6·5527	6·5531	6·5535
1(—3)	1·00(3)	6·3263	6·3277	6·3287	6·3294	6·3300	6·3304	6·3307	6·3310	6·3311	6·3312
2(—3)	5·00(2)	5·6392	5·6393								

M(u, β)—W(u) See Annexure II

u	1/u \ β	72	74	76	78	80	82	84	86	88	90
0		9·9392	9·9940	10·0473	10·0992	10·1498	10·1992	10·2474	10·2944	10·3404	10·3853
1(—6)	1·00(6)	9·7773	9·8276	9·8764	9·9239	9·9700	10·0148	10·0585	10·1011	10·1425	10·1830
2(—6)	5·00(5)	9·7102	9·7586	9·8056	9·8512	9·8955	9·9385	9·9803	10·0210	1·0606	10·0992
4(—6)	2·50(5)	9·6155	9·6613	9·7057	9·7487	9·7904	9·8308	9·8700	9·9081	9·9452	9·9812
6(—6)	1·66(5)	9·5431	9·5869	9·6293	9·6703	9·7101	9·7485	9·7858	9·8220	9·8571	9·8911
8(—6)	1·25(5)	9·4822	9·5244	9·5652	9·6046	9·6426	9·6795	9·7151	9·7497	9·7831	9·8156
1(—5)	1·00(5)	9·4288	9·4696	9·5089	9·5469	9·5835	9·6189	9·6532	9·6863	9·7183	9·7494
2(—5)	5·00(4)	9·2213	9·2566	9·2905	9·3230	9·3542	9·3843	9·4132	9·4410	9·4677	9·4935
4(—5)	2·50(4)	9·9349	8·9630	8·9898	9·0153	9·0396	9·0628	9·0848	9·1059	9·1260	9·1451
6(—5)	1·66(4)	8·7223	8·7455	8·76765	8·7882	8·8076	8·8263	8·8438	8·8603	8·8760	8·8908
8(—5)	1·25(4)	8·5487	8·5682	8·5865	8·6036	8·6197	8·6348	8·6490	8·6623	8·6747	8·6864
1(—4)	1·00(4)	8·4005	8·4170	8·4324	8·4468	8·4601	8·4726	8·4842	8·4949	8·5050	8·5143
2(—4)	5·00(3)	7·8648	7·8727	7·8798	7·8867	7·8920	7·8972	7·9019	7·9061	7·9098	7·9132
4(—4)	2·50(3)	7·2341	7·2362	7·2380	7·2395	7·2408	7·2419	7·2428	7·2436	7·2442	7·2447
6(—4)	1·66(3)	6·8390	6·8396	6·8401	6·8405	6·8408	6·8411	6·8413	6·8414	6·8416	6·8417
8(—4)	1·25(3)	6·5537	6·5539	6·5541	6·5542	6·5543	6·5543	6·5544	6·5544	6·5544	6·5544
1(—3)	1·00(3)	6·3313	6·3314	6·3314	6·3315						

Values of $W(u_A, u_\beta, \Gamma' \gamma')$ (after Streltsova , 1974)

For $\Gamma'=0{\cdot}1$

$1/u_A \backslash \beta$	0·05	0·1	0·2	0·3	0·5	0·75	1·0
$2{\cdot}00\times10^{-1}$	$3{\cdot}70\times10^{-4}$	$5{\cdot}70\times10^{-4}$	$4{\cdot}70\times10^{-4}$	$2{\cdot}90\times10^{-4}$	$1{\cdot}10\times10^{-4}$	$3{\cdot}80\times10^{-5}$	$1{\cdot}70\times10^{-5}$
$4{\cdot}00\times10^{-1}$	$1{\cdot}25\times10^{-2}$	$1{\cdot}21\times10^{-2}$	$7{\cdot}50\times10^{-3}$	$3{\cdot}70\times10^{-3}$	$1{\cdot}10\times10^{-3}$	$3{\cdot}70\times10^{-4}$	$1{\cdot}60\times10^{-4}$
$6{\cdot}00\times10^{-1}$	$3{\cdot}92\times10^{-2}$	$3{\cdot}63\times10^{-2}$	$1{\cdot}84\times10^{-2}$	$8{\cdot}30\times10^{-3}$	$2{\cdot}30\times10^{-3}$	$7{\cdot}60\times10^{-4}$	$3{\cdot}30\times10^{-4}$
$8{\cdot}00\times10^{-1}$	$7{\cdot}31\times10^{-2}$	$6{\cdot}42\times10^{-2}$	$2{\cdot}85\times10^{-2}$	$1{\cdot}22\times10^{-2}$	$3{\cdot}30\times10^{-3}$	$1{\cdot}10\times10^{-3}$	4·[illegible] $\times10^{-4}$
$1{\cdot}00\times10^{0}$	$1{\cdot}09\times10^{-1}$	$9{\cdot}08\times10^{-2}$	$3{\cdot}67\times10^{-2}$	$1{\cdot}52\times10^{-2}$	$4{\cdot}00\times10^{-3}$	$1{\cdot}30\times10^{-3}$	5·[illegible]0× 10[illegible]
$2{\cdot}00\times10^{0}$	$2{\cdot}72\times10^{-1}$	$1{\cdot}82\times10^{-1}$	$5{\cdot}86\times10^{-2}$	$2{\cdot}27\times10^{-2}$	$5{\cdot}80\times10^{-3}$	$1{\cdot}40\times10^{-3}$	7·80× 10[illegible]
$4{\cdot}00\times10^{0}$	$4{\cdot}67\times10^{-1}$	$2{\cdot}53\times10^{-1}$	$7{\cdot}14\times10^{-2}$	$2{\cdot}68\times10^{-2}$	$6{\cdot}70\times10^{-3}$	$2{\cdot}10\times10^{-3}$	$9{\cdot}00\times10^{-4}$
$6{\cdot}00\times10^{0}$	$5{\cdot}68\times10^{-1}$	$2{\cdot}79\times10^{-1}$	$7{\cdot}55\times10^{-2}$	$2{\cdot}81\times10^{-2}$	$7{\cdot}00\times10^{-3}$	$2{\cdot}20\times10^{-3}$	$9{\cdot}30\times10^{-4}$
$8{\cdot}00\times10^{0}$	$6{\cdot}26\times10^{-1}$	$2{\cdot}91\times10^{-1}$	$7{\cdot}73\times10^{-2}$	$2{\cdot}86\times10^{-2}$	$7{\cdot}10\times10^{-3}$	$7{\cdot}10\times10^{-3}$	$9{\cdot}50\times10^{-4}$
$1{\cdot}00\times10^{1}$	$6{\cdot}63\times10^{-1}$	$2{\cdot}99\times10^{-1}$	$7{\cdot}83\times10^{-2}$	$2{\cdot}89\times10^{-2}$	$7{\cdot}20\times10^{-3}$		
$2{\cdot}00\times10^{1}$	$7{\cdot}74\times10^{-1}$	$3{\cdot}11\times10^{-1}$	$8{\cdot}00\times10^{-2}$	$2{\cdot}95\times10^{-2}$	$7{\cdot}30\times10^{-3}$		
$4{\cdot}00\times10^{1}$	$7{\cdot}71\times10^{-1}$	$3{\cdot}16\times10^{-1}$	$8{\cdot}07\times10^{-2}$	$2{\cdot}96\times10^{-2}$	$7{\cdot}30\times10^{-3}$		
$6{\cdot}00\times10^{1}$	$7{\cdot}80\times10^{-1}$	$3{\cdot}18\times10^{-1}$	$8{\cdot}09\times10^{-2}$	$2{\cdot}97\times10^{-2}$			
$8{\cdot}00\times10^{1}$	$7{\cdot}85\times10^{-1}$	$3{\cdot}18\times10^{-1}$	$8{\cdot}69\times10^{-2}$	$2{\cdot}97\times10^{-2}$			
$1{\cdot}00\times10^{2}$	$7{\cdot}87\times10^{-1}$	$3{\cdot}18\times10^{-1}$	$8{\cdot}10\times10^{-2}$				
$2{\cdot}00\times10^{2}$	$7{\cdot}90\times10^{-1}$	$3{\cdot}19\times10^{-1}$	$8{\cdot}10\times10^{-2}$				
$4{\cdot}00\times10^{2}$	$7{\cdot}92\times10^{-1}$	$3{\cdot}19\times10^{-1}$					
$1{\cdot}00\times10^{3}$	$7{\cdot}92\times10^{-1}$	$3{\cdot}19\times10^{-1}$					

$1/u \backslash \beta$	0·05	0·1	0·2	0·3	0·5	0·75	1·0
$1{\cdot}00\times10^{-3}$	$7{\cdot}93\times10^{-1}$	$3{\cdot}20\times10^{-1}$	$8{\cdot}15\times10^{-2}$	$3{\cdot}02\times10^{-2}$	$7{\cdot}70\times10^{-3}$	$2{\cdot}60\times10^{-3}$	$1{\cdot}20\times10^{-3}$
$2{\cdot}00\times10^{-3}$	$7{\cdot}93\times10^{-1}$	$3{\cdot}20\times10^{-1}$	$8{\cdot}15\times10^{-2}$	$3{\cdot}02\times10^{-2}$	$7{\cdot}70\times10^{-3}$	$2{\cdot}60\times10^{-3}$	$1{\cdot}20\times10^{-3}$
$5{\cdot}00\times10^{-3}$	$7{\cdot}93\times10^{-1}$	$3{\cdot}20\times10^{-1}$	$8{\cdot}16\times10^{-2}$	$3{\cdot}03\times10^{-2}$	$7{\cdot}80\times10^{-3}$	$2{\cdot}60\times10^{-3}$	$1{\cdot}20\times10^{-3}$
$1{\cdot}00\times10^{-2}$	$7{\cdot}93\times10^{-1}$	$3{\cdot}20\times10^{-1}$	$8{\cdot}17\times10^{-2}$	$3{\cdot}04\times10^{-2}$	$7{\cdot}99\times10^{-3}$	$2{\cdot}70\times10^{-3}$	$1{\cdot}20\times10^{-3}$
$2{\cdot}00\times10^{-2}$	$7{\cdot}93\times10^{-1}$	$3{\cdot}20\times10^{-1}$	$8{\cdot}19\times10^{-2}$	$3{\cdot}05\times10^{-2}$	$8{\cdot}00\times10^{-3}$	$2{\cdot}89\times10^{-3}$	$1{\cdot}40\times1\ {-3}$
$5{\cdot}00\times10^{-2}$	$7{\cdot}93\times10^{-1}$	$3{\cdot}20\times10^{-1}$	$8{\cdot}24\times10^{-2}$	$3{\cdot}11\times10^{-2}$	$8{\cdot}40\times10^{-3}$	$3{\cdot}10\times10^{-}$	$1{\cdot}69\times10^{-3}$
$1{\cdot}00\times10^{-1}$	$7{\cdot}93\times10^{-1}$	$3{\cdot}31\times10^{-1}$	$8{\cdot}34\times10^{-2}$	$3{\cdot}20\times10^{-2}$	9.10×10^{-3}	$3{\cdot}70\times10^{-}$	$2{\cdot}10\times10^{-3}$
$2{\cdot}00\times10^{-1}$	$7{\cdot}94\times10^{-1}$	$3{\cdot}22\times10^{-1}$	8.53×10^{-2}	$3{\cdot}38\times10^{-2}$	$1{\cdot}05\times10^{-2}$	$4{\cdot}80\times10^{-3}$	$3{\cdot}20\times10^{-3}$
$5{\cdot}00\times10^{-1}$	$7{\cdot}96\times10^{-1}$	$3{\cdot}26\times10^{-1}$	$9{\cdot}09\times10^{-2}$	$3{\cdot}92\times10^{-2}$	$1{\cdot}51\times10^{-2}$	$8{\cdot}80\times10^{-3}$	$7{\cdot}10\times10^{-3}$
$1{\cdot}00\times10^{0}$	$7{\cdot}98\times10^{-1}$	$3{\cdot}32\times10^{-1}$	$1{\cdot}00\times10^{-1}$	$4{\cdot}84\times10^{-2}$	$2{\cdot}33\times10^{-2}$	$1{\cdot}69\times10^{-2}$	$1{\cdot}56\times10^{-2}$
$2{\cdot}00\times10^{0}$	$8{\cdot}03\times10^{-1}$	$3{\cdot}45\times10^{-1}$	$1{\cdot}19\times10^{-1}$	$6{\cdot}74\times10^{-2}$	$4{\cdot}17\times10^{-2}$	$3{\cdot}64\times10^{-2}$	$3{\cdot}73\times10^{-2}$
$5{\cdot}00\times10^{0}$	$8{\cdot}18\times10^{-1}$	$3{\cdot}82\times10^{-1}$	$1{\cdot}74\times10^{-1}$	$1{\cdot}25\times10^{-1}$	$1{\cdot}02\times10^{-1}$	$1{\cdot}00\times10^{-1}$	$1{\cdot}03\times10^{-1}$
$1{\cdot}00\times10^{0}$	$8{\cdot}42\times10^{-1}$	$4{\cdot}39\times10^{-1}$	$2{\cdot}56\times10^{-1}$	$2{\cdot}12\times10^{-1}$	$1{\cdot}86\times10^{-1}$	$1{\cdot}76\times10^{-1}$	$1{\cdot}74\times10^{-1}$
$2{\cdot}00\times10^{-1}$	$8{\cdot}88\times10^{-1}$	$5{\cdot}40\times10^{-1}$	$3{\cdot}87\times10^{-1}$	$3{\cdot}36\times10^{-1}$	$2{\cdot}84\times10^{-1}$	$2{\cdot}57\times10^{-1}$	$2{\cdot}48\ \ 10^{-1}$
$5{\cdot}00\times10^{-1}$	$1{\cdot}01\times10^{0}$	$7{\cdot}57\times10^{-1}$	$5{\cdot}98\times10^{-1}$	$5{\cdot}00\times10^{-1}$	$4{\cdot}01\times10^{-1}$	$3{\cdot}57\times10^{-1}$	$3{\cdot}43\times10^{-1}$
$1{\cdot}00\times10^{-2}$	$1{\cdot}16\times10^{0}$	$9{\cdot}61\times10^{-1}$	$7{\cdot}34\times10^{-1}$	$5{\cdot}98\times10^{-1}$	$4{\cdot}78\times10^{-1}$	$4{\cdot}29\times10^{-1}$	$4{\cdot}13\times10^{-1}$
$2{\cdot}00\times10^{-2}$	$1{\cdot}37\times10^{0}$	$1{\cdot}15\times10^{0}$	$8{\cdot}39\times10^{-1}$	$6{\cdot}81\times10^{-1}$	$5{\cdot}51\times10^{-1}$	$5{\cdot}00\times10^{-1}$	$4{\cdot}83\times10^{-1}$
$5{\cdot}00\times10^{-2}$	$1{\cdot}67\times10^{0}$	$1{\cdot}33\times10^{0}$	$9{\cdot}50\times10^{-1}$	$7{\cdot}80\times10^{-1}$	$6{\cdot}45\times10^{-1}$	$5{\cdot}92\times10^{-1}$	$5{\cdot}75\times10^{-1}$
$1{\cdot}00\times10^{-3}$	$1{\cdot}85\times10^{0}$	$1{\cdot}43\times10^{0}$	$1{\cdot}02\times10^{0}$	$8{\cdot}51\times10^{-1}$	$7{\cdot}15\times10^{-1}$	$6{\cdot}62\times10^{-1}$	$6{\cdot}44\times10^{-1}$
$2{\cdot}00\times10^{-3}$	$1{\cdot}98\times10^{0}$	$1{\cdot}51\times10^{0}$	$1{\cdot}10\times10^{0}$	$9{\cdot}22\times10^{-1}$	$7{\cdot}85\times10^{-1}$	$7{\cdot}31\times10^{-1}$	$7{\cdot}14\times10^{-1}$
$5{\cdot}00\times10^{-3}$	$2{\cdot}11\times10^{0}$	$1{\cdot}61\times10^{0}$	$1{\cdot}19\times10^{0}$	$1{\cdot}01\times10^{0}$	$8{\cdot}77\times10^{-1}$	$8{\cdot}23\times10^{-1}$	$8{\cdot}05\times10^{-1}$
$1{\cdot}00\times10^{-4}$	$2{\cdot}19\times10^{0}$	$1{\cdot}68\times10^{0}$	$1{\cdot}26\times10^{0}$	$1{\cdot}08\times10^{0}$	$9{\cdot}46\times10^{-1}$	$8{\cdot}93\times10^{-1}$	$8{\cdot}74\times10^{-1}$